ORIGINAL PAPER

DOI: 10.26794/2587-5671-2021-25-3-6-19
UDC 336.519.86(045)
JEL G17, H30, H77, C65

Mathematical Models for Implementation of the Concept of Hard Budget Restrictions in the Budgetary System

I.V. Yakovenko
South-Russian State Polytechnic University (NPI) named after M.I. Platova, Novocherkassk, Russia
https://orcid.org/0000-0002-8559-2898

ABSTRACT

The subject of the study is the processes of budget decentralization in the management of public finances, as well as mathematical methods and models for implementing the concept of hard budget restrictions in order to create conditions for the self-development of administrative-territorial units. **The aim** of the study is to develop adaptive economic and mathematical models for implementing the strategy of hard budget constraints implemented in the process of inter-budget regulation. **The relevance** of the study is due to the fact that currently the subject of acute discussion in the scientific community is the self-development of administrative-territorial entities and increasing their financial independence. In this regard, the focus of economic research is focused on the problems of budgetary decentralization as an engine of economic development, as well as the related topics of the use of mathematical tools for modeling decision support in this area. The created models are subject to the requirements of learnability, adaptability to changing conditions of environmental influences, and the ability to operate not only with quantitative, but also with qualitatively defined characteristics. The problem of mathematical modeling is solved by applying an interdisciplinary synthesis of the theories of stochastic automata operating in random environments and fuzzy logic. **The proposed** synthesis of theoretical and methodological devices is the novelty of the research. **As a result**, an economic and mathematical model of a fuzzy automaton is constructed for determining and quantifying the values of the norms for the distribution of tax revenues between budgets of different levels of the budget system. A fuzzy automaton interacts with a simulation model that reproduces budget flows and quantifies the decisions made by the automaton model. **The practical significance** of the research results lies in the program implementation of the developed models and their inclusion in the public finance management circuit. **In the future**, it is planned to create a mathematical model of the collective behavior of fuzzy automata models, the interaction of which solves the problem of coordinating the interests of budgets of different levels of the hierarchy in the distribution of tax revenues.

Keywords: budget decentralization; hard budget constraints; inter-budget regulation; mathematical models; fuzzy automaton

For citation: **Yakovenko I.V. Mathematical models for implementation of the concept of hard budget restrictions in the budgetary system. *Finance: Theory and Practice.* 2021;25(3):6-19. (In Russ.). DOI: 10.26794/2587-5671-2021-25-2-6-19**

INTRODUCTION

At present, territorial self-development is a global problem, which is confirmed by the growing interest of a wide range of researchers in various fields of knowledge. There is a consensus in the global community on the need to solve this problem through the lens of budgetary decentralization in order to create conditions for the emergence of incentives for sub-federal and sub-national authorities to develop the economy in their jurisdictions. In the theory of fiscal federalism, the conceptual framework is a balance of centralized and decentralized relations by determining a compromise between the application of "hard" and "soft" budget constraints. Finding this compromise is a challenging task for financial technologies, which are evolving influenced by digital transformation.

Digital transformation is driving the development of financial innovation, including various financial engineering technologies. These are financial instruments (options, bonds, interest rate swaps, futures contracts, etc.), as well as various engineering developments based on the results of scientific and technological progress.

In the scientific literature, there are different interpretations of the meaning of the phrase "financial engineering". Some researchers, for example, V.I. Flegontov, R.A. Isaev [1, 2], put in it the meaning of a set of measures of financial impact, new schemes for conducting financial transactions aimed at minimizing financial risks.

The author of the article agrees with the point of view of E.F. Sysoeva, D.S. Kozlova [3], N.P. Baryn'kina [4], who interpret "financial engineering" as "a set of intellectual activity based on the achievements of science and technology" (engineering is derived from the Latin "ingenium", meaning ingenuity). According to the author, in the composition of financial engineering technologies, a significant place is occupied by the results of engineering developments used in the financial sector. These developments are influenced by the convergent evolution of computer technology and mathematical tools.

Financial engineering is an important component of the formation of the national economy of any country. Its role is significant in solving the strategic tasks of creating conditions for the endogenous development of territorial economic systems. These problems are solved by looking for internal evolutionary reserves that provide regions and municipalities with a competitive advantage and economic growth.

A wide range of scientific works from different countries is devoted to the development and implementation of economic policies focused on economic growth. At the same time, the subject field of research is expanding by including the methodological and theoretical apparatus of not only economic sciences. Recently, researchers have focused on the convergence of science and technology in order to obtain a synergistic effect in the evolutionary changes of the national economy.

Among the works devoted to solving the problems of managing the development of the national economy, it is worth highlighting the articles by S. Yu. Glaziev, R.M. Nizhegorodtsev, G.L. Kupryashin, N.V. Makogonova, A.V. Sidorov, O.S. Sukharev [5, 6].

Many works of modern foreign scientists are currently devoted to the issues of accelerating economic growth due to the self-development of administrative and territorial units. In their works, they investigate the phenomenon of decentralization of public goods as the most important factor in socio-economic development, based on the theorem of W. Oates. [7–9]. Decentralization of business as a strategy delegates to business entities the right to make independent decisions. This contributes to the creation of conditions for the emergence of incentives for economic agents to search for solutions that allow in the process of economic activity to get as close as possible to the final result, the set goal.

In this context, the studies of G. Everaert and A. Hildebrandt [10] describe the problems associated with the consequences of the application of "soft" budget constraints at the firm level. Attention is paid to the concept of decentralization not only in the field of improving the quality of public services but also in the aspect of stimulating sub-national authorities for the economic development of territories under their jurisdiction. In this regard, the problem arises of the ratio of the use of "soft" and "hard" budget constraints.

The works that reveal the problems of applying "soft" and "hard" budget constraints include articles by D. Chulkov [11], A.O. Hopland [12]. Recently, more and more researchers from different countries are inclined to the need to use mathematical methods to study the phenomenon of decentralization in the theory of fiscal federalism as an evolutionary path of economic development. This is supported by the work of Y. Jin and M. Rider [13], which investigates the impact of decentralization on economic growth based on the production function. Studying in this regard the application of fiscal decentralization policies in China and India based on the equations of equalization and growth, the authors made a fair conclusion that financial equalization does not always have a positive effect on economic growth. Indeed, the decentralization methods must be approached selectively, considering the characteristics of national and sub-national territories. Numerous works are the proof of the viability of issues related to fiscal decentralization in which the authors, relying on economic theories of decentralization, see them as a source of qualitatively new stimulating effects in the behavior of sub-national authorities in search of additional sources of development of administrative and territorial entities. These works include articles by A. Kappeler [14], M. Onofrei, F. Oprea [15], J. Koo, and B. J. Kim [16]. Thus, according to the content of the study, we can conclude that budget decentralization and "hard" budget constraints are not useful for all territories. This point of view is also shared by the author N.E. Barbashova, who believes that "a subsidy to equalize the level of budgetary provision of the territory is an essential element of the system of inter-budgetary transfers" [17]. But, according to N.E. Barbashova, the mechanism of budgetary equalization does not always create dependent sentiments in the territories [17]. The problems of applying the strategy of budgetary decentralization are caused by the heterogeneity of the development of the country's territorial units. To make decisions on the advantages of applying decentralization methods and studying the impact of their consequences on economic growth, it is advisable to use IT technologies with built-in economic and mathematical models. Since currently, active digitalization covers all spheres of the economy, the digital transformation of technologies for the spatial development of administrative and territorial units is the focus of researchers.

The problems of digital transformation of the financial industry as a driver of economic development, causing a change in the models of interaction between participants in the financial sector, are disclosed in the article by I. D. Kotliarov [18]. Among the works of researchers studying the emerging problems of the digital transformation strategy of European financial service providers, the article by S. Chanias, M. Myers, T. Hess [19] should be noted. A feature of the research results of S. Chanias, M. Myers, T. Hess is the presentation of digital transformation in the field of finance as a developing system and its design as a dynamic, iterative learning process and performing the necessary functions.

Adaptive learning models that formally describe the behavior of economic agents in the decision-making process were built by such authors as E.D. Streltsova, I.V. Yakovenko, S. Ziyadin, A. Borodin, S. Suieubaeva, D. Pshembayeva, O. S. Belokrylova, K. A. Belokrylov, S. S. Tsygankov, V.A. Syropyatov [20–25]. These models, built on the basis of the mathematical apparatus of the theory of stochastic automata operating in random environments, as well as fuzzy logic, perform the function of decision support in the process of intergovernmental regulation as a factor of paramount importance in ensuring economic growth.

Analysis of modern scientific sources indicates that the search for ways of territorial development is a promising trend of economic research on the evolution of any country. In view of the current political and economic situation, the problem of providing conditions for the self-development of administrative and territorial units at the expense of internal resources becomes especially urgent. At the same time, one of the criteria for self-development of regions and municipalities is the level of financial independence, achieved with a sufficient number of own sources of income. In the development of financial independence, a special role belongs to inter-budgetary relations, which develop vertically between territories. The processes of inter-budgetary regulation can both promote the intensification of actions of local authorities to develop their own tax base, and slow them down. It is advisable for territories with sufficient tax potential to use "hard" budget constraints, giving them the right to use part of the collected tax revenues when establishing optimal standards for their vertical distribution. The use of such a stimulating function of inter-budgetary regulation significantly increases the authorities' interest in intensifying economic activity.

For territories with a low capacity for self-organization, an effective method of inter-budgetary regulation is "soft" budget constraints through various transfer injections. When focusing on the implementation of the stimulating function of inter-budgetary regulation, the problem arises of determining the optimal proportions of the distribution of receipts from the payment of tax revenues between the budgets of the upper and lower levels of the budget system. The solution to this problem requires a formal description of the processes of expedient behavior of the decision-maker when choosing alternatives using methods of mathematical modeling and subsequent computational experiments.

All this mainstream the content of the research conducted by the author aimed at creating digital technologies to support decision-making on establishing the norms for the share distribution of tax revenues based on learning mathematical models. The relevance of the research carried out in this article lies in the need to put on a digital platform inter-budgetary relation between administrative and territorial entities when implementing the strategy of "hard" budget constraints. At present, this approach is the leading one for all countries of the world. And the transition to digital technologies requires the creation and implementation of mathematical models that describe the behavior of the subject of decision-making. This research is aimed at developing mathematical models in the form of fuzzy automata operating in a random environment. The analytical expressions obtained in the article for the probabilities of the automata choosing their states allow one to carry out computational experiments when choosing alternatives in the process of distributing tax revenues between the budgets of territories of different levels.

The research includes the following sections. In the first section, the urgency of the problem being solved is substantiated and analysis of publications devoted to its solution is given. In the second section, the problem of modeling the decision-making processes on the distribution of tax revenues between the budgets of different levels of the budget system is posed and the results of constructing economic and mathematical models are presented. The third section is devoted to the obtained simulation results and their discussion. The last section outlines the main findings of the study.

MATERIALS AND METHODS

Problem formulation

As mentioned earlier, in [20–22], decision-making models were developed to establish norms for the distribution of tax sources between budgets based on the use of the mathematical apparatus of the theory of stochastic automata operating in fuzzy random environments. In this case, the transitions of automata from state to state were determined either on the basis of the selectivity of the tactics of automata [20–22] or the basis of the equiprobability of their transitions to different states. The disadvantage of the models proposed in [20–22] is the lack of flexibility in their structure in the face of changes in the economic situation of administrative and territorial entities.

In this article, to support decision-making in the implementation of the stimulating function of inter-budgetary regulation, it is proposed to use the synthesis of mathematical apparatus of the theory of stochastic automata and fuzzy algebra. The author has constructed a fuzzy automaton Ω as a mathematical abstraction capable of learning the appropriate behavior of a decision-maker as natural intelligence. According to the theory of stochastic automata [26], the created mathematical model of decision support is immersed in a binary random environment that responds to the actions of the automaton Ω with reactions decomposed into two classes: Win and $Loss$. The set of states of automata is indicated by variables $S(t) = \{s_1(t), s_2(t), ..., s_k(t))\}$. Geometrically, the values $s_i(t) \in S$, $i = \overline{1,k}$ are a set of subsegments of length $\frac{1}{k}$, into which the original segment [0,1] is decomposed as an area of the definition of values $s_i(t) \in S$. Thus, the states will take values equal to $s_i(t) \in S$ will $0, \frac{1}{k}, \frac{2}{k}, ..., \frac{k-1}{k}, 1$.

Modeling

The automaton's behavior tactics are adopted in accordance with [20–22, 26]: in the case of a reaction of the random environment to states $s_i(t) \in S$, belonging to the class Win, the automaton does not leave it, and upon reaction $Loss$ it transforms into any other state $s_j(t) \in S$, $j \neq i$. If the automaton Ω in a state $s_i(t) \in S$ wins, then its input receives a signal "win" with probability p_i. The probability of the automaton losing in a state $s_i(t) \in S$ is indicated by a variable $q_i = 1 - p_i$. The economic meaning of the signals p_i and q_i is described in [32–34] and means, accordingly, the likelihood of a deficit and a surplus in the budget of an administrative and territorial entity. The difference between the model Ω constructed by the author and the previously proposed constructions [20–22] is that the transition matrices of the automaton during the reactions of the class environment Win and $Loss$ are built on the basis of the logical-linguistic analysis developed by L. A. Zadeh [27]. At the same time, territorial economic systems in terms of applying the methods of inter-budgetary regulation are analyzed from the point of view of the ability to self-organize. The need for such an analysis is stated in a previously published article [28], where it is noted that "to administrative and territorial units of different levels of economic development and different ability to self-organize, a differentiated approach should be applied to the choice of a budget regulation strategy" [28]. In this regard, further research is based on the previously proposed [28] decomposition of territories into two classes: *Capable* and *Unable*. It is proposed to include in the *Capable* class territories with an increased ability for self-development, due to the presence of internal resources and competitive advantages. The second class *Unable* includes territories that do not have the ability to self-development [28]. To carry out the classification, the works of such scientists as A. Łuczak, M.A. Just [29], H. Han, S. Trimi [30], A. Hatami-Marbini, F. Kangi [31], K. Palczewski, W. Salabun [32], T. Wu, X. Liu, F. Liu [33], M. Yucesan [34], F. Shen [35]. At the same time, the authors A. Luczak, M.A. Just [29] stated that there is no standard procedure for the classification of territories at different levels of government. In each specific case, it becomes necessary to apply specific methods, indicators, and algorithms that adequately assess the characteristics of territories for evolution at different levels of the administrative structure.

It is noted in [28] that "the systems of indicators, on the basis of which decision-makers or experts assess the level of socio-economic development of a territorial unit, depend on the professional knowledge of specialists, on the possible development scenarios of they work on, as well as on the specifics of the territory's economy". The list of evolution indicators may include such quantitative indicators as the deficit, surplus, budget revenues, and expenditures; per capita gross regional or municipal product; assessment of production potential; the levels of profitability of the main sectors of the economy, etc. [28]. Also, it is possible to focus on qualitatively expressed indicators, which include many different institutional, environmental, and other characteristics.

The solution of the problem of classifying territories according to the ability of self-development requires the use of mathematical methods of multivariate analysis, which will be studied by the author in future works. This article describes the results of constructing a mathematical model as part of the implementation of the "hard" budget constraint strategy to support decision-making regarding the establishment of the shares of distribution of joint taxes between the budgets of the sub-federal and sub-regional levels.

Some aspects of building such a model were outlined in [28], in which analytical expressions are given for the final probabilities of a fuzzy automaton for determining the shares of distribution of tax revenues between the budgets of different levels of the hierarchy and theorems of expedient behavior of the constructed automaton model are proved. This article demonstrates the derivation of analytical expressions for the final probabilities of the automaton model. Thus, for the territory of each class *Capable* and *Unable* it is proposed to apply a qualitatively expressed measure of the expediency of using the tools of inter-budgetary regulation in terms of establishing specific shares of tax deductions. This measure of expediency is described by a linguistic variable $OR = < T(OR), U, M >$, where $T(OR) = \{Capable, \ Unable\}$ — a term-set of a linguistic variable OR, U — its universe, $M = \{\mu_{Cap}, \mu_{Un}\}$ — is a membership function of fuzzy sets of *Capable* and *Unable*, meaning, respectively, the ability and inability of the territorial economic system to self-organize. The universe U is a segment [0,1], from which the norms of the share distribution of tax revenues between the budgets of the territories are taken. Membership functions $\mu^{Cap} : \{S\} \to [0,1]$, $\mu^{Un} : \{S\} \to [0,1]$ are described by equations:

$$\mu^{Un} = \begin{cases} 0, \ s_i < 0; \\ 1 - s_i, \ 0 < s_i < 1; \\ 0, \ s_i > 1; \end{cases} \ (1) \ \mu^{Cap} = \begin{cases} 0, \ s_i < 0; \\ s_i - 0, \ 0 < s_i < 1; \\ 0, \ s_i > 1; \end{cases} \tag{1}$$

Based on this, the automaton Ω is represented by a tuple $\Omega = < \Omega_{un}, \Omega_{Cap} >$, the components of which Ω_{un} and Ω_{Cap} describe its behavior in the linguistic environments *Unable* and *Capable*, respectively. A common feature of automata Ω_{un} and Ω_{Cap} is the identity of their matrices of transitions $\|m_{ij}(1)\|$ from state to state in case of Win:

$$m_{ij}(Win) = \begin{cases} 1, & for \ i = j; \\ 0, & for \ i \neq j \end{cases}. \tag{2}$$

Elements of matrices of transitions $m_{ij}^{Un}(Loss)$ and $m_{ij}^{Cap}(Loss)$ of automata Ω_{un} and Ω_{Cap} in case of $Loss$ are the values of the membership functions μ_{ij}^{Un} and μ_{ij}^{Cap}:

$$m_{ij}^{Cap}(Loss) = \begin{pmatrix} 0 & \frac{2}{k} & \frac{3}{k} & \dots & \frac{k}{k} \\ \frac{1}{k} & 0 & \frac{3}{k} & \dots & \frac{k}{k} \\ \frac{1}{k} & \frac{2}{k} & 0 & \dots & \frac{k}{k} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{1}{k} & \frac{2}{k} & \frac{3}{k} & \dots & 0 \end{pmatrix}; \ m_{ij}^{Un}(Loss) = \begin{pmatrix} 0 & \frac{k-2}{k} & \frac{k-3}{k} & \dots & \frac{k-k}{k} \\ \frac{k-1}{k} & 0 & \frac{k-3}{k} & \dots & \frac{k-k}{k} \\ \frac{k-1}{k} & \frac{k-2}{k} & 0 & \dots & \frac{k-k}{k} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{k-1}{k} & \frac{k-2}{k} & \frac{k-3}{k} & \dots & 0 \end{pmatrix}. \tag{3}$$

Elements of the matrix of transitions $\left\| P_{ij}^{Un} \right\|$ and $\left\| P_{ij}^{Cap} \right\|$ of automata regardless of the input signal Win and $Loss$ are determined based on the expressions

$$P_{ij}^{Un} = m_{ij}^{Un}(Win)p_i + m_{ij}^{Un}(Loss)q_i ;$$

$$P_{ij}^{Cap} = m_{ij}^{Cap}(Win)p_i + m_{ij}^{Cap}(Loss)q_i .$$

The matrices $\left\| P_{ij}^{Un} \right\|$ and $\left\| P_{ij}^{Cap} \right\|$ are as follows:

$$P_{ij}^{Un} = \begin{pmatrix} p_1 & \frac{k-2}{k}q_2 & \frac{k-3}{k}q_3 & \cdots & \frac{k-k}{k}q_k \\ \frac{k-1}{k}q_1 & p_2 & \frac{k-3}{k}q_3 & \cdots & \frac{k-k}{k}q_k \\ \frac{k-1}{k}q_1 & \frac{k-2}{k}q_2 & p_3 & \cdots & \frac{k-k}{k}q_k \\ \cdots & \cdots & \cdots & \cdots & \frac{k-k}{k}q_k \\ \frac{k-1}{k}q_1 & \frac{k-2}{k}q_2 & \frac{k-3}{k}q_3 & \cdots & p_k \end{pmatrix}; \quad P_{ij}^{Cap} = \begin{pmatrix} p_1 & \frac{2}{k}q_1 & \frac{3}{k}q_2 & \cdots & \frac{k}{k}q_k \\ \frac{1}{k}q_1 & p_2 & \frac{3}{k}q_3 & \cdots & \frac{k}{k}q_k \\ \frac{1}{k}q_1 & \frac{2}{k}q_2 & p_3 & \cdots & \frac{k}{k}q_k \\ \cdots & \cdots & \cdots & \cdots & \frac{k-k}{k}q_k \\ \frac{1}{k}q_1 & \frac{2}{k}q_2 & \frac{3}{k}q_3 & \cdots & p_k \end{pmatrix}. \quad (4)$$

In the theory of random processes, it is substantiated that in the case of a finite number of states of the system and provided that the transition from each state to any other is realizable in a finite number of steps, there are final probabilities. The article provides equations for calculating the final probabilities Z_i^{Cap}, Z_i^{Un}, $i = \overline{1,k}$ the automata in each of the states, subject to immersion in the linguistic environments *Capable* and *Unable*. The system of equations for calculating the final probabilities Z_i^{Un} of the stochastic automaton in its states, if the territorial economic system operates in the linguistic environment *Unable*, which means its inability to self-organize, has the form:

$$\begin{cases} Z_1^{Un} = Z_1^{Un}p_1 + Z_2^{Un}\frac{k-2}{k}q_2 + Z_3^{Un}\frac{k-3}{k}q_2 + \ldots + Z_{k-1}^{Un}\frac{k-(k-1)}{k}q_{k-1} + Z_k^{Un}\frac{k-k}{k}q_k \\ Z_2^{Un} = Z_1^{Un}\frac{k-1}{k}q_1 + Z_2^{Un}p_2 + Z_3^{Un}\frac{k-3}{k}q_2 + \ldots + Z_{k-1}^{Un}\frac{k-(k-1)}{k}q_{k-1} + Z_k^{Un}\frac{k-k}{k}q_k \\ Z_3^{Un} = Z_1^{Un}\frac{k-1}{k}q_1 + Z_2^{Un}\frac{k-2}{k}q_2 + Z_3^{Un}p_3 + \ldots + Z_{k-1}^{Un}\frac{k-(k-1)}{k}q_{k-1} + Z_k^{Un}\frac{k-k}{k}q_k \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Z_k^{Un} = Z_1^{Un}\frac{k-1}{k}q_1 + Z_2^{Un}\frac{k-2}{k}q_2 + Z_3^{Un}\frac{k-3}{k}q_3 + \ldots + Z_{k-1}^{Un}\frac{k-(k-1)}{k}q_{k-1} + Z_k^{Un}p_k \end{cases}. \quad (5)$$

After some system transformations, we get:

$$\begin{cases} Z_1^{Un}(1-p_1)+Z_1^{Un}\frac{k-1}{k}q_1=\sum_{i=1}^{k}Z_i^{Un}\frac{k-i}{k}q_i \\ Z_2^{Un}(1-p_2)+Z_2^{Un}\frac{k-2}{k}q_2=\sum_{i=1}^{k}Z_i^{Un}\frac{k-i}{k}q_i \\ Z_3^{Un}(1-p_3)+Z_3^{Un}\frac{k-3}{k}q_3=\sum_{i=1}^{k}Z_i^{Un}\frac{k-i}{k}q_i \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ Z_k^{Un}(1-p_k)+Z_k^{Un}\frac{k-k}{k}q_k=\sum_{i=1}^{k}Z_i^{Un}\frac{k-i}{k}q_i \end{cases}. \tag{6}$$

Thus, we have:

$$Z_1^{Un}q_1+Z_1^{Un}\frac{k-1}{k}q_1=Z_2^{Un}q_2+Z_2^{Un}\frac{k-2}{k}q_2=\dots=Z_k^{Un}q_k+Z_k^{Un}\frac{k-k}{k}q_k. \tag{7}$$

$$Z_2^{Un}=Z_1^{Un}\frac{q_1}{q_2}\frac{2k-1}{2k-2};\ Z_3^{Un}=Z_1^{Un}\frac{q_1}{q_3}\frac{2k-1}{2k-3};\dots;\ Z_k^{Un}=Z_1^{Un}\frac{q_1}{q_k}\frac{2k-1}{2k-k}. \tag{8}$$

We use the normalization condition $\sum_{i=1}^{k}Z_i^{Un}=1$:

$$Z_1^{Un}+Z_1^{Un}\frac{q_1}{q_2}\frac{2k-1}{2k-2}+Z_1^{Un}\frac{q_1}{q_3}\frac{2k-1}{2k-3}+\dots+Z_1^{Un}\frac{q_1}{q_k}\frac{2k-1}{2k-k}=1. \tag{9}$$

Based on this condition, it is determined Z_1^{Un}:

$$Z_1^{Un}=\frac{1}{\sum_{i=1}^{k}\frac{q_1}{q_i}\frac{2k-1}{2k-i}}=\frac{1}{q_1(2k-1)\sum_{i=1}^{k}\frac{1}{q_i(2k-i)}}. \tag{10}$$

Then the values Z_i^{Un}, $i=\overline{1,k}$ are determined based on analytical expressions:

$$Z_2^{Un}=\frac{1}{q_1(2k-1)\sum_{i=1}^{k}\frac{1}{q_i(2k-i)}}\frac{q_1}{q_2}\frac{2k-1}{2k-2}=\frac{1}{q_2(2k-2)\sum_{i=1}^{k}\frac{1}{q_i(2k-i)}}; \tag{11}$$

$$Z_3^{Un}=\frac{1}{q_1(2k-1)\sum_{i=1}^{k}\frac{1}{q_i(2k-i)}}\frac{q_1}{q_3}\frac{2k-1}{2k-3}=\frac{1}{q_3(2k-3)\sum_{i=1}^{k}\frac{1}{q_i(2k-i)}}; \tag{12}$$

$$Z_k^{Un}=\frac{1}{q_1(2k-1)\sum_{i=1}^{k}\frac{1}{q_i(2k-i)}}\frac{q_1}{q_k}\frac{2k-1}{2k-k}=\frac{1}{q_k(2k-k)\sum_{i=1}^{k}\frac{1}{q_i(2k-i)}}. \tag{13}$$

The system of equations for calculating the final probabilities Z_i^{Cap} of the stochastic automaton in its states, if the territorial economic system operates in the *Capable* linguistic environment, which means the ability to self-organize, has the form:

$$\begin{cases} Z_1^{Cap} = Z_1^{Cap} p_1 + Z_2^{Cap} \frac{2}{k} q_2 + Z_3^{Cap} \frac{3}{k} q_2 + \ldots + Z_{k-1}^{Cap} \frac{(1-k)}{k} q_{k-1} + Z_k^{Cap} \frac{k}{k} q_k \\ Z_2^{Cap} = Z_1^{Cap} \frac{1}{k} q_1 + Z_2^{Cap} p_2 + Z_3^{Cap} \frac{3}{k} q_2 + \ldots + Z_{k-1}^{Cap} \frac{(1-k)}{k} q_{k-1} + Z_k^{Cap} \frac{k}{k} q_k \\ Z_3^{Cap} = Z_1^{Cap} \frac{1}{k} q_1 + Z_2^{Cap} \frac{2}{k} q_2 + Z_3^{Cap} p_3 + \ldots + Z_{k-1}^{Cap} \frac{(1-k)}{k} q_{k-1} + Z_k^{Cap} \frac{k}{k} q_k \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Z_k^{Cap} = Z_1^{Cap} \frac{1}{k} q_1 + Z_2^{Cap} \frac{2}{k} q_2 + Z_3^{Cap} \frac{3}{k} q_3 + \ldots + Z_{k-1}^{Cap} \frac{(1)-k}{k} q_{k-1} + Z_k^{Cap} p_k \end{cases} . \quad (14)$$

After transformations, similar for the determining system, we have:

$$\begin{cases} Z_1^{Cap}(1-p_1) + Z_1^{Cap} \frac{1}{k} q_1 = \sum_{i=1}^{k} Z_i^{Cap} \frac{i}{k} q_i \\ Z_2^{Cap}(1-p_2) + Z_2^{Cap} \frac{2}{k} q_2 = \sum_{i=1}^{k} Z_i^{Cap} \frac{i}{k} q_i \\ Z_3^{Cap}(1-p_3) + Z_3^{Cap} \frac{3}{k} q_3 = \sum_{i=1}^{k} Z_i^{Cap} \frac{i}{k} q_i \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Z_k^{Cap}(1-p_k) + Z_k^{Cap} \frac{k}{k} q_k = \sum_{i=1}^{k} Z_i^{Cap} \frac{i}{k} q_i \end{cases} \quad (15)$$

Thus:

$$Z_1^{Cap} q_1 \frac{k+1}{k} = Z_2^{Cap} q_2 \frac{k+2}{k} = \ldots = Z_k^{Cap} q_k \frac{k+k}{k}, \text{ from which}$$

$$Z_2^{Cap} = Z_1^{Cap} \frac{q_1}{q_2} \frac{k+1}{k+2}; \; Z_3^{Cap} = Z_1^{Cap} \frac{q_1}{q_3} \frac{k+1}{k+3}; \; \ldots; \; Z_k^{Cap} = Z_1^{Cap} \frac{q_1}{q_k} \frac{k+1}{k+k}. \quad (16)$$

Using the normalization condition $\sum_{i=1}^{k} Z_i^{Cap} = 1$, we write:

$$Z_1^{Cap} q_1 + Z_1^{Cap} \frac{q_1}{q_2} \frac{k+1}{k+2} + Z_1^{Cap} \frac{q_1}{q_3} \frac{k+1}{k+3} + \ldots + Z_1^{Cap} \frac{q_1}{q_k} \frac{k+1}{k+k} = 1. \quad (17)$$

From this equation we obtain:

$$Z_1^{Cap} = \frac{1}{q_1(k+1) \sum_{i=1}^{k} \frac{1}{q_i(k+i)}}; \quad (18)$$

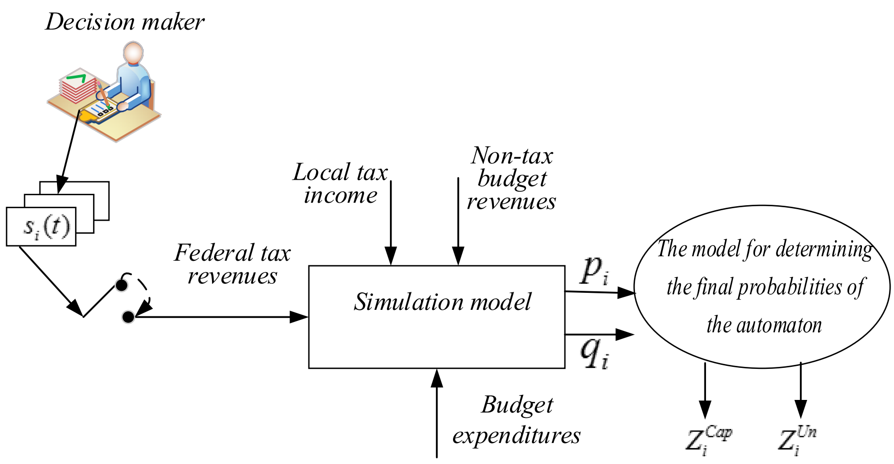


Fig. 1. **Conceptual scheme of interaction of a simulation model with a stochastic automaton**

Source: compiled by the author based on the research results.

$$Z_2^{Cap} = \frac{1}{q_1(k+1)\sum_{i=1}^{k}\frac{1}{q_i(k+i)}}\frac{q_1}{q_2}\frac{k+1}{k+2} = \frac{1}{q_2(k+2)\sum_{i=1}^{k}\frac{1}{q_i(k+i)}}; \quad (19)$$

$$Z_3^{Cap} = \frac{1}{q_1(k+1)\sum_{i=1}^{k}\frac{1}{q_i(k+i)}}\frac{q_1}{q_3}\frac{k+1}{k+3} = \frac{1}{q_3(k+3)\sum_{i=1}^{k}\frac{1}{q_i(k+i)}}; \quad (20)$$

$$Z_k^{Cap} = \frac{1}{q_1(k+1)\sum_{i=1}^{k}\frac{1}{q_i(k+i)}}\frac{q_1}{q_k}\frac{k+1}{k+k} = \frac{1}{q_k(k+k)\sum_{i=1}^{k}\frac{1}{q_i(k+i)}}. \quad (21)$$

Decision-making on the choice of the values of the standards $s_i(t) \in S$, $i = \overline{1,k}$ on the share distribution of funds for paying taxes between the budgets of different levels, is carried out on the basis of using the method of statistical tests for final probabilities Z_i^{Cap}, Z_i^{Un}, $i = \overline{1,k}$.

RESULTS AND ITS DISCUSSION

As part of the analytical expressions derived by the authors for the final probabilities Z_i^{Cap}, Z_i^{Un}, $i = \overline{1,k}$ there are the probabilities of wins p_i and losses q_i of automaton, for the determination of which the author previously proposed a simulation model [20–22].

A conceptual diagram of the interaction of the simulation model with a stochastic automaton for determining the shares of federal tax splitting is shown in *Fig. 1*.

In [28], the results of simulation experiments were illustrated, carried out to determine the values of the probability of wins p_i and losses q_i of the automaton at various values of deductions $s_i(t) \in S$ from personal income tax. In this article, experiments have been carried out to determine the amount of deductions $s_i(t) \in S$ from the tax “Excise taxes on fuels and lubricants”. The experiments were carried out on real information collected for some sub-regions called N 1 and N 2. At the same time, sub-region N 1 is classified as *Capable*, and sub-region N 2 is classified as *Unable*. The experimental results are shown in the *Table*.

Table

Estimates of the probabilities of surplus and deficit of budgets of sub-regions at different values of deductions $s_i(t) \in S$ from the excise tax on fuels and lubricants

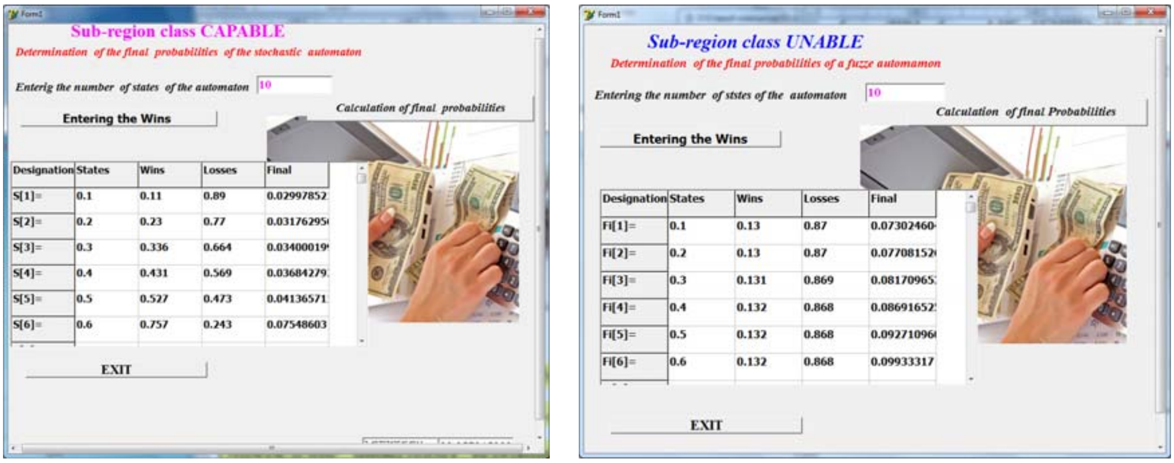
State of automaton $s_i(t) \in S$	Sub-region (capable of self-organization)			Sub-region (unable of self-organization)		
	Surplus Probability Estimate p_i	Deficit Probability Estimate q_i	Final Probability Z_i^{Cap}	Surplus Probability Estimate p_i	Deficit Probability Estimate q_i	Final Probability Z_i^{Un}
$s_1(t) = 0.1$	0.11	0.89	0.029978521	0.13	0.87	0.073024604
$s_2(t) = 0.2$	0.23	0.77	0.031762956	0.13	0.87	0.077081526
$s_3(t) = 0.3$	0.336	0.664	0.034000199	0.131	0.869	0.081709653
$s_4(t) = 0.4$	0.431	0.569	0.036842797	0.132	0.868	0.086916525
$s_5(t) = 0.5$	0.527	0.473	0.041365711	0.132	0.868	0.092710960
$s_6(t) = 0.6$	0.757	0.243	0.075486039	0.132	0.868	0.099333172
$s_7(t) = 0.7$	0.877	0.123	0.140358546	0.133	0.867	0.107097569
$s_8(t) = 0.8$	0.913	0.087	0.187413614	0.133	0.867	0.116022367
$s_9(t) = 0.9$	0.917	0.083	0.186106354	0.134	0.866	0.126716009
$s_{10}(t) = 1$	0.938	0.062	0.236685258	0.134	0.866	0.139387610

Source: compiled by the author based on the research results.

Fig. 2 shows the results of computer processing of experimental data when calculating the final probabilities Z_i^{Cap} и Z_i^{Un}. According to *Fig. 1*, the input of the simulation model receives statistical data characterizing the revenues of the local budget from the payment of local taxes, federal taxes, as well as data on non-tax revenues and expenditures of the local budget.

The probabilities of wins p_i and losses q_i of the automata, which are the output data of the simulation model, were determined on the basis of computer experiments. The decision-maker (DM) should vary the values of the standards for deductions from federal taxes $s_i(t) \in S$. he values obtained at the output of the simulation model p_i and q_i are used to determine the final probabilities Z_i^{Cap} and Z_i^{Un}. The results of experimental studies in determining the final probabilities Z_i^{Cap} and Z_i^{Un}, $i = \overline{1,k}$ are shown in the *Table*.

These *tables* assess the feasibility measures Z_i^{Cap} and Z_i^{Un} the establishment of standard tax deduction rates $s_i(t)$ for territories with a high *Capable* level and a low *Unable* level of self-organization. In this case, the mathematical model gives the following recommendations. For territories of the *Capable* class (that is, those with the ability to self-organize), it is advisable to set the values of the tax deduction rates close to one: for standard rates $s_{10}(t) = 1$, $s_9(t) = 0.9$ and $s_8(t) = 0.8$ final probabilities, respectively, are equal $Z_{10}^{Cap} = 0.23$, $Z_9^{Cap} = 0.18$ and $Z_8^{Cap} = 0.18$. For territories of the *Unable* class (that is, with a low level of self-organization), the model recommends to a greater extent the use of such instruments of interstate regulation as transfer injections than tax deductions. Consequently, the measures of the expediency of establishing standard tax deduction rates close to one are insignificant: the establishment of standard rates $s_{10}(t) = 1$, $s_9(t) = 0.9$ and $s_8(t) = 0.8$, accordingly, is an ineffective solution and is estimated by the values of finale probabilities. $Z_{10}^{Un} = 0.13$, $Z_9^{Un} = 0.12$, $Z_8^{Un} = 0.11$.



a ***b***

Fig. 2. **The results of computer processing of experimental data when calculating the final probabilities Z_i^{Cap} and Z_i^{Un}: *a* – for sub-regions of the class *Capable*; *b* – for sub-regions of the class *Unable***

Source: compiled by the author based on the research results.

CONCLUSIONS

The studies carried out allowed us to draw the following conclusions. Methods of inter-budgetary regulation represent a highly efficient register for managing the evolution of administrative and territorial units. The problem of the implementation of its stimulating function is currently especially relevant. In this regard, research on the creation of economic and mathematical models describing the appropriate behavior of decision-makers when choosing alternatives is becoming particularly relevant.

Earlier, mathematical models were proposed to support decision-making in the strategy of "hard" budget constraints, which have the property of learning and adapting to stochastically changing environmental influence [20–22]. But the recent mobility of the external environment leads to a rapid and frequent change of factors, to the use of approximate initial data, on the basis of which management decisions are made. In financial systems, these factors include the volatility of financial flows. Such instability is caused, for example, by the transition of the territory's economy to a qualitatively new level of development in connection with the emergence of new organizations, enterprises, and industries, the introduction of new technologies, etc. All this creates conditions of uncertainty and requires a quick and adequate response of the financial systems of administrative and territorial entities to maintain and enhance their competitiveness, which sharpens the interest in the use of methods of intellectualization in modeling.

The proposed approach to the convergence of the mathematical apparatus of fuzzy algebra and the theory of stochastic automata makes it possible to use qualitatively expressed characteristics when formalizing decision-making processes within the framework of the strategy of "hard" budget constraints, which forms a new basis for studying decision-making models in a fuzzy environment.

The constructed economic and mathematical models of fuzzy automata are of practical importance in connection with their implementation in software products and the possibility of embedding them into the public finance management scheme at the sub-federal and sub-regional levels. This involves the interaction of fuzzy automata models with database systems operating in public finance departments.

REFERENCES

1. Flegontov V.I. Financial engineering as an instrument of financial economy. *Aktual'nye problemy sotsial'no-ekonomicheskogo razvitiya Rossii*. 2019;(2):85–88. (In Russ.).
2. Isaev R.A. Banking management and business engineering. Moscow: INFRA-M; 2011. 400 p. (In Russ.).
3. Sysoeva E.F., Kozlova D.S. Financial engineering as a process of creating financial innovations. *Natsional'nye interesy: prioritety i bezopasnost' = National Interests: Priorities and Security*. 2010;6(7):51–55. (In Russ.).
4. Baryn'kina N.P. Evolution of the concept of financial engineering in financial science. *Voprosy ekonomiki i prava = Economic and Law Issues*. 2011;(36):101–107. (In Russ.).
5. Glaziev S. Yu., Nizhegorodtsev R.M., Kupryashin G.L., Makogonova N.V., Sidorov A.V., Sukharev O.S. State-level national economic development management (A summary of the Round Table conducted on 26.10.2016). *Gosudarstvennoe upravlenie. Elektronnyi vestnik = Public Administration. E-Journal*. 2017;(60):6–33. (In Russ.).
6. Glaziev S. The world economic crisis as a process of changing technological structures. *Voprosy ekonomiki*. 2009;(3):26–38. (In Russ.). DOI: 10.32609/0042–8736–2009–3–26–38
7. Oates W.E., Schwab R.M. Economic competition among jurisdictions: Efficiency enhancing or distortion inducing? *Journal of Public Economics*. 1988;35(3):333–354. DOI: 10.1016/0047–2727(88)90036–9
8. Oates W.E. An essay on fiscal federalism. *Journal of Economic Literature*. 1999;37(3):1120–1149. DOI: 10.1257/jel.37.3.1120
9. Oates W.E. Toward a second-generation theory of fiscal federalism. *International Tax and Public Finance*. 2005;12(4):349–373. DOI: 10.1007/s10797–005–1619–9
10. Everaert G., Hildebrandt A. On the causes of soft budget constraints: Firm-level evidence from Bulgaria and Romania. In: Advances in the economic analysis of participatory & labor-managed firms. Bingley: Emerald Publishing Ltd.; 2003;7:105–137. DOI: 10.1016/S 0885–3339(03)07007–8
11. Chulkov D. Innovation in centralized organizations: Examining evidence from Soviet Russia. *Journal of Economic Studies*. 2014;41(1):123–139. DOI: 10.1108/JES-05–2011–0057
12. Hopland A.O. Can game theory explain poor maintenance of regional government facilities? *Facilities*. 2015;33(3/4):195–205. DOI: 10.1108/F-08–2013–0062
13. Jin Y., Rider M. Does fiscal decentralization promote economic growth? An empirical approach to the study of China and India. *Journal of Public Budgeting, Accounting & Financial Management*. 2020. DOI: 10.1108/JPBAFM-11–2019–0174
14. Kappeler A. Fiscal externalities in a three-tier structure of government. *Journal of Economic Studies*. 2014;41(6):863–880. DOI: 10.1108/JES-03–2013–0033
15. Onofrei M., Oprea F. Fiscal decentralisation and self-government practices: Southern versus Eastern periphery of the European Union. In: Pascariu G.C., Duarte M.A.P.D.S., eds. Core-periphery patterns across the European Union: Case studies and lessons from Eastern and Southern Europe. Bingley: Emerald Publishing Ltd.; 2017:251–289.
16. Koo J., Kim B.J. Two faces of decentralization in South Korea. *Asian Education and Development Studies*. 2018;7(3):291–302. DOI: 10.1108/AEDS-11–2017–0115
17. Barbashova N.E. Does intergovernmental equalization create disincentives for regional infrastructural development? *Finansy: teoriya i praktika = Finance: Theory and Practice*. 2021;25(1):22–34. (In Russ.). DOI: 10.26794/2587–5671–2021–25–1–22–34
18. Kotliarov I.D. Digital transformation of the financial industry: Substance and trends. *Upravlenets = The Manager*. 2020;11(3):72–81. (In Russ.). DOI: 10.29141/2218–5003–2020–11–3–6
19. Chanias S., Myers M.D., Hess T. Digital transformation strategy making in pre-digital organizations: The case of a financial service provider. *Journal of Strategic Information Systems*. 2019;28(1):17–33. DOI: 10.1016/j.jsis.2018.11.003
20. Streltsova E.D., Dulin A.N., Yakovenko I.V. Perfection of interbudgetary relations as a factor of economic growth of depressed miner territories. *IOP Conference Series: Earth and Environmental Science*. 2019;272(3). DOI: 10.1088/1755–1315/272/3/032164

21. Streltsova E.D.; Yakovenko I.V. Support of decision-making in interbudgetary regulation on the basis of simulation modeling. In: Solovev D.B., ed. Smart technologies and innovations in design for control of technological processes and objects: Economy and production. Proceedings of the international science and technology conference "FarEastCon-2018". Vol. 2. Cham: Springer Nature Switzerland AG; 2019:165–172. DOI:10.1007/978–3–030–18553–4_21
22. Streltsova E.D., Bogomyagkova I.V., Streltsov, V.S. Modeling tools of interbudgetary regulation for mining areas. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*. 2016;(4):123–129.
23. Ziyadin S., Borodin A., Streltsova E., Suieubayeva S., Pshembayeva D. Fuzzy logic approach in the modeling of sustainable tourism development management. *Polish Journal of Management Studies*. 2019;9(1):492–504. DOI: 10.17512/pjms.2019.19.1.37
24. Belokrylova O.S., Belokrylov K.A., Tsygankov S.S., Syropyatov V.A., Streltsova E.D. Public procurement quality assessment of a region: regression analysis. *International Journal of Sociology and Social Policy*. 2021;41(1/2):130–138. DOI: 10.1108/IJSSP-03–2020–0095
25. Belokrylova O.S., Belokrylov K.A., Streltsova E.D., Tsygankov S.S., Tsygankova E.M. Quality evaluation of public procurement: Fuzzy logic methodology. In: Popkova E., ed. Growth poles of the global economy: Emergence, changes and future perspectives. Cham: Springer-Verlag; 2020:823–833. (Lecture Notes in Networks and Systems. Vol. 73). DOI: 10.1007/978–3–030–15160–7_83
26. Tsetlin M.L. Research on the theory of automata and modeling of biological systems, Moscow: Nauka; 1969. 316 p. (In Russ.).
27. Zadeh L.A. The Concept of a linguistic variable and its application to approximate reasoning — I. *Information Sciences*. 1975;8(1):199–249. DOI: 10.1016/0020–0255(75)90036–5. Zadeh L.A. The Concept of a linguistic variable and its application to approximate reasoning — II. *Information Sciences*. 1975;8(4):301–357. DOI: 10.1016/0020–0255(75)90046–8 (Russ. ed.: Zadeh L.A. Ponyatie lingvisticheskoi peremennoi i ego primenenie k prinyatiyu priblizhennykh reshenii. Moscow: Mir; 1976. 168 p.).
28. Yakovenko I. Fuzzy stochastic automation model for decision support in the process inter-budgetary regulation. *Mathematics*. 2021;9(1):67. DOI: 10.3390/math9010067
29. Łuczak A.. Just M. A complex MCDM procedure for the assessment of economic development of units at different government levels. *Mathematics*. 2020;8(7):1067. DOI: 10.3390/math8071067
30. Han H., Trimi S. A fuzzy TOPSIS method for performance evaluation of reverse logistics in social commerce platforms. *Expert Systems with Applications*. 2018;103:133–145. DOI: 10.1016/j.eswa.2018.03.003
31. Hatami-Marbini A., Kangi F. An extension of fuzzy TOPSIS for a group decision making with an application to Tehran stock exchange. *Applied Soft Computing*. 2017;52:1084–1097. DOI: 10.1016/j.asoc.2016.09.021
32. Palczewski K., Sałabun W. The fuzzy TOPSIS applications in the last decade. *Procedia Computer Science*. 2019;159:2294–2303. DOI: 10.1016/j.procs.2019.09.404
33. Wu T., Liu X., Liu F. An interval type-2 fuzzy TOPSIS model for large scale group decision making problems with social network information. *Information Sciences*. 2018;432:392–410. DOI: 10.1016/j.ins.2017.12.006
34. Yucesan M. et al. An integrated best-worst and interval type-2 fuzzy TOPSIS methodology for green supplier selection. *Mathematics*. 2019;7(2):182. DOI: 10.3390/math7020182
35. Shen F. et al. An extended intuitionistic fuzzy TOPSIS method based on a new distance measure with an application to credit risk evaluation. *Information Sciences*. 2018;428:105–119. DOI: 10.1016/j.ins.2017.10.045

ABOUT THE AUTOR

Irina V. Yakovenko — Cand. Sci. (Econ.), Associate Professor of the Department of Mathematics, South-Russian State Polytechnic University (NPI) named after M.I. Platova, Novocherkassk, Russia
iranyak@mail.ru

The article was submitted on 15.02.2021; revised on 28.02.2021 and accepted for publication on 27.03.2021. The author read and approved the final version of the manuscript.