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New Ways to Measure Catastrophic Financial Risks: "*VaR* to the power of *t*" Measures and How to Calculate Them

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ABSTRACT

The work introduces a family of new risk measures, "*VaR* to the power of *t*". The **aim** of the work is to study the properties of this family of measures and to derive formulas to calculate them. The study used **methods** for assessing financial risks by risk measures VaR and ES. As a result, the author proposed a new tool to measure catastrophic financial risks — "*VaR* to the power of *t*". The study proved that for the measuring, it is sufficient to calculate the common risk measure VaR with the confidence probability changed in a certain way. The author **concludes** that this family of measures should find application in solving the problem of penetrating risk events with low probabilities, but with catastrophic financial losses. The study results may be of use to the regulator to assess the capital adequacy of financial institutions. If $t > 1$, these measures prove to be more conservative risk measures of catastrophic losses than the known risk measures VaR, ES and GlueVaR.

Keywords: risk measure VaR; risk measure ES; risk measure VaR squared: $VaR^{(2)}$; risk measures *VaR* to the *t* power: $VaR^{(t)}$; risk measures GlueVaR; confidence probability; probability density distribution; distortion risk measures; risk appetite; subadditivity; tails of distribution

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INTRODUCTION

Financial and insurance risk management practitioners typically have to deal with two opposing demands: on the one hand, they want business units to achieve or outperform the objectives fixed by the firm's executive committee, yet, on the other, they are responsible for controlling their economic risks. Finding a trade-off between these two demands is the challenging task that risk managers face on a daily basis. At the same time, they need to decide how risk should be quantified.

Financial and insurance firms are subject to the capital requirements established by regulators' guidelines and directives. These requirements are typically equal to, or proportional to, a risk measure value that determines a minimum cushion of economic liquidity. The selection of such risk measures and tolerance levels is crucial therefore from the regulators' point of view.

Financial institutions and insurance companies prefer to minimize the level of capital reserves required by solvency regulations, because they must contend with many restrictions on how this capital can be invested and, as such, the return on their capital reserves is usually lower than that provided by other opportunities. For this reason, companies typically favor regulations that impose risk measures and tolerance levels that are not overly conservative.

Managers also prefer simple, straightforward risk measures rather than more complicated alternatives, since they claim that the former are more easily communicated.

From the regulators' perspective, controlling the risk of financial institutions and insurance companies is fundamental in order to protect consumers and investors, which may have conflicting objectives. Strict solvency capital requirements may limit the capacity of firms, but they also reassure consumers and guarantee the position of the financial industry in the economy. Thus, the debate as to what constitutes a suitable risk measure and what represents a suitable tolerance level is interminable, without their apparently having been much investigation as to what might represent an appropriate compromise.

VaR is currently a classic market risk measure widely adopted and developed both in theory and in practice (for example, [1–3]). VaR estimates the threshold that is not overcome in a given (large) percentage of observations over a given period. According to the Basel Committee, it was included as a mandatory risk measure in assessing not only market risk, but also other risks (for example, credit risk and liquidity risk). The VaR measure is used in assessing various risks in corporate governance (for example, [4–5]).

Value-at-Risk (VaR) has been adopted as a standard tool to assess the risk and to calculate capital requirements in the financial industry. However, VaR is known to present a number of pitfalls when applied in practice. A disadvantage when using VaR in the financial context is that the capital requirements for catastrophic losses based on the measure can be underestimated, i.e. the necessary reserves in adverse scenarios may well be less than they should be.

The underestimation of capital requirements may be aggravated when fat-tailed losses are incorrectly modeled by mild-tailed distributions. There are attempts to overcome this kind of model risk when using VaR or, at least, to quantify the risk related to the modelling [6].

A second drawback is that the VaR may fail the subadditivity property. A risk measure is subadditive when the aggregated risk is less than or equal to the sum of individual risks.

Subadditvity is an appealing property when aggregating risks in order to preserve the benefits of diversification. VaR is subadditive for elliptically distributed losses [7]. However, the subadditivity of VaR is not guaranteed for any distribution not included in the class of elliptical [8, 9].

Since the end of the 20th century, a measure of expected shortfall, conditional VaR, a measure of expected tail losses exceeding VaR, often defined as ES (or TVaR) has been widely used in theory and practice of risk management (for example, [1–3]).

The ES measures average losses in the most adverse cases rather than just the minimum loss, as the VaR does. Therefore, capital reserves based on the ES have to be considerably higher than those based on VaR and significant differences in the size of capital reserves can be obtained depending on which risk measure is adopted.

The ES risk measure does not suffer the two drawbacks discussed above for VaR and, as such, would appear to be a more powerful measure for assessing the actual risks faced by companies and financial institutions. However, ES has not been widely accepted by practitioners in the financial and insurance industry. VaR is currently the risk measure contemplated in the European solvency regulation.

In relatively recent papers [10, 11], a new family of risk measures GlueVaR was proposed and examined in the class of distortion risk measures.

The search for various risk measures that satisfy certain needs has a rather long history (general approaches to this problem are presented by [12, 13]). One of the significant classes of examined risk measures is the distortion risk measures introduced by S. Wang [14, 15]. They are closely related to the distortion expectation theory.

Tsanakas and Desli [16] provide a review on how risk measures can be interpreted from several perspectives, and include a clarifying explanation of the relationship between distortion risk measures and distortion expectation theory.

A detailed literature review of distortion risk measures is available in the works by M. Denuit et al., A. Balbas et al. [17, 18].

J. Belles-Sampera et al. [10, 11] define a new family of risk measures, called GlueVaR, within the class of distortion risk measures. The authors find out their relationship with VaR and ES, and receive analytical closed-form expressions for many statistical distributions that are frequently used in financial and insurance applications. Tail-subadditivity is investigated and it is shown that some GlueVaR risk measures satisfy this property. An interpretation in terms of risk attitudes is provided and a discussion is given on the applicability in non-financial problems such as health, safety, environmental or catastrophic risk management.

In work [19], the author introduced the concept of new measure $VaR^{(2)}$. It assesses risks more conservatively than VaR, and even than ES, as some threshold value that cannot be overcome with a given probability (like VaR), and not as an average value from the set of "bad", tail loss values (like ES). For this risk measure, closed computational formulas were obtained in cases of uniform and triangular loss distributions.

The author continued studying $VaR^{(2)}$ risk measure in work [20]. A general, independent of loss distribution formula was obtained for it and expressed it through the common risk measure VaR with the confidence probability changed in a certain way. Moreover, the study investigated the relationships between risk assessments by $VaR^{(2)}$ and other known risk measures, such as ES. It turned out that the ratio under study often depends on the assumption of the loss distribution law, and sometimes on confidence probabilities. It was also revealed that $VaR^{(2)}$ most often provides a more conservative risk assessment than ES [20].

The current work continues the previous research by the author. It introduces the concept of risk measures VaR to any power $t \geq 1$, derives formulas to calculate $VaR^{(t)}$ as the common VaR with the confidence probability changed in a certain way. The work discusses possibilities of practical application of this family of risk measures.

Thus, we propose a new family of risk measures called $VaR^{(t)}$, formulas to calculate them, which lead to the fact that all existing models and tools to calculate the common VaR are also applicable to any measure of the $VaR^{(t)}$ family of measures. The paper gives analytical expressions of $VaR^{(t)}$ closed computational formula for some of the distribution functions most often used in financial and insurance applications. The relationships between $VaR^{(t)}$ at $t > 1$ and risk measures VaR and ES are explained.

This new family of measures is associated with the most popular risk measures and includes a sufficient number of parameters to consider management and regulatory requirements for risk. Therefore, this article is motivated by an attempt to answer the following question: is it possible to develop risk measures that would provide a risk assessment meeting various needs and allowing penetration into the risk assessment of arbitrarily high catastrophe, exceeding the capabilities of both VaR and ES?

The $VaR^{(t)}$ family of risk measures is defined as a function with two parameters: confidence probability p and exponent t. By calibrating these parameters, VaR risk measures can be matched to a wide variety of contexts. In particular, if the level of confidence is fixed, the new family contains risk measures between VaR and ES and can adequately show the risks of moderate catastrophe. However, in certain situations, much more conservative risk measures than even ES may be preferred. We show that these highly conservative risk measures can also be defined using the $VaR^{(t)}$ family. We obtain closed analytical expressions of $VaR^{(t)}$ closed formula, for statistical distributions commonly used in the financial context. These expressions should allow practitioners to make a simple transition from using VaR and ES to risk measures $VaR^{(t)}$.

THE CONCEPT OF RISK MEASURES VAR TO THE POWER OF *n* ($VaR^{(n)}$, *n* — IS A NATURAL NUMBER) AND THE DERIVATION OF COMPUTATIONAL FORMULAS

Work [19] introduced a new risk measure supplementing VaR — VaR squared ($VaR_p^{(2)}$), which tracks rare tail risks associated with serious financial losses.

$VaR_p^{(2)}$ risk measure with confidence probability p (see [19]) is a value that will be exceeded by profit (not exceeded by losses), provided that its threshold value is not exceeded (exceeded) with confidence probability p for a given time.

Work [20] provides a formula to calculate this risk measure.

Let X be the value of the windfall profit for this asset for a given period of time ($-X$ shows the value of the corresponding losses).

The following formula was proved in work [20], which allows calculating $VaR_p^{(2)}$ as VaR with the confidence probability changed in a certain way:

$$VaR_p^{(2)}[X] = VaR_{1-(1-p)^2}[X]. \qquad (1)$$

Thus, to calculate $VaR_p^{(2)}$, we should calculate VaR with the confidence probability $1-(1-p)^2$.

In particular, if the loss distribution law is known (for example, normal), then $VaR_p^{(2)}$ can be calculated by formula (1) with the Monte Carlo method or by the known formula for VaR under this assumption and by formula (1), which will lead to the following result:

$$VaR_p^{(2)} = Vk^{0.1}_{1-(1-p)^2} \cdot \sigma, \qquad (2)$$

where V is the denomination of the position at time 0; σ – is the standard deviation of profitability in the time period over which we estimate $VaR_p^{(2)}$; $k_q^{0.1}$ is the quantile of a standardized distribution of returns with confidence probability q.

If the distribution of returns is not known, $VaR_p^{(2)}$ can be calculated using the empirical loss distribution and formula (1).

In formula (2) we used the formula for calculating the relative VaR, i.e. the maximum deviation in an unfavorable direction from the expected profit with a given probability for a given (unit) time.

The concept of $VaR^{(2)}$ in work [20] was generalized considering that confidence probability p' in determining $VaR^{(2)}$, i.e. the threshold value that the profit will not exceed (the loss will exceed) under the condition of non-exceeding (exceeding) VaR_p with probability p', may differ from p. This risk measure was defined as $VaR_{p,p'}^{(2)}$ and the following formula was obtained:

$$VaR_{p,p'}^{(2)}[X] = VaR_{1-(1-p)(1-p')}[X]. \qquad (3)$$

We introduce the concept of risk measures VaR to the power of n, where n is any natural number. We will introduce these measures inductively, sequentially, moving from VaR to $VaR^{(2)}$, then to $VaR^{(3)}$, and so on, and the we will reach $VaR^{(n)}$. At the same time, we will deal with the sequential derivation of formulas for risk measures VaR to the power of n, $VaR^{(n)}$.

To begin with, we represent the common VaR in the form:

$$VaR_p^{(1)}[X] = VaR_p[X] = VaR_{p_1}[X],$$

$$\text{where } p_1 = 1-(1-p).$$

Then, according to formula (1)

$$VaR_p^{(2)}[X] = VaR_{p_2}[X], \text{where } p_2 = 1-(1-p_1)^2.$$

Then, according to the definition, we assume that VaR squared is just $VaR_{p_2,p}^{(2)}[X]$. So, we get that

$$VaR_p^{(3)}[X] = VaR_{p_2,p}^{(2)}[X] = VaR_{p_3}[X],$$

where according to formula (3) $p_3 = 1-(1-p_2)(1-p)$, but then using formula (1) we have:

$$p_3 = 1-(1-p_2)(1-p) =$$
$$= 1-(1-[1-(1-p)^2])(1-p) = 1-(1-p)^3.$$

In the same way, defining VaR to the power of four as $VaR_{p_3,p}^{(2)}[X]$, we get:

$$VaR_p^{(4)}[X] = VaR_{p_3,p}^{(2)}[X] = VaR_{p_4}[X], \quad \text{where}$$

according to formula (3) $p_4 = 1-(1-p_3)(1-p)$, but then using formula (1) we have:

$$p_4 = 1-(1-p_3)(1-p) =$$
$$= 1-(1-[1-(1-p)^3])(1-p) = 1-(1-p)^4.$$

By proceeding in the same way, we introduce risk measure VaR to the power of n for any natural number n as $VaR_{p_{n-1},p}^{(2)}[X]$, where $p_{n-1} = 1-(1-p)^{n-1}$ and we get:

$VaR_p^{(n)}[X] = VaR_{p_{n-1},p}^{(2)}[X] = VaR_{p_n}[X]$, where according to formula (3) $p_n = 1-(1-p_{n-1})(1-p)$, but then using formula (1) we have:

$$p_n = 1-(1-p_{n-1})(1-p) =$$
$$= 1-(1-[1-(1-p)^{n-1}])(1-p) = 1-(1-p)^n.$$

Thus, we introduced the concept of risk measures VaR to the power of n for any natural number n and obtained a formula that reduces their calculations to calculating the common risk measure VaR with the confidence probability changed in a certain way.

$$VaR_p^{(n)}[X] = VaR_{1-(1-p)^n}[X]. \qquad (4)$$

To calculate $VaR_p^{(n)}$, it is just necessary to calculate VaR with the confidence probability $1-(1-p)^n$.

With risk measures $VaR_p^{(n)}[X]$ the risk manager may delve into studying the left tail of the profit distribution law for confidence probabilities that are multiples of initial confidence probability p, and get information about less probable, but more catastrophic risks.

CONSEQUENCE

For any value of confidence probability $p \in (0,1]$ with unlimited growth of n, the value of risk measure $VaR_p^{(n)}[X]$ unlimitedly approaches the left (right) border of profit distribution carrier X (loss $-X$).

PROOF

This follows from the fact that for the indicated values of p $1-(1-p)^n \to 1$, at $n \to \infty$.

Table 1 provides the table for the calculation formulas for of $VaR_p^{(n)}$, at n = 2, 3, and 4 and certain confidence probabilities.

Table 1 shows that with growth of n, the confidence probability of the corresponding common measure VaR tends to 100%. The faster it is growing, the greater is confidence probability $VaR_p^{(n)}[X]$ with which the risk measure is calculated. Therefore, the values of $VaR_p^{(n)}[X]$ quickly approach the left (right) border of profit distribution carrier X (loss $-X$)., i.e. show losses with increasingly catastrophic and less likely risk events.

Table 1

Expression for $VaR_p^{(n)}[X]$ through the common risk measure VaR at various values of n and confidence probabilities *p*

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p^{(2)}[X]$	$VaR_{99\%}[X]$	$VaR_{99.75\%}[X]$	$VaR_{99.99\%}[X]$
$VaR_p^{(3)}[X]$	$VaR_{99.9\%}[X]$	$VaR_{99.9875\%}[X]$	$VaR_{99.9999\%}[X]$
$VaR_p^{(4)}[X]$	$VaR_{99.99\%}[X]$	$VaR_{99.999\%}[X]$	$VaR_{\approx 100\%}[X]$

Source: the author's calculations.

Table 2

Values $VaR_p^{(n)}[X]$ at various values of *n* and *p*, assuming uniform distribution of variable X at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p[X]$	110	105	101
$VaR_p^{(2)}[X]$	101	100.25	100.01
$VaR_p^{(3)}[X]$	100.1	100.0125	100.0001
$VaR_p^{(4)}[X]$	100.01	100.000625	≈100

Source: the author's calculations.

We will test the results on the known loss distributions and the corresponding numerical examples.

UNIFORM DISTRIBUTION

According to study [21], if profit value *X* is uniformly distributed in interval (*a*, *b*), then for any confidence probability *p*

$$VaR_p[X] = pa + (1-p)b .$$

We rewrite this expression as follows:

$$VaR_p[X] = b - (b-a)p = a + (1-p)(b-a) .$$

Then according to formula (4), we have:

$$VaR_p^{(2)}[X] = VaR_{1-(1-p)^2}[X] =$$

$$= b - (b-a)(1-(1-p)^2) = a + (1-p)^2(b-a) \cdot$$

Note that the expression naturally coincides with the expression obtained by the straightforward conclusion from the definition of $VaR_p^{(2)}[X]$ in work [19] [formula (2)].

Similarly, we get the expression for $VaR_p^{(n)}[X]$:

$$VaR_p^{(n)}[X] = VaR_{1-(1-p)^n}[X] =$$
$$= b-(b-a)(1-(1-p)^n) = a+(1-p)^n(b-a)\cdot$$

I.e.

$$VaR_p^{(n)}[X] = a+(1-p)^n(b-a). \quad (5)$$

We can rewrite formula (5) as follows:

$$VaR_p^{(n)}[X] = (1-(1-p)^n)a+(1-p)^n b,$$

which means that at uniform profit distribution *X,* value $VaR_p^{(n)}[X]$ is presented as a weighted average between the ends of interval (*a, b*), and the weight of the left end rapidly tends to 1 with growth of *n*. Therefore, the value of $VaR_p^{(n)}[X]$ quickly approaches the left (right) border of profit distribution carrier *X* (loss –*X*).

This is illustrated on *Example 1 (Table 2)*.

Example 1

We calculate $VaR_p^{(n)}[X]$ at *n* = 1, ..., 4, *p* = 90%, 95% and 99%, if *a* = 100 units, and *b* = 200 units.

TRIANGULAR DISTRIBUTION

According to study [21], if random variable *X* is subordinate to the triangular distribution with a carrier coinciding with interval (*a, b*) and a vertex, whose projection onto the carrier is represented by point *v*∈(*a*, *b*), then:

$$VaR_p[X] = \begin{cases} a+\sqrt{(1-p)(b-a)(\nu-a)}, \\ \text{if } \nu \ge pa+(1-p)b \\ b-\sqrt{p(b-a)(b-\nu)}, \\ \text{if } \nu \le pa+(1-p)b. \end{cases}$$

We rewrite this expression as follows:

$$VaR_p[X] = \begin{cases} a+\sqrt{(1-p)(b-a)(\nu-a)}, \\ \text{if } \nu \ge a+(1-p)(b-a) \\ b-\sqrt{(1-(1-p))(b-a)(b-\nu)}, \\ \text{if } \nu \le a+(1-p)(b-a). \end{cases}$$

Then according to formula (4),

$$VaR_p^{(2)}[X] = VaR_{p_2}[X], \text{ where } p_2 = 1-(1-p)^2,$$

we have:

$$VaR_p^{(2)}[X] = \begin{cases} a+\sqrt{(1-p_2)(b-a)(\nu-a)}, \\ \text{if } \nu \ge a+(1-p_2)(b-a) \\ b-\sqrt{(1-(1-p_2))(b-a)(b-\nu)}, \\ \text{if } \nu \le a+(1-p_2)(b-a) \end{cases}$$

or

$$VaR_p^{(2)}[X] = \begin{cases} a+\sqrt{(1-p)^2(b-a)(\nu-a)}, \\ \text{if } \nu \ge a+(1-p)^2(b-a) \\ b-\sqrt{(1-(1-p)^2)(b-a)(b-\nu)}, \\ \text{if } \nu \le a+(1-p)^2(b-a). \end{cases}$$

This expression can also be written as follows:

$$VaR_p^{(2)}[X] = \begin{cases} a+(1-p)\sqrt{(b-a)(\nu-a)}, \\ \text{if } \nu \ge a+(1-p)^2(b-a) \\ b-\sqrt{p(2-p)(b-a)(b-\nu)}, \\ \text{if } \nu \le a+(1-p)^2(b-a). \end{cases}$$

Note that the expression naturally coincides with the expression obtained by the straightforward conclusion from the definition of $VaR_p^{(2)}[X]$ in work [19] [formula (2)].

Similarly, we get the expression for $VaR_p^{(n)}[X]$:

$$VaR_p^{(n)}[X] = VaR_{1-(1-p)^n}[X] =$$
$$= \begin{cases} a+\sqrt{(1-p)^n(b-a)(\nu-a)}, \\ \text{if } \nu \ge a+(1-p)^n(b-a) \\ b-\sqrt{(1-(1-p)^n)(b-a)(b-\nu)}, \\ \text{if } \nu \le a+(1-p)^n(b-a). \end{cases} \quad (6)$$

We study the behavior of these risk measures depending on the values of distribution mode ν and confidence probability in *Examples 2–4 (Tables 3–5).*

Example 2

We calculate $VaR_p^{(n)}[X]$ at *n* = 1, ..., 4, *p* = 90%, 95% and 99%, if *a* = 100 units, *b* = 200 units and $\nu = 105$ units.

Table 3

Values $VaR_p^{(n)}[X]$ at various values of *n* and *p*, assuming triangular distribution of variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p[X]$	107.0711	105	102.2361
$VaR_p^{(2)}[X]$	102.2361	101.118	100.2236
$VaR_p^{(3)}[X]$	100.7071	100.25	100.0224
$VaR_p^{(4)}[X]$	100.2236	100.05559	100.0022

Source: the author's calculations.

Table 4

Values $VaR_p^{(n)}[X]$ at various values of *n* and *p*, assuming triangular distribution of variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p[X]$	122.3607	115.8114	107.0711
$VaR_p^{(2)}[X]$	107.0711	103.5355	100.7071
$VaR_p^{(3)}[X]$	102.2361	100.7906	100.0707
$VaR_p^{(4)}[X]$	100.7071	100.1768	100.0071

Source: the author's calculations.

Table 5

Values $VaR_p^{(n)}[X]$ at various values of *n* and *p*, assuming triangular distribution of variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p[X]$	130.8221	121.7945	109.7468
$VaR_p^{(2)}[X]$	109.7468	104.8734	100.9747
$VaR_p^{(3)}[X]$	103.0822	101.0897	100.0975
$VaR_p^{(4)}[X]$	100.9747	100.2437	100.0097

Source: the author's calculations.

Example 3

We calculate $VaR_p^{(n)}[X]$ at $n = 1, \ldots, 4$, p = 90%, 95% and 99%, if a = 100 units, b = 200 units and $\nu = 105$ units.

Example 4

We calculate $VaR_p^{(n)}[X]$ at $n = 1, \ldots, 4$, p = 90%, 95% and 99%, if a = 100 units, b = 200 units and $\nu = 195$ units.

Examples 2–4 show that with growth of n, $VaR_p^{(n)}[X]$ quickly enough tends to the left border of the profit distribution carrier, and the greater the confidence probability p is, the faster this happens. The closer distribution mode ν is to the left border of the profit distribution carrier, the faster $VaR_p^{(n)}[X]$ tends to the left border of the profit distribution carrier for all values of confidence probability p, i.e. the more risky this position is at all levels of catastrophe.

NORMAL DISTRIBUTION

As we know (for example, [1–3]), assuming that the profit distribution is normal, VaR (relative VaR) is calculated by formula

$$VaR_p[X] = V\sigma k_p^{0.1},$$

where V is the denomination of the position at time 0; σ – is the standard deviation of profitability in the time period over which we estimate VaR; $k_q^{0.1}$ is the quantile of a standardized distribution of returns with confidence probability q.

Then according to formula (4), we have

$$VaR_p^{(n)}[X] = V\sigma k_{1-(1-p)^n}^{0.1}. \qquad (7)$$

We investigate the behavior of these risk measures depending on confidence probability p (*Table 6*). Since only quantiles depend on the confidence probability, the example provides the dependence of the corresponding quantiles on confidence probabilities.

We see that for each confidence probability p, the corresponding quantiles increase with growth of n of risk measures $VaR_p^{(n)}[X]$. Thus, for large n, risk measures $VaR_p^{(n)}[X]$ evaluate increasingly catastrophic risks, and the greater confidence probabilities p are, the greater the assessment of such risk measures is.

POLY-VAR RISK MEASURES

We introduce a family of measures generalizing measures $VaR_p^{(n)}[X]$, and will allow the confidence probabilities used in constructing VaR to various powers to vary.

To begin with, we present the common risk measure VaR as follows:

$$VaR_p[X] = VaR_{\tilde{p}_1}, \text{ where } \tilde{p}_1 = p_1 = p = 1-(1-p).$$

Using formula (3), we introduce the concept of poly-VaR squared:

$$VaR_{p_1,p_2}^{(2)}[X] = VaR_{\tilde{p}_2}[X], \text{ where } \tilde{p}_2 = 1-(1-p_1)(1-p_2).$$

Thus, poly-VaR to the third power is defined as follows:

$$VaR_{p_1,p_2,p_3}^{(3)}[X] = VaR_{\tilde{p}_2,p_3}^{(2)}[X] = VaR_{\tilde{p}_3}[X],$$

where

$$\tilde{p}_3 = 1-(1-\tilde{p}_2)(1-p_3) = 1-(1-[1-(1-p_1)(1-p_2)])(1-p_3) = 1-(1-p_1)(1-p_2)(1-p_3).$$

Further, poly-VaR to the power of n is defined as follows:

$$VaR_{p_1,p_2,\ldots,p_n}^{(n)}[X] = VaR_{\tilde{p}_{n-1},p_n}^{(2)}[X] = VaR_{\tilde{p}_n}[X],$$

where

$$\tilde{p}_n = 1-(1-\tilde{p}_{n-1})(1-p_n) = 1-(1-[1-(1-p_1)(1-p_2)\ldots(1-p_{n-1})])(1-p_n) = 1-(1-p_1)\ldots(1-p_n).$$

That is, the formula for poly-VaR to the power of n is as follows:

$$VaR_{p_1,p_2,\ldots,p_n}^{(n)}[X] = VaR_{1-(1-p_1)(1-p_2)\ldots(1-p_n)}[X], \qquad (8)$$

expressing it through the common risk measure VaR with the confidence probability recalculated in a certain way.

VAR RISK MEASURE TO ANY VALID POWER $t \geqslant 1$, $VaR_p^{(t)}[X]$

Any real number $t \geq 1$ can be unambiguously represented as follows $t = k + \alpha$, where k is a

Table 6

Values $VaR_p^{(n)}[X]$ (through values of corresponding $k_q^{0.1}$) at various values of *n* and *p*, assuming normal distribution of variable *X*

	p = 90%	*p* = 95%	*p* = 99%
$k_p^{0.1}$ (VaR_p)	1.2816	1.6449	2.3263
$k_{1-(1-p)^2}^{0.1}$ $(VaR_p^{(2)})$	2.3264	2.8070	3.7190
$k_{1-(1-p)^3}^{0.1}$ $(VaR_p^{(3)})$	3.0902	3.6623	4.7534
$k_{1-(1-p)^4}^{0.1}$ $(VaR_p^{(4)})$	3.7190	4.3687	5.6120

Source: the author's calculations.

Table 7

Values $VaR_p^{(t)}[X]$ at various values of *t* and *p*, assuming uniform distribution of variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p[X]$	110	105	101
$VaR_p^{(1.1)}[X]$	109.1	104.525	100.901
$VaR_p^{(1.5)}[X]$	105.5	102.625	100.505
$VaR_p^{(1.9)}[X]$	101.9	100.725	100.109

Source: the author's calculations.

natural number; and α — is a real number, with $0 \le \alpha < 1$. Obviously, k is the integer part of t, and α is its fractional part.

Then we can determine VaR to any valid power $t \ge 1$, $VaR_p^{(t)}[X]$ as follows

$$VaR_p^{(t)}[X] = VaR^{(k+1)}_{\underbrace{p,p,\ldots,p}_{k},\alpha p}. \quad (9)$$

In particular, applying (9) and (8), we have:

$$VaR_p^{(1+\alpha)}[X] = VaR^{(2)}_{p,\alpha p}[X] = VaR_{1-(1-p)(1-\alpha p)}[X] \quad (10)$$

and

$$VaR_p^{(2+\alpha)}[X] = VaR^{(3)}_{p,p,\alpha p}[X] = VaR_{1-(1-p)^2(1-\alpha p)}[X] \quad (11)$$

etc.,

$$VaR_p^{(t)}[X] = VaR_p^{(k+\alpha)}[X] = VaR_{1-(1-p)^k(1-\alpha p)}[X]. \quad (12)$$

Table 8

Values $VaR_p^{(t)}[X]$ at various values of *t* and *p*, assuming uniform distribution of variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p^{(2)}[X]$	101	100.25	100.01
$VaR_p^{(2.1)}[X]$	100.9	100.226	100.009
$VaR_p^{(2.5)}[X]$	100.6	100.131	100.005
$VaR_p^{(2.9)}[X]$	100.2	100.036	100.001

Source: расчеты автора / the author's calculations.

With risk measures $VaR_p^{(t)}[X]$, the risk manager may delve into studying the left tail of the profit distribution law for confidence probabilities that are multiples of initial confidence probability *p*, and get very detailed information about less probable, but more catastrophic risks.

UNIFORM DISTRIBUTION (VAR TO A FRACTIONAL POWER)

Applying formulas (10) and (11) in the case of a uniform distribution, we have:

$$VaR_p^{(1+\alpha)}[X] = a + (1-p)(1-\alpha p)(b-a)$$

and

$$VaR_p^{(2+\alpha)}[X] = a + (1-p)^2(1-\alpha p)(b-a).$$

***Example 5** (Table 7)*

We calculate $VaR_p^{(t)}[X]$ at *t* = 1; 1.1; 1.5; 1.9, and *p* = 90%, 95% and 99%, if *a* = 100 units, *b* = 200 units.

***Example 6** (Table 8)*

We calculate $VaR_p^{(t)}[X]$ at *t* = 2; 2.1; 2.5; 2.9, and *p* = 90%, 95% and 99%, if *a* = 100 units, *b* = 200 units.

Examples 5 and 6 show that at growing α, risk measures $VaR_p^{(1+\alpha)}[X]$ and $VaR_p^{(2+\alpha)}[X]$ tend to the left border of the profit distribution carrier, and the greater the confidence probability *p* is, the faster this happens. However, this happens more slowly than when moving from $VaR_p[X]$ to $VaR_p^{(2)}[X]$, and, accordingly, from $VaR_p^{(2)}[X]$ to $VaR_p^{(3)}[X]$ (compare with *Example 1*). That is, applying VaR risk measures to the power of $(1+\alpha)$ and $(2+\alpha)$ at various α, the risk manager, depending on the risk appetite of his company, can rather subtly examine the risks in the left tail of the profit distribution.

TRIANGULAR DISTRIBUTION (VAR TO A FRACTIONAL POWER)

Applying formulas (10) and (11) in the case of a uniform distribution, we have:

$$VaR_p^{(1+\alpha)}[X] = \begin{cases} a + \sqrt{(1-p)(1-\alpha p)(b-a)(\nu-a)}, \\ \text{if } \nu \geq a + (1-p)(1-\alpha p)(b-a) \\ b - \sqrt{(1-(1-p))(1-\alpha p)(b-a)(b-\nu)}, \\ \text{if } \nu \leq a + (1-p)(1-\alpha p)(b-a) \end{cases}$$

and

$$VaR_p^{(2+\alpha)}[X] =$$

$$= \begin{cases} a + \sqrt{(1-p)^2(1-\alpha p)(b-a)(\nu-a)}, \\ \text{if } \nu \geq a + (1-p)^2(1-\alpha p)(b-a) \\ b - \sqrt{(1-(1-p)^2(1-\alpha p))(b-a)(b-\nu)}, \\ \text{if } \nu \leq a + (1-p)^2(1-\alpha p)(b-a) \end{cases}$$

***Example 7a** (Table 9)*

We calculate $VaR_p^{(t)}[X]$ at *t* = 1; 1.1; 1.5; 1.9, and *p* = 90%, 95% and 99%, if *a* = 100 units, *b* = 200 units and $\nu = 105$ units.

***Example 7b** (Table 10)*

We calculate $VaR_p^{(t)}[X]$ at *t* = 1; 1.1; 1.5; 1.9, and *p* = 90%, 95% and 99%, if *a* = 100 units, *b* = 200 units and $\nu = 105$ units.

Table 9

Values $VaR_p^{(t)}[X]$ at various values of *t* and *p*, assuming triangular distribution of variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p[X]$	107.5338	105	102.2361
$VaR_p^{(1.1)}[X]$	107.0726	104.7566	102.1225
$VaR_p^{(1.5)}[X]$	105.2503	103.6228	101.5890
$VaR_p^{(1.9)}[X]$	103.0822	101.9039	100.7382

Source: the author's calculations.

Table 10

Values $VaR_p^{(t)}[X]$ at various values of *t* and *p*, assuming triangular distribution of variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p[X]$	122.3607	115.8114	107.0711
$VaR_p^{(1.1)}[X]$	121.3007	115.0416	106.7119
$VaR_p^{(1.5)}[X]$	116.5831	111.4564	105.0249
$VaR_p^{(1.9)}[X]$	109.7468	106.0208	102.3345

Source: the author's calculations.

Table 11

Values $VaR_p^{(t)}[X]$ at various values of *t* and *p*, assuming triangular distribution of variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p[X]$	130.8221	121.7945	109.7468
$VaR_p^{(1.1)}[X]$	129.4024	120.7334	109.2518
$VaR_p^{(1.5)}[X]$	122.8583	115.7916	106.9264
$VaR_p^{(1.9)}[X]$	113.4350	108.2991	103.2179

Source: the author's calculations.

Table 12

Values $VaR_p^{(t)}[X]$ at various values of *t* and *p*, assuming triangular distribution of a variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p^{(2)}[X]$	103.0206	101.1180	100.2236
$VaR_p^{(2.1)}[X]$	102.9766	101.0636	100.2125
$VaR_p^{(2.5)}[X]$	102.8005	100.8101	100.1589
$VaR_p^{(2.9)}[X]$	100.9747	100.4257	100.0738

Source: the author's calculations.

Table 13

Values $VaR_p^{(t)}[X]$ at various values of *t* and *p*, assuming triangular distribution of variable *X* at interval (*a*, *b*)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p^{(2)}[X]$	107.0711	103.5355	100.7071
$VaR_p^{(2.1)}[X]$	106.7454	103.3634	100.6712
$VaR_p^{(2.5)}[X]$	105.2440	102.5617	100.5025
$VaR_p^{(2.9)}[X]$	103.0822	101.3463	100.2335

Source: the author's calculations.

Table 14

Values $VaR_p^{(t)}[X]$ at various values of *t* and *p*, assuming triangular distribution of variable *X* at interval (a, b)

	p = 90%	*p* = 95%	*p* = 99%
$VaR_p^{(2)}[X]$	109.7468	104.8734	100.9747
$VaR_p^{(2.1)}[X]$	109.2978	104.6361	100.9252
$VaR_p^{(2.5)}[X]$	107.2284	103.5311	100.6926
$VaR_p^{(2.9)}[X]$	104.2485	101.8557	100.3118

Source: the author's calculations.

Table 15

Values $VaR_p^{(t)}[X]$ (through values of corresponding $k_q^{0.1}$) at various values of t and *p*, assuming normal distribution of variable *X*

	p = 90%	*p* = 95%	*p* = 99%
$k_p^{0.1}$ (VaR_p)	1.281552	1.644854	2.326348
$k_{1-(1-p)(1-0.1p)}^{0.1}$ $(VaR_p^{(1.1)})$	1.334622	1.692766	2.365207
$k_{1-(1-p)(1-0.5p)}^{0.1}$ $(VaR_p^{(1.5)})$	1.598193	1.939011	2.572387
$k_{1-(1-p)(1-0.9p)}^{0.1}$ $(VaR_p^{(1.9)})$	2.074855	2.4446632	3.064547

Source: the author's calculations.

Example 7c *(Table 11)*

We calculate $VaR_p^{(t)}[X]$ at *t* = 1; 1.1; 1.5; 1.9, and *p* = 90%, 95% and 99%, if *a* = 100 units, *b* = 200 units and $\nu = 195$ units.

Examples 7a, 7b, and *7c* show that at growing α, risk measures $VaR_p^{(1+\alpha)}[X]$ tend to the left border of the profit distribution carrier, and the greater confidence probability *p* is, the faster this happens. However, this happens more slowly than when moving from $VaR_p[X]$ to $VaR_p^{(2)}[X]$. Moreover, the closer distribution mode ν is to the left border of the profit distribution carrier, the faster risk measures $VaR_p^{(1+\alpha)}[X]$ at growing α tend to the left border of the profit distribution carrier at all *p*. That is, this position is all the more risky. Applying VaR risk measures to powers $(1+\alpha)$ at various α, the risk manager, depending on the risk appetite of his company, can rather subtly examine the risks in the left tail of the profit distribution.

Example 8a *(Table 12)*

We calculate $VaR_p^{(t)}[X]$ at *t* = 2; 2.1; 2.5; 2.9, and *p* = 90%, 95% and 99%, if *a* = 100 units, *b* = 200 units and $\nu = 105$ units.

Example 8b *(Table 13)*

We calculate $VaR_p^{(t)}[X]$ at *t* = 2; 2.1; 2.5; 2.9, and *p* = 90%, 95% and 99%, if *a* = 100 units, *b* = 200 units and $\nu = 150$ units.

Example 8c *(Table 14)*

We calculate $VaR_p^{(t)}[X]$ at *t* = 2; 2.1; 2.5; 2.9, and *p* = 90%, 95% and 99%, if *a* = 100 units, *b* = 200 units and $\nu = 195$ units.

Examples 8a, 8b, and *8c* show that at growing α, risk measures $VaR_p^{(2+\alpha)}[X]$ tend to the left border of the profit distribution carrier, and the greater confidence probability *p* is, the faster this happens. However, this happens more slowly than when moving from $VaR_p^{(2)}[X]$ to $VaR_p^{(3)}[X]$. The closer distribution mode ν is to the left border of the profit distribution carrier, the faster risk measures $VaR_p^{(2+\alpha)}[X]$ at growing α tend to the left border of the profit distribution carrier at all *p*. That is, this position is all the more risky. Applying VaR risk measures to powers $(2+\alpha)$ at various α, the risk manager, depending on the risk appetite of his company, can rather subtly examine the risks in the left tail of the profit distribution.

NORMAL DISTRIBUTION (VAR TO A FRACTIONAL POWER)

Applying formulas (9) and (10) in the case of a normal distribution, we have:

$$VaR_p^{(1+\alpha)}[X] = V\sigma k_{1-(1-p)(1-\alpha p)}^{0.1}$$

and

$$VaR_p^{(2+\alpha)}[X] = V\sigma k_{1-(1-p)^2(1-\alpha p)}^{0.1}.$$

We study the behavior of these risk measures, depending on the confidence probability in *Tables 15* and *16*. Since only quantiles depend on the confidence probability, the examples provide precisely the dependence of the corresponding quantiles on confidence probabilities.

Table 16

Values $VaR_p^{(t)}[X]$ (through values of corresponding $k_q^{0.1}$) at various values of *t* and *p*, assuming normal distribution of variable *X*

	p = 90%	*p* = 95%	*p* = 99%
$k^{0.1}_{1-(1-p)^2}$ $(VaR_p^{(2)})$	2.326348	2.807034	3.719016
$k^{0.1}_{1-(1-p)^2(1-0.1p)}$ $(VaR_p^{(2.1)})$	2.361524	2.839036	3.74527
$k^{0.1}_{1-(1-p)^2(1-0.5p)}$ $(VaR_p^{(2.5)})$	2.542699	3.008547	3.888177
$k^{0.1}_{1-(1-p)^2(1-0.9p)}$ $(VaR_p^{(2.9)})$	2.894304	3.379946	4.24561

Source: the author's calculations.

Tables 15 and *16* show that for each confidence probability *p*, the corresponding quantiles increase with the growth of α, risk measures $VaR_p^{(1+\alpha)}[X]$ and $VaR_p^{(2+\alpha)}[X]$ increase. Thus, at large α, these risk measures rather subtly assess the increasingly catastrophic risks of various levels of catastrophe, and the greater confidence probabilities *p* are, the greater the assessment of such risk measures is.

CLARIFYING RISK ASSESSMENTS USING RISK MEASURE "VAR TO THE POWER OF ..." ADDING INCREASINGLY SMALL FRACTIONS TO THE POWER

Suppose that the risk manager assessed the asset risk using $VaR_p(X)$. However, in some time, s/he had to check the left tail of the profit distribution on the asset a little further to protect her/himself from slightly less frequently observed threats. Thus, s/he calculated risk measure $VaR_p^{(1+\frac{1}{2})}(X)$. Further circumstances may make her/him check the left tail of the profit distribution on the asset even farther to protect her/himself from even less frequently observed threats — and s/he calculated risk measure $VaR_p^{(1+\frac{1}{2}+\frac{1}{3})}(X)$. This may lead to the situation when calculation and application risk measures such as $VaR_p^{(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n})}(X)$. May be of practical interest in risk management. Applying formula (7), we have the following formula to calculate these risk measures in the form of common risk measures VaR with a specially selected confidence probability:

$$VaR_p^{(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n})}(X) = VaR_{\tilde{p}_n}, \qquad (13)$$

where

$$\tilde{p}_n = 1-(1-p)(1-\frac{1}{2}p)(1-\frac{1}{3}p)...(1-\frac{1}{n}p). \qquad (14)$$

We are interested in questions that have both theoretical and practical meaning: how deeply can one investigate with the help of such measures all kinds of risks (catastrophic) that can be observed in the left tail of the profit distribution on the asset? Is it possible to cover 100% of all the risks possible for this asset with the help of this sequence of risk measures?

To do this, we first try to investigate the asymptotic behavior of confidence probabilities $\tilde{p}_n$ at unlimited increase of *n*.

Note that these probabilities can be as follows

$$\tilde{p}_n = 1 - e^{x_n}, \text{ where}$$

$$x_n = \ln[(1-p)(1-\frac{1}{2}p)(1-\frac{1}{3}p)\dots(1-\frac{1}{n}p)] =$$
$$= \ln(1-p) + \ln(1-\frac{1}{2}p) + \ln(1-\frac{1}{3}p) + \dots + \ln(1-\frac{1}{n}p).$$

Remember that function ln(1 + x) is expanded in a Taylor series as follows:

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots,$$

converging for all $x \in (-1,1]$, and we apply this expansion to each member of the expression for x_n

$$\ln(1-p) = -p - \frac{1}{2}p^2 - \frac{1}{3}p^3 - \frac{1}{4}p^4 - \dots$$

$$\ln(1-\frac{1}{2}p) = -\frac{1}{2}p - \frac{1}{2}\frac{1}{2^2}p^2 - \frac{1}{3}\frac{1}{2^3}p^3 - \frac{1}{4}\frac{1}{2^4}p^4 - \dots$$

$$\ln(1-\frac{1}{3}p) = -\frac{1}{3}p - \frac{1}{2}\frac{1}{3^2}p^2 - \frac{1}{3}\frac{1}{3^3}p^3 - \frac{1}{4}\frac{1}{3^4}p^4 - \dots$$

etc.

$$\ln(1-\frac{1}{n}p) = -\frac{1}{n}p - \frac{1}{2}\frac{1}{n^2}p^2 - \frac{1}{3}\frac{1}{n^3}p^3 - \frac{1}{4}\frac{1}{n^4}p^4 - \dots$$

Substituting all these expansions into the expression for x_n and making a reduction of such terms in powers of p, we have:

$$x_n = -p(1+\frac{1}{2}+\dots+\frac{1}{n}) - \frac{p^2}{2}(1+\frac{1}{2^2}+\dots+\frac{1}{n^2}) -$$
$$-\frac{p^3}{3}(1+\frac{1}{2^3}+\dots+\frac{1}{n^3}) - \dots$$
$$-\frac{p^s}{s}(1+\frac{1}{2^s}+\frac{1}{3^s}+\dots+\frac{1}{n^s}) + \dots$$

Marking through

$\varsigma_n(s) = \sum_{k=1}^{n}\frac{1}{k^s}$, при s = 1, 2, …, the expression for x_n can be as follows:

$$x_n = -p\varsigma_n(1) - \frac{p^2}{2}\varsigma_n(2) - \frac{p^3}{3}\varsigma_n(3) - \dots -$$
$$-\frac{p^s}{s}\varsigma_n(s) - \dots = -\sum_{s=1}^{\infty}\frac{p^s}{s}\varsigma_n(s).$$

Note that values $\varsigma(s)$ are partial sums of a series that determines the value of the famous Riemann zeta function:

$\varsigma(s) = \sum_{k=1}^{\infty}\frac{1}{k^s}$, which in our case is considered only for the natural values of argument s. As we know (for example, [22]), this function takes a finite value at s = 2, 3, …, however, its value is infinite (the series diverges) at s = 1.

This means that all values $\varsigma_n(s)$ at s = 2, 3, … tend to a finite limit at $n \to \infty$, but $\varsigma_n(s)$ tends to +∞ at $n \to \infty$.

Wherein

$$x_n = -p\varsigma_n(1) - \sum_{s=2}^{\infty}\frac{p^s}{s}\varsigma_n(s)$$

since $\varsigma_n(s) < \varsigma(s)$, a $\varsigma(s) \le \varsigma(2)$, and $s \ge 2$ we have

$$\sum_{s=2}^{\infty}\frac{p^s}{s}\varsigma_n(s) < \sum_{s=2}^{\infty}\frac{p^s}{s}\varsigma(s) \le \varsigma(2)(\sum_{s=1}^{\infty}\frac{p^s}{s} - p) <$$
$$< \varsigma(2)(\sum_{s=1}^{\infty}p^s - p) = \varsigma(2)(\frac{p}{1-p} - p) = \frac{\pi^2}{6}\frac{p^2}{1-p} < \infty.$$

Here, we used the formula for the sum of an infinite decreasing geometric progression, as well as the well-known Euler's identity, which states that $\varsigma(2) = \frac{\pi^2}{6}$ (for example, [22]).

Thus, $x_n \to -\infty$ at $n \to \infty$, and therefore, with $\tilde{p}_n = 1 - e^{x_n}$, пwe have $\tilde{p}_n \to 1$ at $n \to \infty$.

This means that a gradual increase in confidence probability with decreasing probabilities $p, \frac{1}{2}p, \dots, \frac{1}{n}p, \dots$ when calculating risk measures $VaR_p^{(1+\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{n})}(X)$, leads to full coverage of the left tail of the asset profit distribution, and the value of these measures tends to the left end of probability distribution carrier X.

CONCLUSIONS

The paper introduces a new family of risk measures, called VaR to the power of t, $VaR^{(t)}$. Expressions obtained for this family are easily applicable in practice, as well as closed analytical expressions for many statistical distributions that are often used in financial and insurance applications.

$VaR^{(t)}$ family of measures can help regulators and practical risk managers. Risk measures $VaR^{(t)}$ should improve regulatory methods in calculating capital requirements, as they may include more information about the relationship of agents with risk positions. Including quality information in decision-making tools is important for risk managers, and risk measures $VaR^{(t)}$ may play a key role in achieving this goal.

By calibrating parameters, one can compare risk measures $VaR^{(t)}$ with a wide variety of contexts. In particular, with a fixed level of confidence, the new family contains risk measures that are between VaR and ES measures and can adequately reflect average catastrophic risks of loss. However, in certain situations, more conservative risk measures than ES may be preferred. We show that such extremely conservative risk measures can also be determined by means of $VaR^{(t)}$ family. The conservatism of risk measures $VaR_p^{(t)}[X]$introduced in the work increases with the growth of t, and at large values of $t > 1$ they are more conservative compared to the known measures VaR and ES. These measures can be applied by cautious investors, who are afraid of possible very bad investment results. Although these results are very unlikely, in their opinion, they are quite possible in these circumstances. Research and assessment of such risks can be carried out using sequential calculation of $VaR_p^{(t)}[X]$ with increasing values of t. The way to calculate risk measures $VaR_p^{(t)}[X]$ will depend on many investor preferences, including their risk appetite.

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