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Analytical Generalization of the Depreciation Multiplier as a Factor of Extended Reproduction of Fixed Assets Depending on the Age Structure

L.A. Antonov

Surgut state University, Surgut, Russia https://orcid.org/0000-0001-7899-6176

ABSTRACT

The **subject** of the study is the expansion multiplier as a quantitative characteristic of the depreciation expansive effect in models of the extended reproduction of fixed assets. The aim of the study is to identify the impact of the age structure of fixed assets on the expansion multiplier value factoring in various methods of calculating depreciation. The research methods include mathematical and computer modelling, as well as deductive logic as a process of reasoning from more general to more specific. The high depreciation of fixed assets of Russian enterprises and the high demand for depreciation as a source of funds for their renewal and extended reproduction ensure the relevance of the study. The results of the study include the models of extended reproduction of fixed assets due to the expansive depreciation effect using various methods of accruing depreciation; demonstrate the relationship between the age structure of fixed assets and the expansion multiplier. Generalizing the methods of depreciation in terms of their impact on the expansion multiplier values provides the scientific novelty of the research. This paper introduces the expansion multiplier calculation formulas for general and special accelerated depreciation and limits of multiplicative potential of accelerated depreciation. The conclusions of the research work illustrate the possibility of using the expansion multiplier to plan and optimize the depreciation policies of organizations, as well as to evaluate the multiplier of fixed assets in various economies. The authors identified the problems of applying the expansive depreciation effect as a means of extended reproduction of fixed assets in practice, as a result of the limitations of its theoretical models. In overcoming the identified limitations, areas of technical theoretical research are proposed.

Keywords: fixed assets; depreciation; expansion multiplier; depreciation multiplier; the Lohmann-Ruchti effect; expansive depreciation effect; extended reproduction; capital investment; accelerated depreciation; deceleration rate

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INTRODUCTION

According to various sources, the degree of fixed assets depreciation of Russian enterprises remains at a high level. Their renewal is an important condition for increasing the competitiveness of domestic enterprises and their products both in the home and global markets. Finding funds aimed at updating the fleet of fixed assets is an urgent problem for many enterprises but political contradictions, high macroeconomic volatility, and business activity decrease as a result of various unpredictable factors (pandemic, emergencies, etc.) will only aggravate the business situation. The difficult conditions in which business entities find themselves as a result of the action of both economic factors and factors of a different nature, stimulate the search for sources of financing for their growth that go beyond the classical options -- loans and profits. Depreciation is one such source.

Depreciation is one of the most underestimated and understudied economic concepts. For a long time (often still) it was considered as a means of *simple reproduction* of fixed assets, with the help of which new similar equipment can be purchased to replace worn-out equipment. Thus, the "immortality" of the organization is achieved and at the same time, the principle of going concern in accounting is observed.

At the beginning of the XX century, the theory and practice of using *accelerated* (over straight-line method) depreciation methods were developing. However, the conscious and purposeful use of accelerated depreciation with the aim of using it as a source of capital investments was not carried out immediately. This was preceded by a long period of its use as a temporary tool to support the economy in wartime emergencies in England and the United States [1, 2].

Today, depreciation in advanced economies is the main source of the reproduc-

 $^{\rm l}$ Chichkin A. Wear and tear. Rossiyskaya Gazeta — Economy. Nº 143(5519). URL: https://rg.ru/2011/07/05/iznos.html (accessed on 10.06.2020).

tion of fixed assets. Since the middle of the twentieth century, the share of depreciation in investment funds has been growing and its importance has been increasing. For example, in the USA, Germany, and Japan, depreciation is about 70, 65, and 50%, respectively [3].

In Russia, organizations have the right to charge depreciation with one of the following methods: straight-line method; diminishing balance method; method of writing off the value by the sum of the number of years of useful economic life; method of writing off the cost in proportion to the volume of products (works).2 At the same time, accelerated methods of depreciation are not widely used in our country by economic entities. About threequarters of enterprises use the straight-line depreciation method.³ This is probably due to both the general inertia of business entities in the area of accounting practices they use and lack of knowledge on this issue. These and other factors increase the interest of scientists in the study of such an economic phenomenon as depreciation.

EXPANSION MULTIPLIER FOR STRAIGHT-LINE METHOD TO CALCULATE DEPRECIATION

For the first time, K. Marx and F. Engels considered depreciation as a source of funds that go beyond simple reproduction. They revealed the fact that if you use the straight-line method of calculating depreciation with the annual reinvestment of the depreciation fund in production, an increase in the volume of fixed assets occurs. Limiting himself to primary observation, Marx did not develop this concept. Only since the end of the first half of the XX century, this effect began to find its description and substantiation in the works of such

² Accounting Regulations (PBU) 6/01 clause 18 "Accounting for fixed assets". URL: http://www.consultant.ru/document/cons_doc_LAW_31472/71350ef35fca8434a702b24b27e57b60e1 162f1e/ (accessed on 10.06.2020).

³ Methods for calculating the depreciation of fixed assets. Website Assistentus. URL: https://assistentus.ru/osnovnye-sredst-va/sposoby-nachisleniya-amortizacii/ (accessed on 10.06.2020).

Table 1

Model of extended reproduction of fixed assets using the straight-line depreciation method, thousand rubles

| Original cost of fixed assets | 100 | 25 | 31.25 | 39.06 | 48.83 | 36.04 | 38.79 | 40.68 | 41.08 |
|-------------------------------|-----|------|-------|-------|--------|--------|-------|--------|--------|
| Fixed assets No. Year | FA1 | FA2 | FA3 | FA4 | FA5 | FA6 | FA7 | FA8 | FA9 |
| 1 | 25 | | | | | | | | |
| 2 | 25 | 6.25 | | | | | | | |
| 3 | 25 | 6.25 | 7.81 | | | | | | |
| 4 | 25 | 6.25 | 7.81 | 9.77 | | | | | |
| 5 | | 6.25 | 7.81 | 9.77 | 12.21 | | | | |
| 6 | | | 7.81 | 9.77 | 12.21 | 9.01 | | | |
| 7 | | | | 9.77 | 12.21 | 9.01 | 9.7 | | |
| 8 | | | | | 12.21 | 9.01 | 9.7 | 10.17 | |
| 9 | | | | | | 9.01 | 9.7 | 10.17 | 10.27 |
| Total original cost | 100 | 125 | 156.2 | 195.3 | 144.14 | 155.17 | 162.7 | 164.31 | 156.59 |

scientists as N. Grozdov, B. Horvat, M. Lohmann, G. Ruchti, M. Feldman, etc. This phenomenon and its quantitative measurement received various names and were associated with such terms, as the "Lohmann-Ruchti Effect", "expansion multiplier", "depreciation multiplier", etc. [4, 5].

Lohmann-Ruchti effect (expansion effect) — an economic effect that is manifested in the ability of depreciation to produce an extended reproduction of assets, limited in volume and in time, without using other sources of financing. This effect occurs when depreciation charges are periodically reinvested in the purchase of new items of fixed assets instead of accumulating them. Thus, the depreciation fund is transformed from a passive means of accumulation into an active means of production, and simple reproduction into extended ones. The quantitative expression of the expanded reproduction of fixed assets is the ex-

pansion multiplier (coefficient), the formula (1) [6, 7]:

$$M_{exp} = \frac{\sum OC_{FA}}{\sum OC_0} = \frac{N}{N_0},$$
 (1)

where $M_{\rm exp}$ — expansion multiplier; N or $\Sigma OC_{\rm FA}$ — stable value of the equipment park or the total original cost of fixed assets, formed as a result of the annual reinvestment of depreciation charges; $N_{\rm 0}$ or $\Sigma OC_{\rm 0}$ — the original value of the equipment park or the total original cost of fixed assets.

Thus, the expansion multiplier shows how much the original cost of fixed assets in operation (the so-called total original cost) will increase as compared to its base level.

The model of extended reproduction of fixed assets using the straight-line depreciation method (*Table 1*) shows depreciation charges of a conditional organization

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by years (lines) and fixed assets (columns), which acquired a fixed asset in the first year with an initial cost of 100 thousand rubles, useful life (UL) 4 years. The organization carries out expanded reproduction, reinvesting the amount of accumulated depreciation once a year. The depreciation method is a straight-line method. The input parameter of the model is highlighted in color, which is necessary for its construction and calculation of the amounts of depreciation charges.

Thus, the amount of accrued depreciation in the current year is used by the organization to acquire a new item of fixed assets in the next year, the depreciation amount of the i-year is equal to the original cost of i + 1 item of fixed assets.

Reaching a peak in the fourth year, the total original cost starts to decrease and subsequently fluctuates around some constant value. Moreover, the total original cost at any given time is greater than the amount invested in the purchase of the first fixed asset. So, in the ninth year, the total original cost of existing fixed assets will be 156.59 thousand rubles.

We calculate the values of the total original cost for each year as compared to the original of the first fixed asset. To do this, we will divide the total original cost of each year by 100 thousand rubles. (a basic indicator of dynamics).

After a certain number of years since the acquisition of the first item of fixed assets, the value of the basic indicator of dynamics, similar to the total original cost, stabilizes and varies around a certain constant value (*Table 2*). The value tends to the *expansion multiplier* of the item of fixed assets of the conditional organization.

Thus, the organization, without using the sources of external investment and without spending its own financial resources, due to depreciation is able to increase its fixed assets both in monetary and natural value, thereby carrying out expanded reproduction limited in volume and time.

The expansion multiplier is a relative measure, so a change in the acquisition value of the underlying asset does not affect its value. The factor influencing the expansion multiplier in the models of expanded reproduction of fixed assets is a useful life. Thus, when modeling the expanded reproduction of fixed assets with different useful economic lives, an increase in useful life ensures the expansion multiplier value increase.

Table 3 shows the dependence of the expansion factor on the useful economic life of fixed assets. The data from the table in the form of a graph are presented.

Figure 1 demonstrates that with an increase in the useful life, the value of the expansion multiplier also increases, but the growth of the curve slows down and it tends to a certain maximum value.

A large number of economists have contributed to the development of this area of economic science. The search for a theoretical substantiation of the expansion multiplier and a general formula for its calculation has become the object of a number of studies since the second quarter of the XX century. Thus, N. Grozdov, and later B. Horvat investigated the expansion multiplier using a straight-line method of calculating depreciation. One of the results of these studies was the formula for calculating the multiplier depending on the useful life, published by B. Horvat, the formula (2) [8, 9]:

$$M_{\rm exp} = 2\frac{UL}{UL + 1}. (2)$$

This formula, also known as the "Horvat's depreciation multiplier", allows us calculating the expansion multiplier value without building a model, and drawing a conclusion about the multiplicative potential of the straightline depreciation method [10–13].

$$\lim_{UL \to \infty} M_{\exp} = \lim_{UL \to \infty} 2 \frac{UL}{UL+1} = 2.$$
 (3)

Table 2

The values of basic indicators of the dynamics of the total original cost

| Year | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | ••• | 19 | œ |
|-----------------------------|---|------|------|------|------|------|------|------|-----|-----|-----|
| Basic indicator of dynamics | 1 | 1.25 | 1.56 | 1.95 | 1.44 | 1.55 | 1.63 | 1.64 | | 1.6 | 1.6 |

Table 3
Expansion multiplier value depending on the useful life of fixed assets (1–49 years) when using the straight-line depreciation method

| × | ×1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | 0 | _ | 1 | 1.333 | 1.5 | 1.6 | 1.667 | 1.714 | 1.75 | 1.778 | 1.8 |
| | 1 | 1.818 | 1.833 | 1.846 | 1.857 | 1.867 | 1.875 | 1.882 | 1.889 | 1.895 | 1.9 |
| | 2 | 1.905 | 1.909 | 1.913 | 1.917 | 1.920 | 1.923 | 1.926 | 1.929 | 1.931 | 1.933 |
| | 3 | 1.935 | 1.938 | 1.939 | 1.941 | 1.943 | 1.944 | 1.946 | 1.947 | 1.949 | 1.95 |
| | 4 | 1.951 | 1.952 | 1.953 | 1.955 | 1.956 | 1.957 | 1.957 | 1.958 | 1.959 | 1.96 |

Source: author's calculations.

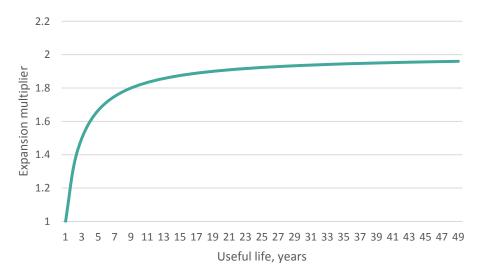


Fig. 1. The values of the expansion multiplier depending on the useful life of fixed assets when using the straight-line depreciation method

Source: compiled by the author based on Table 3.

With an increase in the UL, the expansion multiplier tends to two when using the straight-line method of calculating depreciation, the formula (3).

These and other studies were based on the assumption that the depreciation fund is reinvested once a year. Later, M. Feldman ob-

tained a formula for calculating the expansion factor depending on the amount of equipment, which made it possible to more accurately calculate the result, not relying on the assumption of annual reinvestment of funds, but taking into account the real possible frequency of renewal of fixed assets.

Although the results of these and other studies helped to understand the value of depreciation as a means of expanded reproduction of fixed assets better, they were limited to the straight-line method of its accrual, which was relevant for its time. Today, the legislation of many countries, including Russia, allows the use of accelerated methods.

Such scientists as Berg, Waegenaere, Wielhouwer, Gazzola, Beretta, Mella, Lemarchand, Nikitin, Breif, Anton, etc. studies the problem of the depreciation multiplier [14–17].

Further studies of the theory of depreciation are necessary for the possibility of practical implementation of its multiplicative potential.

DECELERATION FACTOR

Different methods of calculating depreciation imply different methods for calculating the amount of depreciation by years of useful life. The diminishing-balance method, in contrast to the straight-line depreciation method, is calculated with the acceleration factor. It increases the annual depreciation rate and helps to speed up the transfer of the original cost of fixed assets to the cost of finished goods. The legislation establishes the procedure for calculating the amount of depreciation by years of useful life, based on the residual value of an item of fixed assets, the formula (4):

$$D_n = RV_{FA} \times N \times K, \tag{4}$$

where D_n — the amount of depreciation of the n-th year of the useful life; RV_{FA} — residual value an item of fixed assets; N — depreciation rate; K — acceleration factor.

However, this calculation method is not always convenient and cannot be sufficiently formalized. For the convenience of calculations, we use the *deceleration rate*.

The deceleration coefficient is an indicator reflecting how the amount of depreciation charges will change in the next year as compared to the previous one (chain indicator of dynamics). For exam-

ple, for a straight-line method of calculating depreciation, this coefficient always equals to one, the formula (5):

$$Dec_{Lin} = \frac{D_{n+1}}{D_n} = \frac{OC_{FA} \times N}{OC_{FA} \times N} = 1,$$
 (5)

where Dec_{Lin} — deceleration coefficient for a straight-line method of calculating depreciation.

The deceleration coefficient for a diminishing-balance depreciation method is different, the formulas (6), (7):

$$Dec_{DB} = \frac{D_{n+1}}{D_n} = \frac{\left(RV_{FA} - RV_{FA} \times N \times K\right) \times N \times K}{RV_{FA} \times N \times K} = (6)$$
$$= 1 - N \times K.$$

Or:

$$Dec_{DB} = 1 - \frac{K}{UL},\tag{7}$$

where Dec_{DB} — deceleration rate for a diminishing-balance depreciation method; RV_{FA} — residual value of an item of fixed assets; UL — useful life of an item of fixed assets.

Obviously, the deceleration factor does not depend on the original cost of the item of fixed assets and can be calculated without it on the basis of the acceleration factor and the useful life of an item of fixed assets (depreciation rate).

Table 4 shows the values of the deceleration rate for acceleration factors from 1 to 3 in increments of 0.2 and useful lives from 1 to 15 years. Along with the values of the deceleration rate of the method diminishing-balance method, the table also includes the value of the deceleration factor for the straight-line method of depreciation (Lin).

When using a deceleration factor to calculate the depreciation rates from the original cost, the following rules are applied:

1) the share of depreciation of the original cost of the first year is calculated as the difference between the unit and the deceleration rate;

Table 4
The values of deceleration rates for different pairs of acceleration factor and useful life when using the diminishing-balance depreciation method

| | | Lin | | | | Dimini | shing ba | lance: acc | eleration | factor | | | |
|-------------|----|-----|------|------|------|--------|----------|------------|-----------|--------|------|------|------|
| | | Lin | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 | 2.2 | 2.4 | 2.6 | 2.8 | 3 |
| | 1 | | - | _ | _ | - | - | _ | - | - | _ | _ | - |
| | 2 | | 0.5 | 0.4 | 0.3 | 0.2 | 0.1 | _ | - | - | _ | _ | - |
| | 3 | | 0.67 | 0.6 | 0.53 | 0.47 | 0.4 | 0.33 | 0.27 | 0.2 | 0.13 | 0.07 | - |
| | 4 | | 0.75 | 0.7 | 0.65 | 0.6 | 0.55 | 0.5 | 0.45 | 0.4 | 0.35 | 0.3 | 0.25 |
| | 5 | | 0.8 | 0.76 | 0.72 | 0.68 | 0.64 | 0.6 | 0.56 | 0.52 | 0.48 | 0.44 | 0.4 |
| | 6 | | 0.83 | 0.8 | 0.77 | 0.73 | 0.7 | 0.67 | 0.63 | 0.6 | 0.57 | 0.53 | 0.5 |
| ife | 7 | | 0.86 | 0.83 | 0.8 | 0.77 | 0.74 | 0.71 | 0.69 | 0.66 | 0.63 | 0.6 | 0.57 |
| Jseful life | 8 | 1 | 0.88 | 0.85 | 0.83 | 0.8 | 0.78 | 0.75 | 0.73 | 0.7 | 0.68 | 0.65 | 0.63 |
| Use | 9 | | 0.89 | 0.87 | 0.84 | 0.82 | 0.8 | 0.78 | 0.76 | 0.73 | 0.71 | 0.69 | 0.67 |
| | 10 | | 0.9 | 0.88 | 0.86 | 0.84 | 0.82 | 0.8 | 0.78 | 0.76 | 0.74 | 0.72 | 0.7 |
| | 11 | | 0.91 | 0.89 | 0.87 | 0.85 | 0.84 | 0.82 | 0.8 | 0.78 | 0.76 | 0.75 | 0.73 |
| | 12 | | 0.92 | 0.9 | 0.88 | 0.87 | 0.85 | 0.83 | 0.82 | 0.8 | 0.78 | 0.77 | 0.75 |
| | 13 | | 0.92 | 0.91 | 0.89 | 0.88 | 0.86 | 0.85 | 0.83 | 0.82 | 0.8 | 0.78 | 0.77 |
| | 14 | | 0.93 | 0.91 | 0.9 | 0.89 | 0.87 | 0.86 | 0.84 | 0.83 | 0.81 | 0.8 | 0.79 |
| | 15 | | 0.93 | 0.92 | 0.91 | 0.89 | 0.88 | 0.87 | 0.85 | 0.84 | 0.83 | 0.81 | 0.8 |

- 2) the share of depreciation of the original cost of the n- th year following the first (except for the last) is calculated as the product of the share of the previous n-1 year and the deceleration rate;
- 3) the share of depreciation of the original cost of the last year is calculated as the difference between the unit and the sum of shares for previous years (the share of the residual value).

Thus, the calculation of the share of depreciation of the original cost of the n- th year (except for the last) can be expressed by the formula (8):

$$\%D_n = (1 - Dec) \times Dec^{n-1}.$$
 (8)

And the share of depreciation of the last year — to the formula (9):

$$\%D_{UL} = 1 - \sum_{n=1}^{UL-1} (1 - Dec) \times Dec^{n-1} = Dec^{UL-1}, \quad (9)$$

where $%D_n$ — the share of depreciation of the original cost of n-th year of the useful life.

For the sake of convenience in calculations, the last two formulas can be expressed by one (Heaviside step function), the formula (10):

$$%D_n = (1 - Dec) \times Dec^{n-1} \times 0^{0^{UL-n}} + Dec^{UL-1} \times 0^{UL-n}.$$
 (10)

The use of the deceleration factor for calculating depreciation charges seems preferable, since the number of required settlement operations can be reduced, and instead of calculating the absolute amounts of depreciation charges for years of useful life, calculate the shares of depreciation charges of the original cost.

Table 5 contains the calculated shares of depreciation charges of the original cost for

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 ${\it Table~5}$ Depreciation share of the original cost by useful life of assets, depending on the deceleration rate

| K_{dec} | | | | | Useful | life, year | | | | |
|-----------|-----|------|-------|-------|--------|------------|-------|-------|-------|-------|
| dec | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0.1 | 0.9 | 0.09 | 0.009 | | | | | | | |
| 0.2 | 0.8 | 0.16 | 0.032 | | | | | | | |
| 0.3 | 0.7 | 0.21 | 0.063 | 0.019 | | | | | | |
| 0.4 | 0.6 | 0.24 | 0.096 | 0.038 | 0.015 | | | | | |
| 0.5 | 0.5 | 0.25 | 0.125 | 0.063 | 0.031 | 0.016 | | | | |
| 0.6 | 0.4 | 0.24 | 0.144 | 0.086 | 0.052 | 0.031 | 0.019 | | | |
| 0.7 | 0.3 | 0.21 | 0.147 | 0.103 | 0.072 | 0.050 | 0.035 | 0.025 | 0.017 | 0.012 |

some deceleration factors (from 0.1 to 0.7 in increments of 0.1). It provides the possibility of applying deceleration factors for fixed assets with specified useful lives. The shares of depreciation charges of the original cost with the same deceleration rates are equal, only the useful life and the duration of depreciation are different.

The highlighted cells of the table show the limits of the useful life in relation to a specific value of the deceleration coefficient. Thus, the useful life of fixed assets with a deceleration factor of 0.1 can range from 2 to 3 years, and the deceleration rate of fixed assets with a useful life of more than four years will always be greater than 0.3. The data from the above table matches the *Table*. 4.

The deceleration rate for the straight-line method and the method of diminishing-balance has the same economic value — it shows the size of the share of depreciation of the future period relative to the previous one. The straight-line method and the method of diminishing-balance are calculated using various methods, and it is a complex task to compare them. The use of the deceleration factor makes it possible to reduce the methods of calculating depreciation to a single comparable indicator [18].

MODELING OF EXTENDED REPRODUCTION OF FIXED ASSETS WHEN USING THE DIMINISHING-BALANCE DEPRECIATION METHOD

We model the extended reproduction of fixed assets by years and items of fixed assets for the diminishing-balance method of depreciation. Suppose there is a fixed asset with an original value of 100 thousand rubles, with a useful life of 4 years. The acceleration factor for the diminishing-balance depreciation method equals three.

Modeling was carried out in a Microsoft Excel 2019spreadsheet. For more accuracy, the depreciation charges were calculated up to the year 100th. The expansion multiplier was found as the quotient of dividing the total original cost of year 100 by the total original cost of the first year of the fixed asset.

We find the deceleration rate for the model under consideration, the formula (11), and the share of depreciation of the original cost (*Table 6*).

$$Dec = 1 - \frac{K}{UL} = 1 - \frac{3}{4} = 0.25. \tag{11}$$

We construct a table reflecting the structure and dynamics of depreciation charges by year and by items of fixed assets (*Table 7*).

Table 6
Depreciation share of the original cost by useful life of assets when using the diminishing-balance depreciation method

| Year | 1 | 2 | 3 | 4 |
|---|------|--------|-----------|-----------|
| Depreciation share of the original cost | 0.75 | 0.1875 | 0.046 875 | 0.015 625 |

Table 7
Model of extended reproduction of fixed assets when using the diminishing-balance depreciation method, thousand rubles

| Original costs of fixed assets | 100 | 75 | 75 | 75 | 75.39 | 75.29 | ••• | 75.29 |
|--------------------------------|-------|-------|-------|-------|--------|--------|-----|--------|
| Fixed assets No. Year | FA1 | FA2 | FA3 | FA4 | FA5 | FA6 | | FA100 |
| 1 | 75 | | | | | | | |
| 2 | 18.75 | 56.25 | | | | | | |
| 3 | 4.69 | 14.06 | 56.25 | | | | | |
| 4 | 1.56 | 3.52 | 14.06 | 56.25 | | | | |
| 5 | | 1.17 | 3.52 | 14.06 | 56.54 | | | |
| 6 | | | 1.17 | 3.52 | 14.14 | 56.47 | | 56.47 |
| 7 | | | | 1.17 | 3.53 | 14.12 | | 14.12 |
| 8 | | | | | 1.18 | 3.53 | | 3.53 |
| 9 | | | | | | 1.18 | | 1.18 |
| Total original cos | 100 | 175 | 250 | 325 | 300.39 | 300.68 | | 301.17 |

Source: author's calculations.

The depreciation charges of the first three years are constant and equal to depreciation of the first year of operation of the item of fixed assets. Based on such an observation, one can make a hasty conclusion about the invariability of the depreciation costs when using the diminishing-balance method but further calculations show that this is not the case. As in the case of using the straight-line method, the depreciation fluctuates over the years and tends to a certain fixed value.

$$M_{\rm exp} = \frac{301.17}{100} = 3.012. \tag{12}$$

The expansion multiplier achieved by the organization on the 100th year is 3.012, the formula (12), which is 1.88 times more than when a straight-line method of calculating depreciation is used by the organization. Modelling the depreciation costs in a similar way, we find the values of the expansion multiplier for different pairs of acceleration factors and useful life of assets.

FINANCETP.FA.RU • 49

The values of the expansion multiplier for different pairs of acceleration factors and useful life of assets

| | | | Acceleration factor | | | | | | | | | | | |
|-------------|----|-------|---------------------|-------|-------|-------|-------|-------|-------|-------|-------|--|--|--|
| | | 1 | 1.4 | 1.6 | 1.8 | 2 | 2.2 | 2.4 | 2.6 | 2.8 | 3 | | | |
| | 2 | 1.333 | 1.538 | 1.667 | 1.818 | 2.000 | _ | _ | _ | _ | - | | | |
| | 3 | 1.421 | 1.650 | 1.781 | 1.923 | 2.077 | 2.243 | 2.419 | 2.606 | 2.801 | 3.000 | | | |
| | 4 | 1.463 | 1.704 | 1.838 | 1.981 | 2.133 | 2.294 | 2.463 | 2.640 | 2.823 | 3.012 | | | |
| | 5 | 1.487 | 1.736 | 1.872 | 2.017 | 2.169 | 2.328 | 2.495 | 2.668 | 2.847 | 3.031 | | | |
| | 6 | 1.504 | 1.757 | 1.895 | 2.040 | 2.192 | 2.352 | 2.517 | 2.689 | 2.866 | 3.048 | | | |
| | 7 | 1.515 | 1.772 | 1.911 | 2.057 | 2.210 | 2.369 | 2.534 | 2.705 | 2.881 | 3.061 | | | |
| Useful life | 8 | 1.523 | 1.783 | 1.923 | 2.069 | 2.223 | 2.382 | 2.547 | 2.717 | 2.892 | 3.072 | | | |
| Usefu | 9 | 1.530 | 1.791 | 1.932 | 2.079 | 2.233 | 2.392 | 2.557 | 2.727 | 2.901 | 3.080 | | | |
| | 10 | 1.535 | 1.798 | 1.939 | 2.087 | 2.241 | 2.400 | 2.565 | 2.735 | 2.909 | 3.087 | | | |
| | 11 | 1.540 | 1.803 | 1.945 | 2.093 | 2.247 | 2.407 | 2.572 | 2.741 | 2.915 | 3.093 | | | |
| | 12 | 1.543 | 1.808 | 1.950 | 2.098 | 2.253 | 2.412 | 2.577 | 2.747 | 2.920 | 3.098 | | | |
| | 13 | 1.546 | 1.812 | 1.954 | 2.103 | 2.257 | 2.417 | 2.582 | 2.751 | 2.925 | 3.102 | | | |
| | 14 | 1.549 | 1.815 | 1.958 | 2.107 | 2.261 | 2.421 | 2.586 | 2.755 | 2.929 | 3.106 | | | |
| | 15 | 1.551 | 1.818 | 1.961 | 2.110 | 2.265 | 2.425 | 2.589 | 2.759 | 2.932 | 3.109 | | | |

The diminishing-balance method of calculating depreciation has the greatest multiplicative effect as compared to other methods and ranges from 1.33 to 3 and more (*Table 8*). In addition, unlike other methods, in which the values of the expansion multiplier are rigidly tied to the useful life, the diminishing-balance method allows changing the expansion multiplier over a wide range of values.

MODELLING OF EXTENDED REPRODUCTION OF FIXED ASSETS WHEN USING THE SUM OF YEARS DIGIT METHOD OF CALCULATING DEPRECIATION

Similarly, we model the extended reproduction of fixed assets for the depreciation method according to the sum of years of useful life. We leave the conditions unchanged.

The legislation establishes the procedure for calculating the depreciation charges when using the sum of the years' method, based on the original cost of an item of fixed assets, the formula (13):

$$D_n = OC_{FA} \times \frac{t}{T},\tag{13}$$

where t — years, un-depreciated useful life; T — sum of the number of years (cumulative).

Having calculated the share of depreciation of the original cost by years of useful life (*Table 9*), we will construct a model for extended reproduction of fixed assets, reflecting the structure and dynamics of depreciation charges by years and fixed assets when using the depreciation method by the sum of the number of years of useful life (*Table 10*).

We find the value of the expansion multiplier, the formula (14):

Table 9

Depreciation share of the original cost by useful life of assets when an organization uses
the sum of years method of calculating depreciation

| Year | 1 | 2 | 3 | 4 |
|---|-----|-----|-----|-----|
| Depreciation share of the original cost | 0.4 | 0.3 | 0.2 | 0.1 |

Table 10

Model of extended reproduction of fixed assets when using the sum of years of useful life method of calculating depreciation, thousand rubles

| Original cost of fixed asset | 100 | 40 | 46 | 50.4 | 51.96 | 49.91 | 50 |
|------------------------------|-----|-----|------|-------|--------|--------|-----------|
| Fixed asset No. Year | FA1 | FA2 | FA3 | FA4 | FA5 | FA6 | FA100 |
| 1 | 40 | | | | | | |
| 2 | 30 | 16 | | | | | |
| 3 | 20 | 12 | 18.4 | | | | |
| 4 | 10 | 8 | 13.8 | 20.16 | | | |
| 5 | | 4 | 9.2 | 15.12 | 19.64 | | |
| 6 | | | 4.6 | 10.08 | 14.73 | 19.96 | 20 |
| 7 | | | | 5.04 | 9.82 | 14.97 | 15 |
| 8 | | | | | 4.91 | 9.98 | 10 |
| 9 | | | | | | 4.99 | 5 |
| Total original cost | 100 | 140 | 186 | 236.4 | 188.36 | 201.37 | 200 |

Source: author's calculations.

$$M_{\rm exp} = \frac{200}{100} = 2. \tag{14}$$

The expansion multiplier equals 2, which is 1.25 times more than when the company uses the straight-line depreciation method, but 1.5 times less than when the company uses the diminishing-balance method.

Modeling the expanded reproduction of fixed assets, we find the values of the expansion multiplier depending on the asset's useful life.

Table 11 shows the values of the expansion multiplier for different values of the useful life. For clarity, *Table 11* data are presented as a graph.

Figure 2 clearly demonstrates how with an increase in the useful life, the value of the expansion multiplier also increases, but the growth of the curve slows down, and it tends to a certain maximum value. With equal useful life, the value of the expansion multiplier when using the sum of years method is higher than when using the straight-line depreciation method [19].

Table 11
The values of the expansion multiplier depending on the useful life of assets (1-49 years) when using the sum of years method of calculating depreciation

| ×10 ×1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|------|-------|
| 0 | - | 1 | 1.5 | 1.8 | 2 | 2.143 | 2.25 | 2.333 | 2.4 | 2.455 |
| 1 | 2.5 | 2.538 | 2.571 | 2.6 | 2.625 | 2.647 | 2.667 | 2.684 | 2.7 | 2.714 |
| 2 | 2.727 | 2.739 | 2.75 | 2.76 | 2.769 | 2.778 | 2.786 | 2.793 | 2.8 | 2.806 |
| 3 | 2.813 | 2.818 | 2.824 | 2.829 | 2.833 | 2.838 | 2.842 | 2.846 | 2.85 | 2.854 |
| 4 | 2.857 | 2.86 | 2.864 | 2.867 | 2.87 | 2.872 | 2.875 | 2.878 | 2.88 | 2.882 |

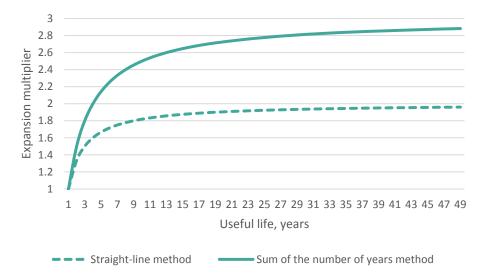


Fig. 2. The values of the expansion multiplier depending on the useful life of assets when using the sum of years method of calculating depreciation

Source: compiled by the author based on Table 11.

The considered models of extended reproduction of fixed assets allow us to make a number of important observations:

— Extended reproduction, carried out at the expense of the depreciation fund, is limited both in terms of the time of its implementation and in volume. Different methods of depreciation are characterized by different values of the expansion multiplier. Thus, the diminishing-balance method has the greatest multiplicative effect of the considered methods of depreciation, and the straight-line depreciation method — the least.

The result of the expansive effect of depreciation is ensured by the fact that both the depreciation charges amounts equate to each other, and the total original cost is equally distributed across all items of fixed assets in operation.

THE IMPACT OF THE AGE STRUCTURE OF FIXED ASSETS ON THE VALUE OF THE EXPANSION MULTIPLIER

The straight-line depreciation models of domestic and foreign authors, as well as the accelerated depreciation models, previously dis-

Table 12 Combinations that correspond to the methods of calculating depreciation before and after extended reproduction

| After Before | Straight-line | Diminishing-balance | Sum of years |
|---------------------|---------------|---------------------|--------------|
| Straight-line | Lin-Lin | Lin-DB | Lin-SOY |
| Diminishing-balance | DB-Lin | DB-DB | DB-SOY |
| Sum of years | SOY-Lin | SOY-DB | SOY-SOY |

Source: compiled by the author.

cussed in this paper (standard models), clearly demonstrate the dependence of the expansion multiplier on factors such as useful life and acceleration factor. However, they are not sufficient to be relevant to the real aspects of the economy. Thus, the standard models are based on the assumption that fixed assets are considered new assets that have just come into operation. In a situation close to the real one at the moment when the organization decides to start the implementation of expanded reproduction, the same type of fixed assets of the enterprise can be characterized by different age structure and original (residual) costs. Models that consider this and a number of other factors described below will be called regressive, and the depreciation multiplier, respectively, the regressive expansion multiplier.

The meaning of the regressive expansion multiplier is important and worth mentioning. If, when considering standard models, the amount of accumulated depreciation of fixed assets attributed to the original object was absent (due to its novelty), then it takes place when considering regressive models. This raises the following question: is it necessary to take into account the amount of accumulated depreciation as a result of simple reproduction for the purpose of calculating the expansion multiplier in regression models? Thoughtful reasoning leads the researcher to the conclusion that there is no need to take it into account. The amounts of the original cost of these assets in the process of operation through depreciation increased the cost of finished goods and ultimately accumulated in the form of cash on the current account. If the enterprise has established control over the targeted spending of the accumulated depreciation fund, the accumulated depreciation will be reinvested in full. However, since they are associated with retained earnings of the organization, it is rational to take into account only the "potential" depreciation charges for calculating the regressive expansion multiplier, i.e. those that will accrue to items of fixed assets in future periods. The funds of the current account, albeit "depreciation", invested in production for the acquisition of fixed assets, in essence do not differ from other sources of financing and have an equal maximum multiplicative potential of new fixed assets.

The expansion multiplier is a relative indicator and when the absolute value of the original cost changes within the framework of standard models, its value does not change. Regressive models are characterized by several items of the same type of fixed assets with the same useful lives, but different original cost and years of their operation. This leads to the fact that the age structure and the original cost of fixed assets in regressive models become a factor of the expansion multiplier.

Additionally, the enterprise in the process of carrying out its economic activities may change the methods of calculating the depreciation of fixed assets. Taking this fact into account should also be reflected in re-

Table 13 Comparative characteristics of standard and regression models of extended reproduction of fixed assets

| Standard models | Regression models |
|---|--|
| Fixed assets are new and their operational cycle starts from the first year | Fixed assets are characterized by a certain age structure: there are both new and old ones |
| The depreciation method has not been changed | The depreciation method after the expanded reproduction start may change |
| Extended reproduction is carried out from the first year | Extended reproduction is carried out starting from the second year or later |

Source: compiled by the author.

Table 14 **Example of a regression model of extended reproduction of fixed assets, thousand rubles**

| | Before | | | | After | | | | |
|-------------------------|--------|-----|------|-----|-------|-------|-------|-----|-------|
| Original cost | 75 | 0 | 25 | 0 | 25 | 16.25 | 20.25 | ••• | 18.75 |
| Fixed asset No. Year | FA1 | FA2 | FA3 | FA4 | FA5 | FA6 | FA 7 | | FA100 |
| 1 | 18.75 | 0 | 6.25 | 0 | | | | | |
| 2 | | 0 | 6.25 | 0 | 10 | | | | |
| 3 | | | 6.25 | 0 | 7.5 | 6.5 | | | |
| 4 | | | | 0 | 5 | 4.88 | 8.1 | | 7.5 |
| 5 | | | | | 2.5 | 3.25 | 6.08 | | 5.63 |
| 6 | | | | | | 1.63 | 4.05 | | 3.75 |
| 7 | | | | | | | 2.03 | | 1.88 |
| Total original cost | 75 | 75 | 100 | 100 | 50 | 66.26 | 61.52 | | 75 |

Source: author's calculations.

gressive models that claim to be "close to the real economy". As a result of the three considered methods of calculating depreciation, the number of their combinations corresponding to the method of calculating depreciation before and after extended reproduction is $3^2 = 9$. For brevity, we denote each method of calculating depreciation: Lin — straight-line method of calculating depreciation; DB — diminishing-balance method of calculating depreciation; SOY — a method of calculating depreciation according to the sum of the number of years in an asset's useful life (*Table 12*).

Thus, for the purpose of overcoming the white spots of the theory of depreciation in

Table 15

The effect of changes in the year of operation on the expansion multiplier in absolute and relative values for nine different situations when the methods of calculating depreciation change

| | | Absolute values | | | | | Relative v | /alues | |
|---------|-----------------|-----------------|-------|-------|-------|-------|------------|--------|------|
| No. u/l | Year Changes | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| 1 | Lin-Lin | 0.4 | 0.8 | 1.2 | 1.6 | 25% | 50% | 75% | 100% |
| 2 | Lin-DB | 0.753 | 1.506 | 2.259 | 3.012 | 25% | 50% | 75% | 100% |
| 3 | Lin-SOY | 0.5 | 1 | 1.5 | 2 | 25% | 50% | 75% | 100% |
| 4 | DB-Lin | 0.025 | 0.1 | 0.4 | 1.6 | 1.56% | 6.25% | 25% | 100% |
| 5 | DB-DB | 0.047 | 0.188 | 0.753 | 3.012 | 1.56% | 6.25% | 25% | 100% |
| 6 | DB-SOY | 0.031 | 0.125 | 0.5 | 2 | 1.56% | 6.25% | 25% | 100% |
| 7 | SOY-Lin | 0.16 | 0.48 | 0.96 | 1.6 | 10% | 30% | 60% | 100% |
| 8 | SOY-DB | 0.301 | 0.904 | 1.807 | 3.012 | 10% | 30% | 60% | 100% |
| 9 | SOY-SOY | 0.2 | 0.6 | 1.2 | 2 | 10% | 30% | 60% | 100% |

terms of the expansion multiplier, it is proposed to build and analyze regressive models of extended reproduction of fixed assets, which have a number of distinctive features (*Table 13*).

Table 14 presents an example of building a regression model of expanded reproduction corresponding to the combination "Lin–SOY". By the time the expanded reproduction begins, the organization has two items of fixed assets (FA) with a useful life of 4 years: FA1 with an original cost of 75 thousand rubles in the fourth year of operation and FA3 with an original cost of 25 thousand rubles in the second year of operation.

There is no extended reproduction of fixed assets until the fourth year, and depreciation is not carried over to the original cost of fixed assets in the following year. From the moment the item of fixed assets FA5 is put into operation, the enterprise begins to carry out expanded reproduction and transfer the amounts of accrued depreciation to the original cost of newly introduced fixed assets.

$$M_{\rm exp} = \frac{75}{100} = 0.75. \tag{15}$$

For the regressive model of expanded reproduction considered as an example, the value of the expansion multiplier is 0.75, the formula (15). Compared to the standard model of expanded reproduction, when using the depreciation method based on the sum of the number of years of an asset's useful life, this is more than 2.5 times lower value of the expansion multiplier. Thus, as a result of expanded reproduction, the total original cost will decrease in the future. This is due to the fact that fixed assets in the process of obsolescence lose their multiplicative potential relative to the multiplicative potential of new items of fixed assets (hence the name of this type of model).

The analysis of regressive models revealed that in the case when fixed assets are distributed over the years in equal shares, the implementation of expanded reproduction using the same method of depreciation does not give a multiplier effect and the expansion multiplier is equal to 1. Any process of expanded repro-

Table 16

Dynamics of depreciation costs for the total original cost depreciation in the first year of the useful life of assets

| FA No. Year | FA1 | FA2 | FA3 | FA4 | FA5 | FA6 |
|----------------|-----|-----|-----|-----|-----|-----|
| 1 | 1 | | | | | |
| 2 | 0 | 1 | | | | |
| 3 | 0 | 0 | 1 | | | |
| 4 | 0 | 0 | 0 | 1 | | |
| 5 | | 0 | 0 | 0 | 1 | |
| 6 | | | 0 | 0 | 0 | 1 |

Source: compiled by the author.

duction considered earlier, both within the framework of standard and regression models, led to a situation where their total original cost was equally distributed among all items of fixed assets.

We consider the effect of changing the year of operation of a fixed asset on the regressive expansion multiplier. To do this, we will find its values for each year of the useful life of assets, excluding the influence of other years of the useful economic life, sequentially substituting an original non-zero cost in each of the cells, nullifying all others. The data from nine models are presented in a more convenient form (Table 15). We also calculate the basic indicator of the dynamics by years of operation. As a comparison base, we take the values of the expansion multiplier for new fixed assets in the first year of operation (the far right column), the values of the regressive expansion multiplier of which are equal to the expansion multiplier for standard models of expanded reproduction of fixed assets.

Table 15 data show that:

• The depreciation method used by the organization prior to the commencement of expanded reproduction determines how the values of the regressive expansion factor by years of useful life will relate to the base value of the expansion multiplier. The table can be conditionally divided into three groups with

the same relative values of the expansion multiplier (1-3; 4-6; 7-9). Percentage relative to the baseline expansion multiple is a fraction of the residual value relative to the original cost.

• The method of depreciation used by the organization after the start of expanded reproduction determines the maximum value of the expansion multiplier. The table can be conditionally divided into three groups according to the equality of the maximum value of the expansion multiplier: 1, 4, 7; 2, 5, 8 and 3, 6, 9.

Thus, the data of the *Table*. 15 allow us to conclude that only residual value takes part in the regressive models of expanded reproduction. The total original cost can be calculated as the product of the residual value and the expansion multiplier (for the standard model) corresponding to the depreciation method after the start of expanded reproduction. In turn, the expansion multiplier for the regression model can be calculated using the formula (16):

$$\overline{M_{exp}} = \frac{RV_{FA} \times M_{exp}}{OC_{FA}} = \overline{LC} \times M_{exp}, \tag{16}$$

where $\overline{M_{\rm exp}}$ — regressive expansion multiplier; $M_{\rm exp}$ — expansion multiplier value for the standard model; LC — life coefficient.

Table 17

Dynamics of depreciation costs for the total original cost depreciation in the second year of the useful life of assets

| FA No. Year | FA1 | FA2 | FA3 | FA4 | FA5 | FA6 | FA7 | FA8 |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0 | | | | | | | |
| 2 | 1 | 0 | | | | | | |
| 3 | 0 | 0 | 0 | | | | | |
| 4 | 0 | 0 | 1 | 0 | | | | |
| 5 | | 0 | 0 | 0 | 0 | | | |
| 6 | | | 0 | 0 | 1 | 0 | | |
| 7 | | | | 0 | 0 | 0 | 0 | |
| 8 | | | | | 0 | 0 | 1 | 0 |

Source: compiled by the author.

Table 18

Dynamics of depreciation costs for the total original cost depreciation in the third year of the useful life of assets

| FA No. Year | FA1 | FA2 | FA3 | FA4 | FA5 | FA6 | FA7 | FA8 |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | 0 | | | | | | | |
| 2 | 0 | 0 | | | | | | |
| 3 | 1 | 0 | 0 | | | | | |
| 4 | 0 | 0 | 0 | 0 | | | | |
| 5 | | 0 | 0 | 0 | 0 | | | |
| 6 | | | 0 | 1 | 0 | 0 | | |
| 7 | | | | 0 | 0 | 0 | 0 | |
| 8 | | | | | 0 | 0 | 1 | 0 |

Source: compiled by the author.

Thus, having identified the patterns of the impact of the age structure of fixed assets on the expansion multiplier, for the purposes of further analysis, we can limit ourselves to the study of three models corresponding to three different methods of depreciation, which, in turn, correspond to the methods used by the organization after the start of expanded reproduction. If the residual and original costs provide information support for accounting

and are known for the organization, then the value of the expansion multiplier is the subject of theoretical research.

THEORETICAL BASIS AND ANALYTICAL GENERALIZATION OF THE EXPANSION MULTIPLIER

The three considered cases of expanded reproduction due to the multiplicative effect of the depreciation of fixed assets have different eco-

FINANCETP.FA.RU • 57

nomic effects. With all the differences in the methodology for calculating depreciation, the difference between them comes down to one thing: different methods of calculating depreciation in different proportions distribute the original cost over the years of the useful life of assets.

Each next year, the accumulated amount of depreciation charges is reinvested in production. The amounts of the original cost are different from year by year, which cannot be applied to the proportions of its distribution in the form of depreciation charges between the years of the useful life — they are always the same and are determined by the method of calculating depreciation.

We assume the possibility of using the depreciation method, in which the shares of the original cost are randomly distributed over the years of the useful life. Let there be a fixed asset with a useful life of 4 years. We consider four cases.

In the first case (*Table 16*), the organization depreciates the entire amount of the original cost in the first year of operation of the items of fixed assets. In this case, the organization can reinvest the entire original cost of the fixed asset every year. The result of this in the future will be the simultaneous operation of four items of fixed assets at the enterprise.

In the second case (*Table 17*), the organization depreciates the entire amount of the original cost in the second year of operation of the items of fixed assets, reinvests the entire original cost of the fixed asset every two years, since the depreciation is not charged in the first year of operation of fixed assets. This is reflected by the columns corresponding to the fixed asset items with even numbers since the basis for their calculation is zero. The result of this in the future will be the simultaneous operation of two items of fixed assets at the enterprise.

In the third case (*Table 18*), the organization depreciates the entire amount of the original cost for the third year of operation of the items of fixed assets, reinvesting the en-

tire original cost of the fixed asset every three years. The result of this in the future will be the simultaneous presence of one or two items of fixed assets at the enterprise. Thus, in the sixth and seventh year one — the fourth item of fixed assets — will be in operation, and in the eighth year — two: the fourth and seventh. On average, this gives 1.33 items of fixed assets, existing simultaneously in the future.

In the last, fourth case, the organization depreciates the entire amount of the original cost for the fourth year of operation of fixed assets. The result of this in the future will be the simultaneous operation at the enterprise of only one item of fixed assets.

Obviously, the simultaneous presence of the n-th number of fixed assets at the enterprise is nothing more than a quantitative measurement of expanded reproduction using the expansion multiplier. Thus, isolated from the influence of other years of useful life, the values of the expansion multiplier are in the range from 1 to useful economic life and can be calculated using the formula (17):

$$I_n = \frac{UL}{n},\tag{17}$$

where I_n — expansion multiplier of the n-th year, isolated from the impact of other years of the useful life; n — year of useful life.

Since in standard methods of calculating depreciation, the amount of the original cost is distributed in a certain proportion between all the useful lives, the found values of the isolated expansion multiplier require averaging using a harmonic weighted average, the weights in which will be the shares of depreciation of the original cost, the formula (18):

$$M_{\text{exp}} = \frac{1}{\frac{\%D_1}{I_1} + \frac{\%D_2}{I_2} + \dots + \frac{\%D_{UL}}{I_{UL}}} = \frac{UL}{\sum_{i=1}^{UL} i \times \%D_i}, (18)$$

where $%D_n$ — the share of depreciation of the original cost of fixed assets of the n-th year.

The resulting formula for calculating the expansion multiplier is the most general and is suitable for calculating its value for any method of calculating depreciation. It allows us to derive special formulas for calculating the expansion multiplier for the diminishingbalance method and for the sum of the number of years of the useful life of assets.

For the formula for calculating the expansion multiplier of the diminishing-balance method, we use the deceleration factor, the formulas (19), (20):

$$M_{\exp.DB} = \frac{1}{\frac{(1 - Dec) \times Dec^{0}}{UL/1}} + \frac{\sum_{i=1}^{UL} Dec^{i-1}}{\sum_{i=1}^{UL} Dec^{i-1}} + \frac{(1 - Dec) \times Dec^{1}}{UL/2} + \dots + \frac{Dec^{UL-1}}{UL/UL}$$
(19)

or

$$M_{\text{exp.}DB} = \frac{UL \times (Dec - 1)}{Dec^{UL} - 1}.$$
 (20)

We find the value to which the expansion multiplier tends for the diminishing-balance depreciation method with an increase in the useful life, the formula (21):

$$\lim_{UL \to \infty} M_{\exp.DB} = \lim_{UL \to \infty} \frac{UL \times (Dec - 1)}{Dec^{UL} - 1} =$$

$$= \lim_{UL \to \infty} \frac{UL \times \left(\left(1 - \frac{K}{UL} \right) - 1 \right)}{\left(1 - \frac{K}{UL} \right)^{KL} - 1} = \frac{Ke^K}{e - 1}, \quad (21)$$

where K — acceleration factor; e — Euler's number.

For the maximum permissible acceleration factor established by law (K = 3), the multiplier will be:

$$\lim_{UL \to \infty} M_{\exp.\overline{M}B(=3)} = \frac{3e^3}{e^3 - 1} \approx 3.157.$$
 (22)

Similarly, we derive a special formula for calculating the expansion multiplier for the method by the sum of the number of years of the useful life of assets, the formula (23):

$$M_{\text{exp.}SNY} = \frac{1}{\frac{UL}{UL/1} + \frac{UL-1}{UL/2} + \dots + \frac{1}{UL/UL}} = 3\frac{UL}{UL+2}. (23)$$

We find the value to which the expansion multiplier tends for the depreciation method based on the sum of the number of years of the useful life with an increase in the useful economic life of assets, the formula (24):

$$\lim_{UL \to \infty} M_{\exp.SNY} = \lim_{UL \to \infty} 3 \frac{UL}{UL + 2} = \lim_{UL \to \infty} 3 \frac{1}{1 + \frac{2}{UL}} = 3.$$
 (24)

Thus, if the general formula for calculating the expansion multiplier allows us to find the value of the expansion multiplier based on the annual shares of depreciation of the original cost, special formulas can be used both to calculate the value of the expansion multiplier and to find the limits of the multiplicative potential of methods for calculating depreciation.

CONCLUSIONS

All three considered methods of calculating depreciation have different potential for extended reproduction. The maximum values of the expansion multiplier are: for the straight-line depreciation method — 2; for the diminishing-balance method — 3.157 and for the method of depreciation according to the sum of the number of years in an asset's useful life—3. The method of diminishing-balance has the greatest potential and flexibility for the purposes of implementing expanded reproduction. Unlike other methods of calculating depreciation, it is characterized by the maximum possible expansion multiplier and allows wide adjustment of its value.

The obtained general and special formulas for calculating the expansion multiplier are critical for planning and forecasting the reproduction of fixed assets by business entities. They can be used to form and optimize the depreciation policy of enterprises. At the same time, the assumptions on which they are based limit the scope of their application to theoretical models and those close to real estimates of the expanded reproduction.

The main assumption that distorts the expansion multiplier in theoretical models is the fact that the depreciation fund is reinvested

annually. Though such a policy can be implemented by the enterprise in practice, it is not preferable. In a real economy, an enterprise can implement this process both more often and less often. More often, if for a period of time less than a year, the organization is able to accumulate funds sufficient to purchase a new item of fixed assets. In this case, newly acquired items of fixed assets generate a stream of depreciation charges, which increase expanded reproduction. Later, if the funds accumulated by the organization within a year are not enough to purchase one item of fixed assets. It is also true that the expansion multiplier depends not so much on the useful life as on the number of fixed assets of the enterprise.

Another assumption that distorts the expansion multiplier is that the depreciation fund may be used to implement expanded reproduction, becoming a means for acquiring fixed assets in volume and value in proportion to the current (assuming that the production structure of the organization is optimal). Excessive acquisitions of production assets with high expansion potential and small acquisitions of assets with low expansion potential at the same time will make some production assets ineffective, bringing the production structure out of its optimal state. To overcome this, it is preferable to use the funds accumulated as a result of depreciation of some funds for the acquisition of others in order to ensure their proportional commissioning, which, in turn, raises questions about the quantitative characteristics of such replacement reproduction and its impact on the final expansion multiplier.

These and other factors reduce accuracy and limit the use of the expansion multiplier in enterprises. Overcoming them is an important scientific, methodological, and practical task but it is beyond the scope of this article.

Expanded reproduction of fixed assets, realized through depreciation, imposes a number of restrictions on its implementation. However, it should be considered that with the same depreciation methodology, it can be implemented only once. At the end of the expansion cycle, the amounts of fixed assets received will be split equally among items of fixed assets on all years of useful life from 1 to the last year of useful life. In such conditions, additional expanded reproduction already carried out without changing the method of depreciation using the method with a large multiplicative potential is impossible.

Multiplicative models and conclusions based on them allow to approximately estimate the multiplicative potential of expanded reproduction of fixed assets at different scales of the economy. Thus, according to the degree of depreciation of fixed assets (at the enterprise, in the industry, in the state, etc.), the prospects for the implementation of expanded reproduction can be judged and a quantitative estimation can be given.

The expansion multiplier is a quantitative characteristic of the expansive effect of depreciation, the effect that is a consequence of the essence of fixed assets — their long-term use. This natural effect today is increasingly reflected in the implementation of expanded reproduction, which was not evident for economists of the past, who considered depreciation as a means of simple reproduction.

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ABOUT THE AUTHOR

Leonid A. Antonov — lecturer, Surgut State University, Surgut, Russia leonid.surgu@mail.ru

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