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Russian Ruble Exchange Rate: Modeling of Comparative Medium-Term and Long-Term Dynamics

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ABSTRACT

The **subject** of the study is the dynamic mechanism of the formation of the exchange rate of the Russian ruble in a multi-level system of economic fundamental determinants-aggregates in the context of the independent floating rate of the national currency. The **aim** of the study is to develop the author's theoretical and methodological conceptual approach to modeling the dynamics of the equilibrium exchange rate based on international flows (IFEER) and to develop a new model of the Russian ruble exchange rate dynamics on its basis. The **methodological base** of the research includes system analysis, fundamental methods of economic theory, classical methods of mathematical analysis, and economic and statistical analysis, and the provisions of national accounting. The paper presents data on the verification of the results of modeling medium-term equilibrium dynamics. At the same time, the author pays considerable attention to the mathematical modeling of the long-term dynamics of the ruble exchange rate in comparison with the medium-term equilibrium dynamics and the mathematical analysis of internal functional relationships in modern conditions, which determines the **scientific novelty and relevance** of the study. Based on the conducted mathematical modeling, the author **concludes** about the trends of a stronger ruble exchange rate in the long run, while maintaining the current long-term trends.

Keywords: mathematical modeling; Russian ruble exchange rate; balance of payments; nonlinear long-term dynamics

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INTRODUCTION

Exchange rates in modern open economies are parallel or secondary macroeconomic policy objectives to be adjusted. At the same time, due to the systemic impact on other economic variables, they act as key instruments of foreign exchange and monetary policy. This is aimed at transferring the real sectors of the economy to a sustainable growth trajectory, as well as at the transition from monetary positions to regulating balances of payments and inflation rates in countries. These questions are deeply investigated in the works of D.E. Sorokin, S.V. Shmanev, I.L. Yurzinova, A.K. Bedrintsev [1, 2], L.A. Strizhkova [3], Ya.M. Mirkin [4] and others.

The ruble exchange rate at the microeconomic level is one of the most important factors that have a significant impact on the motivation of economic entities through the mechanism of forming international relative competitive advantages, on international trade flows and by creating favorable investment climate within the country for capital flows. At the same time, and importantly, these economic aggregates are the determining factors in the dynamics of the exchange rate.

After the completion of the almost complete liberalization of the exchange rate and the transition of the Bank of Russia to the inflation targeting regime according to the classification of the International Monetary Fund, the regime of the Russian ruble exchange rate became de facto characterized by an independent floating rate of the national currency. Even in these conditions, this classification type of the exchange rate regime is also characterized by the high importance of the exchange rate policy of the Central Bank and close attention to the dynamics of both the nominal and real exchange rates.

Mathematical methods are widely used in modeling exchange rates in the works of R. Dornbusch [5], J. Frenkel [6], A. Stockman [7], R. Mundell [8], M. Mussa [9], M. Obstfeld and K. Rogoff [10] et al (including from the point of view of portfolio balance in the works of R. Driskill [11], L. Taylor [12]), domestic scientists S. Yu. Glazyev [13], A. Yu. Kuzmin [14, 15] et al. In particular, R.V. Ivanov [16] draws attention to the use of mathematical methods for modeling the valuations of financial instruments considering the currency component.

However, aspects of the long-term dynamics of exchange rates seem to be insufficiently studied. At the same time, it is necessary first of all to pay attention to the fundamental factors of dynamics, which is emphasized in the studies of J. Williamson [17], C. Engel, N. Mark and K. West. [18], L. Killian [19], P. Clark and R. MacDonald [20] et al.

This paper presents a systematic approach aimed at studying the behavior of the Russian ruble, and is based on the concept of modeling exchange rates based on the International Flows Equilibrium Exchange Rate (IFEER) developed by the author to model comparative medium-term and long-term dynamics.

CURRENCY DYNAMICS MODELING: IFEER'S CONCEPTUAL FRAMEWORK

From the point of view of long-term modeling, the following approach deserves attention. One of the cornerstone equations of an open economy concerns the balance of payments equilibrium. At the same time, it is assumed that there will be no interventionist actions on the part of the monetary authorities in the regulation of floating exchange rates. This equation is expressed in local currency:

$$(eE - I) = (K^{-} - eK^{+}).$$

On the left is the current account balance, on the right is the capital account balance, e — exchange rate, E — exports of goods and services, I — imports of goods and services, K^{-}, K^{+} — capital outflow and inflow, respectively.

Having selected and carried out mathematical transformations, we get the national currency rate:

$$e = (I + K^{-}) \big/ (E + K^{+}).$$

This approach has several disadvantages. The exchange rate here is determined from a macroeconomic point of view. However, it practically does not have a base in the foreign exchange market in the form of real transactions. Moreover, in the world practice of national accounting, when using economic and statistical information to calculate macroeconomic aggregates at the balance of payments level, different values of national currencies can be used for exactly a certain period of time. Russia is no exception here.

Initially, all actual market transactions at nominal exchange rates $e_i, i \in (1, N)$ in the domestic foreign exchange market that have occurred during a specified period of time are considered.

Let us denote e_i, D_i, R_i in i-the operation: the nominal exchange rate, the amount in a certain foreign currency and the amount in the national currency, respectively.

These variables are related by the ratios: $e_i D_i = R_i$ and, therefore, $e_i = R_i / D_i$.

Moreover, the contribution of each transaction is different. It depends on the volume of the transaction. It should be noted that a significant part of transactions on the Russian market is carried out in US dollars. At the same time, conversion operations in other currencies, such as the euro, Canadian dollar, British pound sterling, are directly linked to the current exchange rate through a system of cross rates on both the international and national markets. Thus, for the purposes of modeling, the US dollar and its direct quotes against the Russian ruble are further considered as a foreign currency.

To study the medium-term and long-term dynamics of the Russian ruble exchange rate, it is proposed to determine the exchange rate *e* as the average weighted by volumes in foreign currency value of the exchange rates *N* of market transactions $e_i, i \in (1, N)$ conducted for a certain period of time:

$$e = \sum_{i=1}^{N} \frac{D_i}{\sum_{j=1}^{N} D_j} \times e_i \,. \qquad (1)$$

It can then be obtained by summing over *i*:

$$e = \sum_{i=1}^{N} \frac{D_i}{\sum_{j=1}^{N} D_j} \times \frac{R_i}{D_i} = \sum_{i=1}^{N} R_i \Big/ \sum_{j=1}^{N} D_j \,.$$

The final formula disaggregating flows across the balance of payments accounts can be represented as

$$e = \frac{\sum R^{CA} + \sum R^{K}}{\sum D^{CA} + \sum D^{K}}, \qquad (2)$$

Where the *CA* and *K* indices, refer, respectively, to funds flowing through the current account and the capital account. The actions of the monetary authorities in the form of foreign exchange interventions will be considered in aggregates with *K*.

For convenience we denote:

$$\sum R_i^{CA} = I \quad \sum D_i^{CA} = E,$$

$$\sum R_i^{K} = K^{-}, \sum D^{K} = K^{+}.$$

Then in the dynamic aspect (2) will be as

$$e_t = (I_t + K_t^{-}) \big/ (E_t + K_t^{+}),$$

where *E* — supply from the export of foreign currency; *I* — demand in national currency from imports for foreign currency; K^{-}, K^{+} — the value, respectively, of the outflow and inflow of capital between countries for the time period *t*.

It should be emphasized that functional dependence (2) is natural from an economic point of view:

$$e = e^{*(-1)} = f_e(I^{\uparrow}, (K^{-})^{\uparrow}, E^{\downarrow}, (K^{+})^{\downarrow}).$$

Hereinafter, the upper sign “↑” or “↓” for this factor means that the function is strictly increasing or decreasing, respectively.

For example, in our case, in terms of partial derivatives:

$$\partial f_e(I\ ,K^+,E\ ,K^-)\big/\partial K^+ < 0.$$

MODELING THE DYNAMICS OF THE RUBLE EXCHANGE RATE: BALANCE OF CURRENT OPERATIONS

Consider a dynamic two-period model in periods *t* — 1, *t*. Within this framework, the volume of currency *E* in dollar prices during the period *t*, delivered to the domestic foreign exchange market in the form of proceeds from the export of goods and services:

$$E_t = P_t^* k_E (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\delta} (e_{t-1}^R)^z, \qquad (3)$$

where $k_E = const$; *Q* is the level of real gross domestic product (GDP as a representative of total production); P_t^* — the level of average actual export prices; P_{t-1} — the level of consumer prices index (CPI); and the indices *t* — 1, *t* indicate successive simulation periods, *x* and δ — are the adjustable model parameters. Let us discuss further the properties of the parameters *x* and δ.

The *Z*-score is the rate of reaction to changes in the terms of trade. Within the framework of this model, the volume of currency supplied to the domestic market in period *t* is directly determined by the physical volume of exports in actual export prices P_t^* for the same period. At the same time, the volume itself also depends on the terms of trade, which are represented by a value $e^R{}_{t-1} = e_{t-1} P_{t-1}^* \big/ P_{t-1}$ in the form of an adjusted nominal exchange rate for the ratio of internal and external prices, and is determined by the decisions of exporters in the previous period *t* — 1. This dependence was studied in detail in various aspects in the author’s work [21], where it found its empirical confirmation in different periods of time.

A part of dependence (3) $k_E(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\delta}$ asserts that physical exports are part of the total output, which is averaged in a dynamic sense — the exponents Q_t are represented by weights:

$$\frac{1}{x+1} + \frac{x}{x+1} = 1.$$

The exponent $\delta \geq 0$ reflects the “slightly larger” growth in exports over imports, with non-negativity constraints as a function of total output. This is due to limited domestic demand and, therefore, the need to realize the growing total production volume precisely through exports. At the same time, the method of averaging the total output itself will not have a significant effect on the modeling result due to the insignificance of the volatility in the medium term of the variable *Q* in comparison with possible dynamic changes in other macroeconomic determinants used in modeling.

At the same time, dependence (3) also has a natural functional character from an economic point of view in relation to the entire system of the main listed factors of currency formation:

$$E_t = f_{E_t}(P_t^{*\uparrow}, Q_t^{\uparrow}, Q_{t-1}^{\uparrow}, e_{t-1}^{R\uparrow}).$$

In the economic scientific literature, a fact is known: the choice of export pricing in the currency of the consumer or producer will have a significant impact on the transmission mechanism for the transmission of exogenous shocks as a whole to the exchange rate. It should be emphasized that the real pricing mechanisms for the export products of Russian companies based on the prices of the main world commodity exchanges denominated in US dollars (in this case, in foreign currency) are of particular importance from the point of view of the modeling performed.

When modeling the dependence of imports of goods and services, let us assume that residents direct to consumption of imports in period *t* a part of their income, represented by both current income and income in the previous time period $t-1$ in domestic prices P_t. The initial functional dependence has the following form, considering the reaction y to a change in the terms of trade of the previous period:

$$I_t = P_t \, k_I (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})(e_{t-1}^R)^y , \qquad (4)$$

where $z-y=x$. In fact, within the framework of this model, the indicators z and y in formulas (3) and (4) determine the indicator x, introduced earlier in the same formulas.

МОДЕЛИРОВАНИЕ ДИНАМИКИ MODELING THE DYNAMICS OF THE RUBLE EXCHANGE RATE DYNAMICS: CAPITAL FLOWS AND OPERATIONS WITH FINANCIAL INSTRUMENTS

In modern conditions of medium-term and long-term changes in the current account balance of the country's balance of payments, capital movement plays a significant role, being one of the most important factors in the behavior of the ruble exchange rate, which clearly has a great influence on the results of formula (2).

However, the problem of this difficult to predict and rather an unstable determinant of the formation of the exchange rate will be solved at the formal logical level within the framework of the model under consideration by putting forward a number of hypotheses about the dynamics of capital flows.

For the functional dependence of capital outflow, we will accept the hypothesis — this is a part of the average total income of microeconomic agents inside the country in domestic prices P_t, withdrawn for the purpose of savings abroad, considering the relative international competitive advantages of the previous period:

$$K_t^- = P_t \, k_{K^-} (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})(e_{t-1}^R)^y . \qquad (5)$$

With regard to the amount of capital inflows, we will accept the following hypothesis: this is a function that increases in terms of the real aggregate product since international investors and speculators want to buy part of it at domestic prices P_t and under the terms of trade. The latter is explained by the fact that with an increase in the USD/RUR exchange rate and a fall in the national currency, investment conditions for non-residents are improving.

Therefore, the dependence of the capital inflow must satisfy the following condition:

$$K_t^+ = f_{K_t^+}(P_t^{*\uparrow}, Q_t^{\uparrow}, Q_{t-1}^{\uparrow}, e_{t-1}^{R\,\uparrow}).$$

Based on this, we assume:

$$K_t^+ = P_t^* k_{K^+} (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\theta} (e_{t-1}^R)^z , \qquad (6)$$

where θ — an adjustable parameter.

An increase in capital inflow in the form of dependence (6) with GDP growth to a more than proportional (indicator θ in $k_{K^+}(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\theta}$ in a case $\theta \geq 0$) degree is associated with a general improvement in the investment climate in Russia with an increase in economic growth and an inflow of direct and portfolio investments and the expected positive effect of import substitution.

Substituting formulas (3)–(6) into (2), we obtain:

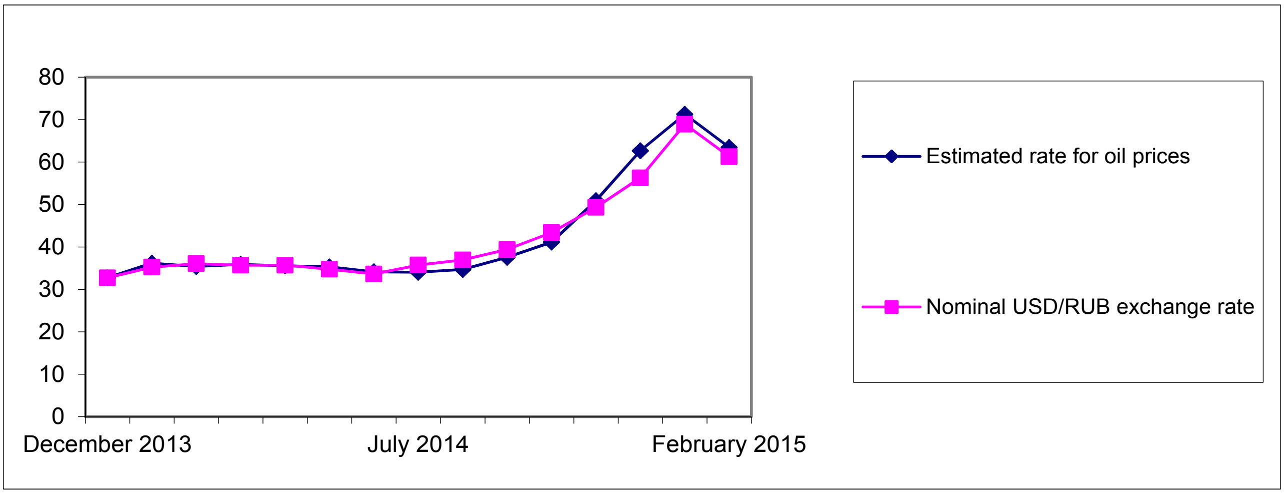

Fig. **Estimated and nominal US dollar/ruble exchange rates in 2013–2015**
Source: author's calculations, monthly data.

$$e_t = \frac{k_I P_t (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})(e^R_{t-1})^y + k_{K^-} P_t (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})(e^R_{t-1})^y}{P_t^* k_E (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\delta}(e^R_{t-1})^z + P_t^* k_{K^+}(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\theta}(e^R_{t-1})^z} =$$

$$= \frac{P_t (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})(e^R_{t-1})^y (k_I + k_{K^-})}{P_t^* k_E (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\theta}(e^R_{t-1})^z (k_E (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\delta-\theta} + k_{K^+})} =$$

$$= \frac{P_t (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})(k_I + k_{K^-})}{P_t^* (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\theta}(e^R_{t-1})^x (k_E (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\delta-\theta} + k_{K^+})} =$$

$$= \frac{P_t (k_I + k_{K^-})}{P_t^* (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\theta}(e_{t-1} {P^*_{t-1}}/{P_{t-1}})^x (k_E (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\delta-\theta} + k_{K^+})} = \qquad (7)$$

$$= \frac{(k_I + k_{K^-})}{({P_t^*}/{P_t})(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\theta}(e_{t-1})^x ({P^*_{t-1}}/{P_{t-1}})^x (k_E (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\delta-\theta} + k_{K^+})}.$$

Using the properties of the indicators $\delta \approx \theta$ and the greater dynamic stability of the averaged term $(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\delta-\theta}$ in comparison with the volatility of internal and external prices, in the medium term, we set the term as constant k':

$$\frac{(k_I + k_{K^-})}{(k_E (Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\delta-\theta} + k_{K^+})} = (k')^{x+1} = const\,.$$

We rewrite (7) as

$$e_t\,(e_{t-1})^x=\left(k'\frac{P_t}{P_t^*}Q_t^{-\theta/_{x+1}}\right)\left(k'\frac{P_{t-1}}{P_{t-1}^*}Q_{t-1}^{-\theta/_{x+1}}\right)^x .$$

After re-designating the exponent $\theta'=\theta/_{x+1}$ and the time division of the variables involved in the process, we dynamically extend the model to a multi-period case and obtain the time dependence of the ruble exchange rate on the system of the main accepted fundamental internal and external economic determinants:

$$e(t,Q(t),P(t),P^*(t))=e_t=$$
$$=k'\frac{P_t}{P_t^*}Q_t^{-\theta/_{x+1}}=k'\frac{P_t}{P_t^*}Q_t^{-\theta'} . \qquad (8)$$

For the purposes of model verification, one of the most suitable is the period of the financial crisis of 2014–2015, which is associated with the significant rapid depreciation of the ruble.

In this study, we will use the methodology and results of the author's work [22] (initial data have been updated). The consumer price index is used as the determinant of the *P* model, the real GDP index is used as the determinant of *Q* (according to the Federal State Statistics Service[1]), the price of the Intercontinental Exchange Brent crude oil is used as a determinant P_t^* (according to Bloomberg, information terminal).

The *Figure* (the author's calculations, monthly data) shows the dynamics of the calculated exchange rate of the ruble according to the main research formula (8) compared with the nominal exchange rate of the US dollar to the ruble at the end of the period (according to the data of the Bank of Russia[2]). As a result of numerical simulation by the least squares method with normalization of the nominal header, the parameter value is set $\theta'=0{,}45$.

The average value of the normalized deviations and the average of the absolute normalized deviations of the nominal and calculated rates were 3 and 0.3%, respectively, which confirms the high quality of the model (8).

MODELING RUBLE EXCHANGE RATE: LONG-TERM DYNAMICS

For the purpose of this study, it is necessary to make the following assumption — in the long term, the coefficient k_E in the functional dependence (3) ceases to be a constant and becomes a dynamic function $k_E(t)$, which is important for further modeling in the long term:

$$E_t=P_t^*k_E(t)(Q_t^{\frac{1}{x+1}}Q_{t-1}^{\frac{x}{x+1}})^{1+\delta}(e_{t-1}^R)^y . \qquad (9)$$

Let us discuss further the properties of the introduced function $k_E(t)$. Over the past three decades, changes in world prices for Russian export goods have had an extremely significant impact on the entire macroeconomic dynamics. And it is very important that this remains in the long term one of the fundamental basic drivers of Russia's macroeconomic

[1] URL: https://rosstat.gov.ru/folder/10705 (accessed on 11.02.2021).

[2] URL: http:// www.cbr.ru/statistics (accessed on 11.02.2021).

dynamics. Indeed, it is at this stage, in addition to our country, that real opportunities arise for restructuring the entire export structure for the coming years.

It should be noted here that the author, assessing this scenario as very likely, belongs to that part of the expert economic community that considers changes in the structure of Russian exports towards an increase in sales of intermediate products and its real diversification as an international competitive advantage of our country in the long term. When implementing this strategy, it is necessary to consider the global trends in the excess of the growth rates of prices for intermediate goods over the growth rates of prices for raw materials. In the long term, this will undoubtedly lead to a significant increase in the volume of currency in dollar prices, which will enter the domestic foreign exchange market in the form of receipts from the export of goods and services.

All of the above allows us to consider formula (9) exactly in the variant of imposing a constraint $k_E^{/}(t) > 0$. There is a strict increase in the function $k_E(t)$ with respect to *t*.

Likewise, in the long term, the investment climate in Russia is bound to improve. This should lead to a significant increase in capital inflows due to an increase in direct and portfolio investments that will flow to the domestic foreign exchange market, and allows one to consider formula (6) in the version of a strict increase $k_{K^+}(t)$ with respect to *t* and setting a limitation here $k_{K^+}^{/}(t) > 0$.

Therefore, the dependence of capital inflows must satisfy the following condition:

$$K_t^+ = f_{K_t^+}(k_{K^+}(t)^{\uparrow}, P_t^{*\uparrow}, Q_t^{\uparrow}, Q_{t-1}^{\uparrow}, e_{t-1}^{R\uparrow}).$$

Accordingly, based on this:

$$K_t^+ = P_t^* k_{K^+}(t)(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\theta}(e_{t-1}^R)^z. \quad (10)$$

Substituting formulas (4), (5), (9), (10) into (2), similarly to calculations (7), we obtain:

$$e_t = \frac{P_t(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})(k_I + k_{K^-})}{P_t^*(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{1+\theta}(e_{t-1}^R)^x (k_E(t)(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\delta-\theta} + k_{K^+}(t))} =$$
$$= \frac{P_t(k_I + k_{K^-})}{P_t^*(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\theta}(e_{t-1} P_{t-1}^* / P_{t-1})^x (k_E(t)(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\delta-\theta} + k_{K^+}(t))}. \quad (11)$$

Next, we introduce the function $K(t)$:

$\dfrac{(k_I + k_{K^-})}{(k_E(t)(Q_t^{\frac{1}{x+1}} Q_{t-1}^{\frac{x}{x+1}})^{\delta-\theta} + k_{K^+}(t))} = (K(t))^{x+1}$. (12) Formula (11) considering (12) in this case is transformed: $e_t(e_{t-1})^x = \left(K(t)\dfrac{P_t}{P_t^*} Q_t^{-\theta/x+1}\right)\left(K(t)\dfrac{P_{t-1}}{P_{t-1}^*} Q_{t-1}^{-\theta/x+1}\right)^x$.

Let us extend the model to a multi-period case and, after temporarily dividing the variables, we obtain a dynamic dependence of the ruble exchange rate over time on the main fundamental external and internal macroeconomic factors:

$$e_t = e(K(t), Q(t), P(t), P^*(t)) = \\ = K(t)\frac{P_t}{P_t^*} Q_t^{-\theta/x+1} = K(t)\frac{P_t}{P_t^*} Q_t^{-\theta'}. \quad (13)$$

As a result, it is important to note the key properties of the function $K(t)$ in formula (13): due to $\delta \approx \theta$ (since the "additional" inflow of capital for GDP growth should primarily serve the growth of exports of goods and services) sufficient stability in comparison with other members $(Q_{t-1}^{x/x+}Q_t^{1/x+1})$ a strict increase in internal functions $k_E(t)$ and $k_{K^+}(t)$ guarantees a strict decrease in the key function $K(t)$ (12) with respect to t: $K'(t) < 0$.

As a consequence, this indicates a tendency for the ruble to strengthen in the long term as compared to the equilibrium medium-term dynamics.

CONCLUSIONS

The ruble exchange rate is one of the most important factors that have a significant impact on the motivation of economic entities through the mechanism of forming international relative competitive advantages, on international trade flows, and by creating a favorable investment climate within the country on capital flows.

It is important to note that these economic aggregates are the determining factors in the dynamics of the exchange rate itself. At the same time, after the completion of the almost complete liberalization of the exchange rate and the transition of the Bank of Russia to the inflation targeting regime according to the classification of the International Monetary Fund, the regime of the Russian ruble exchange rate de facto began to be characterized by the independent floating of the national currency.

The above aspects determined the course of mathematical modeling of the ruble exchange rate and led to the development of the author's conceptual approach to modeling the dynamics of the equilibrium exchange rate based on international flows equilibrium exchange rate (IFEER).

Considerable attention is paid to the mathematical modeling of the long-term dynamics of the ruble exchange rate in comparison with the medium-term equilibrium dynamics and the mathematical analysis of internal functional relationships, on the basis of which a conclusion is made about the tendencies of the strengthening of the ruble exchange rate in the long term.

The paper presents data on the verification of the results of modeling medium-term equilibrium dynamics. At the same time, economic verification of internal relationships and model results in the long term will require further accumulation of economic statistics and may become the subject of future research.

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