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Multivariate Asymmetric GARCH Model with Dynamic Correlation Matrix

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ABSTRACT

This study examines the problem of modeling the joint dynamics of conditional volatility of several financial assets under an asymmetric relationship between volatility and shocks in returns (leverage effect). We propose a new multivariate asymmetric conditional heteroskedasticity model with a dynamic conditional correlation matrix (DCC-EGARCH). The proposed method allows modelling the joint dynamics of several financial assets taking into account the leverage effect in the financial markets. DCC-EGARCH model has two main advantages over previously proposed multivariate asymmetric specifications. It involves a substantially simpler optimization problem and does away with the assumption of conditional correlation time invariance. These features make the model more suitable for practical applications. To study the properties of the obtained estimators, we conducted a simulated data analysis. As a result, we found statistical evidence in favor of the developed DCC-EGARCH model compared with the symmetric DCC-GARCH process in case of considering data with the presence of the leverage effect. Further, we applied the proposed method to analyze the joint volatility of Rosneft stock returns and Brent oil prices. By estimating the DCC-EGARCH model, we found statistical evidence for both the presence of the leverage effect in the oil price data and the presence of the dynamic correlation structure between the time series, which motivates the practical application of the proposed method.

Keywords: financial markets; financial assets; volatility modelling; EGARCH; DCC-GARCH; leverage effect; conditional correlation; multivariate GARCH models; joint volatility dynamics

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INTRODUCTION

Volatility is one of the main indicators characterizing the behavior of assets in financial markets. This indicator is expressed as a standard deviation of the return of the considered financial instruments and is an indicator of the level of risk of assets or a portfolio of securities in the aggregate [1, 2]. For this reason, various financial market participants are interested in volatility modeling in order to conduct an effective risk management policy [3]. One of the most well-known methods for modeling conditional volatility is the family of GARCH processes. However, in the modern world, the hedging process is closely related to modeling the variety of assets included in a portfolio of securities, while one-dimensional GARCH processes allow us to consider the dynamics of assets only separately [4]. This reason served as a stimulus for the development of a class of multivariate GARCH models, which task is to jointly model the dynamics of the volatility of several financial instruments.

Since the main area of application of GARCH processes is modeling the dynamics of financial time series, as various modifications of GARCH models were developed, researchers set themselves the goal of integrating the behavior of financial assets into the methods being developed. One of the most interesting and widely studied stylized facts in financial markets is the asymmetric relationship between the return of assets and their volatility, also known in the literature as the leverage effect [5]. The essence of this feature is that the market reacts more inertially to negative shocks in returns than to positive ones [5, 6]. In the literature, there are several approaches explaining the causes of the leverage effect [6]. For example, according to [7, 8], negative shocks in returns lead to an increase in the financial leverage of issuing companies, which increases the risk level of issued shares and, as a result, leads to an increase in their volatility. In addition, the leverage effect may arise as a result of the cognitive characteristics of investors in accordance with the prospect theory of Kahneman and Tversky [9], people tend to perceive losses more critically, due to which, in the event of negative shocks to profitability, investors may resort to the mass asset disposal, thereby causing an increase in volatility.

Due to the fact that the standard GARCH model is a symmetric model and does not consider the leverage effect, over time, the authors developed asymmetric modifications of the GARCH models, the main contribution to the development of which was successfully made in studies [6, 10, 11].

However, the stratum devoted to the development of multivariate asymmetric GARCH processes is little studied in the modern literature, which is the subject of this study.

Existing methods include the asymmetric BEKK model[1] [12], the GJR[2]-BEKK [13] specification, as well as generalizations of the asymmetric EGARCH process to the multivariate case proposed in [14, 15]. However, these models are characterized by the presence of the "curse of dimensionality" phenomenon, since they require the simultaneous estimation of a large number of unknown parameters. In turn, the proposed multivariate EGARCH specifications [14, 15] contain a strict unrealistic assumption that the correlation matrix is constant over time, which makes it difficult to apply them to real data.

This study proposes an alternative asymmetric multivariate conditional heteroskedasticity model with a dynamic correlation matrix over time, hereinafter referred to as the DCC-EGARCH model. The proposed specification allows modeling the joint dynamics of the conditional volatilities of several assets with the possibility of considering the leverage effect. The developed method is implemented by adapting the asymmetric EGARCH process [6] to the multivariate case, using the DCC-GARCH specification [16] as the basis. The advantage of the proposed DCC-EGARCH model compared to analogs is the significantly lower complexity of the optimization problem due to the possibility of estimating the parameters using the two-step procedure of Engle [16], Newey and McFadden [17], which avoids the "curse of dimensionality". In addition, due to the use of the DCC specification, the proposed method weakens the assumption about the invariance of the conditional correlation matrix with respect to time, which is typical for earlier generalizations of the EGARCH processes to the multivariate case proposed in [14, 15].

To study the properties of the proposed method, this paper uses the simulated data analysis. As a result, statistical evidence was found for the advantage of using the DCC-EGARCH model over the symmetrical DCC-GARCH process when considering leveraged data. In particular, based on the analysis of simulations, the developed specification was able to provide more efficient estimators compared to the classical DCC-GARCH model. In addition, the proposed DCC-EGARCH specification is being used to study real data, which are Rosneft stocks returns and the time series of changes in Brent oil prices. Based on the results of the analysis, statistical evidence was found both in favor of the presence of a significant effect of asymmetry in the case of considering the time series of oil prices and in favor of the presence of a dynamic correlation structure between assets, which justifies the use of the proposed method on real data.

1. LITERATURE REVIEW

1.1. Exponential Generalized Autoregressive Conditional Heteroskedasticity Model (EGARCH)

One of the best known asymmetric GARCH processes is the exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model proposed by Nelson [6]. Let ε_t be a random shock in the return of the asset under consideration. Assume that σ_t^2 is the conditional variance of ε_t, and therefore cannot be negative. A similar constraint in the GARCH model was met by defining the unconditional variance as a linear combination of positive random variables using positive coefficients.

[1] The abbreviation BECK consists of the first letters of the names of its authors: Baba, Engel, Kraft and Kroner [21].

[2] Similarly to BEKK, the model is designated by the names of the authors: Glosten, Jagannathan and Runkle [10].

When developing the EGARCH model, Nelson [6] proposed another elementary transformation to fulfill this condition: representing the logarithm of the conditional variance as a linear function of time and previous values of independent, equally distributed random shocks z_t. The application of this specification provides non-negative conditional variance values without the need to impose any restrictions on the process parameters. Further, in order to be able to consider the asymmetric relationship between the returns and volatility of a financial asset in the model, it became necessary to set the dependence of the conditional variance logarithm in such a way that its value depended both on the magnitude of the shock z_t, and on its sign [6]. As an appropriate specification, Nelson [6] proposed that the logarithm of the conditional variance be given as a linear combination of z_t and $|z_t|$. Then the final specification of the EGARCH process can be written using the following system of equations:

$$\varepsilon_t \mid \psi_{t-1} \sim N\left(0, \sigma_t^2\right), \quad (1)$$

$$y_t = \mu + \varepsilon_t, \quad (2)$$

$$\varepsilon_t = z_t \sigma_t, \quad (3)$$

$$\ln\left(\sigma_t^2\right) = \omega + \alpha \mid z_{t-1} \mid + \gamma z_{t-1} + \beta \ln\left(\sigma_{t-1}^2\right), \quad (4)$$

where ψ_t denotes the information available in the period t, where $t \in N$. Random shocks z_t are independent and identically distributed standard normal random variables. The estimated parameters are μ, ω, α, γ and β. In this case, the coefficients ω, α, γ and β, which are responsible for the dynamics of the conditional variance, are of the greatest interest.

A feature of the model is the presence of a coefficient γ, responsible for the leverage effect, which allows the specification of the conditional variance process to react asymmetrically to positive and negative shocks in financial asset returns [6]. Thus, if $\gamma > 0$, then the increase of $ln\left(\sigma_{t+1}^2\right)$ is positive when the return value is higher than its expected value; and vice versa — if the return value turned out to be lower than the expected value, then the increase in volatility will be less than in the first case. Similarly, if $\gamma < 0$, then the increase in conditional variance will be more significant in the case of negative shocks of return and weaker if the return value exceeds the expected value [6].

Similar to other GARCH family models, the unknown parameters in the EGARCH specification are usually estimated using the maximum likelihood method under the assumption of a normal distribution of random shocks. Then the likelihood function to be maximized takes the following form:

$$L\left(\mu, \omega, \alpha, \beta, \varepsilon\right) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{\varepsilon_t^2}{2\sigma_t^2}\right),$$

where variance σ_t^2 is described by equation (4) of the EGARCH model, and T is the number of time periods present in the data.

1.2. Dynamic Conditional Correlation Model (DCC-GARCH)

A significant contribution to the development of multivariate GARCH processes was made by Engle's study [16], in which he proposed a generalized model of conditional heteroskedasticity with a dynamic correlation matrix (DCC[3]-GARCH). This specification is a generalization of the CCC[4]-GARCH [18], weakening the premise of the invariance of correlation over time, which, according to [4, 16, 19], is rigid and may often not agree with real data.

Let us denote the returns N of different assets as a vector y_t. Then the DCC-GARCH process has the following specification:

$$y_t = \mu + \varepsilon_t, \ \varepsilon_t \sim N\left(0, H_t\right),$$

$$\varepsilon_t = H_t^{1/2} z_t,$$

$$H_t = D_t R_t D_t,$$

where H_t is the $N \times N$ conditional covariance matrix of ε_t at time t. D_t is the $N \times N$ diagonal matrix of conditional standard deviations ε_t at

[3] Dynamic Conditional Correlation.
[4] Constant Conditional Correlation.

time t. The matrix R_t is a time-dynamic $N \times N$ correlation matrix of standardized residuals at time t, and z_t — a vector of independent, identically distributed, standard normal random variables [16].

Also note that the elements of the diagonal matrix D_t — conditional standard deviations determined by univariate GARCH processes:

$$\sigma_{ti}^2 = \omega_i + \alpha_i \varepsilon_{(t-1)i}^2 + \beta_i \sigma_{(t-1)i}^2,$$

where the index i denotes the number of assets, $i \in \{1,\ldots,m\}$. The parameters α and β denote the contribution of the ARCH and GARCH parts to the formation of the conditional variance, respectively.

Note that the matrix R_t is a conditional correlation matrix of standardized residuals ϵ_t, which implies [16]:

$$\epsilon_t = D_t^{-1} \varepsilon_t \sim N(0, R_t).$$

To comply with the condition of strict positive definiteness of the covariance matrix and ensure correlation values that do not exceed one in absolute value, Engel [16] proposed to specify the matrix R_t as follows:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1},$$

$$Q_t = (1-a-b)\bar{Q} + a\epsilon_{t-1}\epsilon_{t-1}^T + bQ_{t-1},$$

where $\bar{Q}$ is the unconditional covariance matrix of standardized residuals ϵ_t,[5] a and b are the estimated parameters, and Q_t^* — is the diagonal matrix consisting of the square roots of the diagonal elements of the matrix Q_t:

$$Q_t^* = \begin{pmatrix} \sqrt{q_{1,1,t}} & 0 & \cdots & 0 \\ 0 & \sqrt{q_{2,2,t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sqrt{q_{n,n,t}} \end{pmatrix}.$$

Also note that in order to fulfill the condition of positive definiteness of the conditional covariance matrix H_t the following restrictions are imposed on the parameters a and b [16]:

$$a \geq 0, \quad b \geq 0,$$

$$a + b < 1.$$

As a rule, the parameters of the DCC-GARCH model are estimated via the maximum likelihood method under the assumption of a joint normal distribution of random shocks. Then the logarithm of the maximized likelihood function can be written as follows:

$$\ln L = -\frac{1}{2}\sum_{t=1}^{T}\begin{pmatrix} n \times \ln(2\pi) + 2\ln(|D_t|) + \\ + \varepsilon_t^T D_t^{-1} D_t^{-1} \varepsilon_t - \epsilon_t^T \epsilon_t + \ln(|R_t|) + \\ + \epsilon_t^T R_t^{-1} \epsilon_t \end{pmatrix}.$$

It is easy to see that in the case of a large covariance matrix; the direct maximization of the likelihood function becomes a difficult task [16]. As an alternative method for obtaining parameter estimates, the two-step procedure proposed by Newey and McFadden [17] is used. The use of this method makes it possible to significantly simplify the optimization problem while maintaining the consistency of the estimators.

Let us designate the vector of estimated parameters of the matrix D as θ, and the vector of matrix parameters R as Δ: i.e. $\theta = (\mu, \omega, \alpha, \beta)$, $\Delta = (a, b)$. Then the logarithm of the likelihood function can be represented as the sum of the contributions of volatility and correlation [16, 20]:

$$\ln L(\theta, \Delta) = \ln L_V(\theta) + \ln L_C(\theta, \Delta).$$

The first step of the procedure is to maximize the part of the likelihood function that reflects the contribution of volatility, i.e. $\hat{\theta} = \operatorname{argmax}\{\ln L_V(\theta)\}$. At the same time, we note that maximization $\ln L_V(\theta)$ implies a separate estimation of the parameters of univariate GARCH processes for each of the assets. At the second step of the procedure, the second part of the likelihood function is maximized, due to which estimates of the parameters a and b, which are responsible for the dynamics of the change in the conditional

[5] An estimate of the unconditional covariance matrix may be obtained as $\bar{Q} = \frac{1}{T}\sum_{t=1}^{T} \epsilon_t \epsilon_t^T$.

correlation, can be obtained. In this case, instead of the vector of true parameters θ, its estimate found at the first step is substituted: $max \ln L_C(\theta, \Delta)$. Note that under certain regularity conditions [16, 17], obtaining consistent estimators at the first step ensures that consistent estimates are also obtained at the second step of the method.

1.3. Asymmetric Multivariate GARCH models

Due to the presence of a stylized fact about an asymmetric relationship between the volatility of assets and returns in financial markets [5], the development of multivariate GARCH models entailed the need to integrate them into account for asymmetric dependence by extending asymmetric univariate GARCH models to the multivariate case.

One of the first such generalizations was the asymmetric BEKK model proposed by Kroner and Ng [12]. This specification repeats the BEKK model [21], except that the conditional covariance matrix dynamics equation also includes an additional quadratic form that determines the asymmetry effect. This element depends on the pairwise product of vectors that reflect negative shocks in returns:

$$H_t = CC^T + A^T \varepsilon_{t-1} \varepsilon_{t-1}^T A + B^T H_{t-1} B + G^T \eta_{t-1} \eta_{t-1}^T G,$$

where $\eta_{it} = max[0, -\varepsilon_{it}]$, and $\eta_t = [\eta_{1t}, \ldots, \eta_{Nt}]^T$, matrices C, A, B and G are $N \times N$ matrices of estimated parameters that satisfy the following conditions:

- C — the lower triangular matrix;
- A, B and G — diagonal matrices, where the matrix G reflects the effect of the asymmetric response of variance to shocks in returns.

Note that the imposed restriction on the diagonal form of the matrices under consideration gives rise to the premise that the variances depend only on the eigen squares of the residuals, and the covariances depend solely on the past values of the cross products of the residuals, which may not agree with the real data [21–23]. However, if this assumption is weakened, the method suffers from the "curse of dimensionality" phenomenon. That is, in the case of considering a large number of time series, the optimization problem is characterized by high complexity due to the large number of estimated parameters, which is a disadvantage of this type of model in comparison with the DCC specification. In addition to the presented asymmetric BEKK model [12], its modification GJR-BEKK [13] was subsequently developed, which contains a binary switch variable that reflects the impact of positive and negative shocks on conditional volatility. However, using the GJR-BEKK specification in a similar way to the asymmetric BEKK model leads to the "curse of dimensionality" problem.

Alternative specifications adapting univariate asymmetric GARCH models to the multivariate case have been proposed by Koutmos and Booth [14] and Jane and Ding [15]. In these studies, generalizations of the asymmetric EGARCH process to the multivariate case were presented. At the same time, both the model [14] and [15] require the fulfillment of a rigid premise that the conditional correlation matrix is invariant with respect to time. This premise may often be inconsistent with real data [4, 16], which is a significant limitation of the proposed methods. For example, statistical evidence was found in favor of a difference in the correlation between the US S&P 500 and the Japanese Nikkei 225 in the periods before and after the global financial crisis, since the correlation between these financial time series before the crisis was determined by normal market conditions, while after the crisis these conditions were violated [4].

This study proposes a generalization of the EGARCH model to the multivariate case using the DCC specification. The advantage of the developed method in comparison with analogs lies in the possibility of taking into account the leverage effect, weakening the premise of the invariance of the correlation structure with respect to time. At the same time, the use of a two-step procedure for estimating the parameters of the DCC specification makes it possible to provide a significantly lower complexity of the optimization problem without imposing additional restrictions, in contrast to the previously proposed asymmetric BEKK specifications.

2. METHODOLOGY

2.1. The data generating process in the DCC-EGARCH model

Let the logarithmic return of each of the considered N assets at the moment of time t be denoted as a vector y_t. Then the equation of return takes the form:

$$y_t = \mu + \varepsilon_t, \varepsilon_t \sim N(0, H_t).$$

Specifying random shocks based on the classical GARCH process [24] and applying the DCC specification [16] to pass to the multivariate case, we obtain:

$$\varepsilon_t = H_t^{1/2} z_t,$$

$$H_t = D_t R_t D_t,$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1},$$

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha \epsilon_{t-1}\epsilon_{t-1}^T + \beta Q_{t-1},$$

where all notations are similar to the DCC-GARCH process.

A feature of the DCC-EGARCH model presented in this paper is that the D_t matrix elements are generated not by using univariate symmetric GARCH processes, but by using an asymmetric EGARCH model [6], i.e.

$$\sigma_t = \sqrt{\exp\left(\omega + \alpha |z_{t-1}| + \gamma z_{t-1} + \beta \ln\left(\sigma_{t-1}^2\right)\right)},$$

where the parameter γ reflects the effect of asymmetry.

In view of the fact that maximizing the total likelihood function is a complex optimization problem due to a large number of estimated parameters, the current study uses a two-step estimation procedure proposed in [16, 17]. Due to its implementation, an increase in the number of estimated parameters because of considering the asymmetric effect of variance on shocks in returns does not lead to even higher computational complexity. This feature of the proposed method is an advantage relative to asymmetric BEKK models [12, 13], which are characterized by high optimization complexity due to a large number of estimated parameters.

2.2. Two-Step Estimation Procedure for the DCC-EGARCH Model

According to [16], the first step in the estimation procedure for a model with dynamic correlation is to maximize the logarithm of the likelihood function, which reflects the contribution of volatilities. Note that in this study, the EGARCH model is adapted to the bivariate[6] case. Then the first step is to evaluate two univariate EGARCH processes, assuming a normal distribution of random shocks:

$$\ln L_V(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{N}\left(\ln(2\pi) + \ln\left(\sigma^2_{i,t}\right) + \frac{\varepsilon^2_{i,t}}{\sigma^2_{i,t}}\right),$$

where $\sigma^2_{i,t} = \exp\left(\omega + \alpha |z_{t-1}| + \gamma z_{t-1} + \beta \ln\left(\sigma^2_{t-1}\right)\right)$, and θ is the vector of estimated parameters of the EGARCH process: $\theta = (\mu, \omega, \alpha, \beta, \gamma)$.

The second step of the estimation procedure for the DCC model [16] involves maximizing the part of the likelihood function that reflects the contribution of the correlation, assuming a joint normal distribution of random shocks, i.e.:

$$\ln L_C(\theta, \Delta) = -\frac{1}{2}\sum_{t=1}^{T}\left(-\epsilon_t^T \epsilon_t + \ln(|R_t|) + \epsilon_t^T R_t^{-1} \epsilon_t\right),$$

where Δ is the vector of parameters responsible for the changes in the correlation matrix over time: $\Delta = (a, b)$.

Note that in the case of adapting the EGARCH model to the multivariate case using the DCC specification, the changes relative to the symmetric DCC-GARCH model concern only the implementation of the first step of the procedure. Thus, at the first step, the parameters of each of the univariate EGARCH processes are estimated, including the coefficient γ, which is responsible for the leverage effect. By finding these estimates in the first step, matrices of estimated standard deviations D_t, a matrix of estimated values of standardized residuals, as well as an estimate of the unconditional covariance matrix $\bar{Q} = E(\epsilon_t \epsilon_t^T)$ may be calculated [16]. Then the second step of the procedure for estimating the asymmetric

[6] This study focuses on the analysis of a bivariate specification. In this case, the specifications of N-dimensional models and the corresponding likelihood functions are similar to the two-dimensional case.

DCC-EGARCH model is to maximize the likelihood function with the substitution of the estimates found in the first step, due to which estimates of the parameters a and b, which determine the change in the conditional correlation matrix over time, may be found.

3. ANALYSIS OF THE SIMULATED DATA

3.1. Description of the simulated data

In order to study the properties of obtained estimators using the developed DCC-EGARCH method, this study uses the analysis of simulated data. The generated data is a bivariate sample consisting of 300 observations,[7] and the true simulation parameters are given in *Table 1*. Note that the first five parameters are different for each of the two processes, since they determine the dynamics of the conditional variance for each equation, following the specification of EGARCH processes. At the same time, the parameters a and b are given for the entire process as a whole, since they determine the change in the correlation matrix of shocks over time. The values of the true parameters were chosen in compliance with the stationarity conditions of the process [6, 16, 24]. The parameters responsible for the leverage effect and the dynamics of the correlation matrix were determined to be sufficiently large in absolute value in order to ensure their significant effect on the generated process, while the stationarity conditions were also met.

For the purpose of preliminary analysis of the generated data according to the DCC-EGARCH process, the following is a graphical analysis of the dynamics of the true conditional variance and the time-varying correlation between random shocks. *Fig. 1* shows the dependence of volatility on the previous value of random shocks for each of the two considered time series.

Note that the asymmetry effect is clearly present on the graph in accordance with the data generation process of the DCC-EGARCH model. Volatility increases more inertially with negative shocks in returns than with positive ones. Accordingly, it is expected that by applying the proposed DCC-EGARCH model, the leverage effect present in the data can be captured.

Table 1

True simulation parameters

Parameter	First process	Second process
μ	0.5	0.3
α	0.15	0.25
β	0.7	0.5
ω	0.001	0.005
γ	–0.4	–0.3
a	0.5	
b	0.2	

Source: compiled by the authors.

Fig. 2 shows the dynamics of changes in the correlation over time between the series under consideration. Note that for the chosen true values of the parameters a and b the range of correlation change is quite large,[8] while the dynamics of its change are very intense.

3.2. Comparison of DCC-EGARCH with symmetric DCC-GARCH model

To analyze the advantage of the proposed DCC-EGARCH model, this section compares the symmetric DCC-GARCH specification and the asymmetric DCC-EGARCH specification. The purpose of this analysis is to identify the advantages of considering the asymmetry effect in the case of applying models on data characterized by the presence of a leverage effect. That is, it is important to find out how critical it is to apply a symmetric model to data with asymmetric effects and, therefore, whether there is a need to develop and apply asymmetric specifications for multivariate GARCH models.

Table 2 shows the average estimates of the unknown parameters over 100 simulations

[7] Such a sample size is due to the approximation to the real conditions that arise when evaluating financial time series, which, as a rule, have a large number of structural breaks in their structure, which often does not allow using larger samples.

[8] $\rho \in [-0.80; 0.94]$.

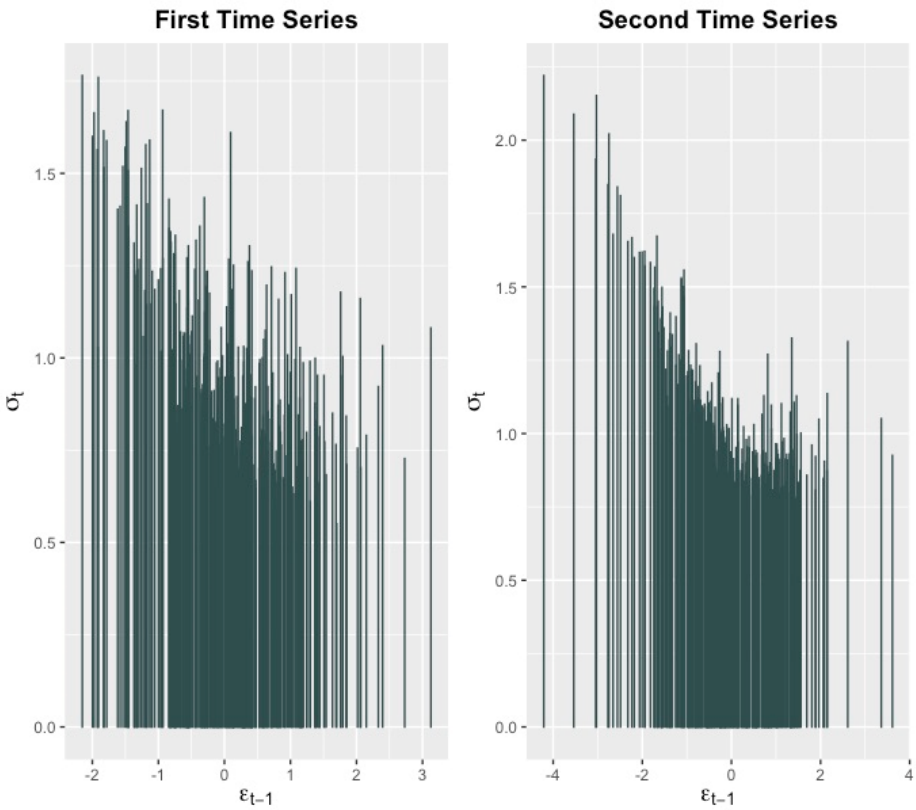

Fig. 1. **Volatility response to shocks in returns**

Source: compiled by the authors.

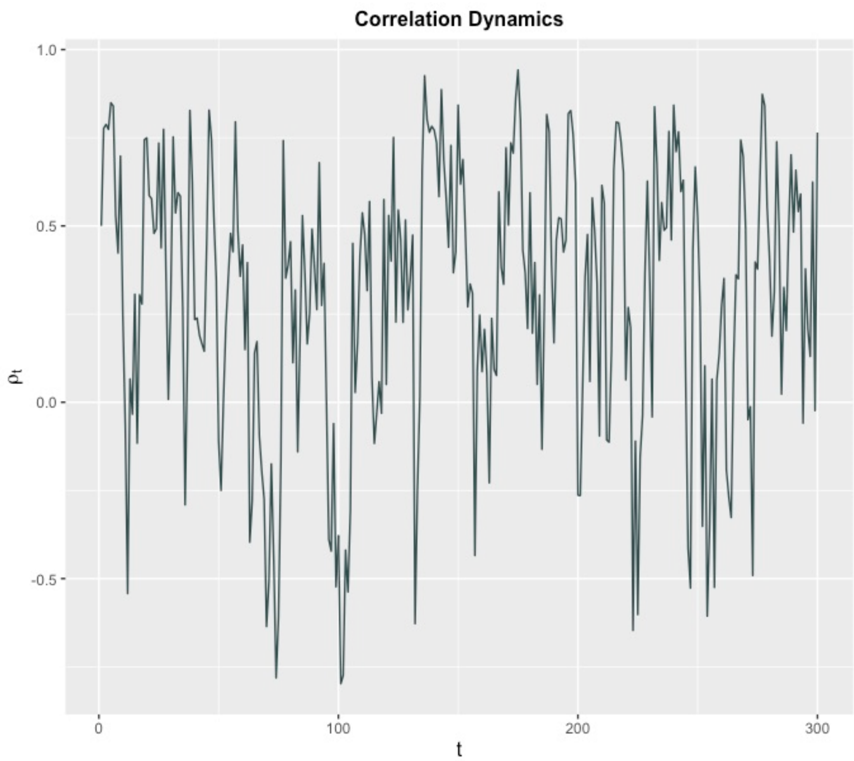

Fig. 2. **Dynamics of correlation over time**

Source: compiled by the authors.

Table 2

The comparison of DCC-EGARCH and DCC-GARCH models

	DCC-EGARCH	RMSE	DCC–GARCH	RMSE
$\widehat{\mu}_1$	0.50043	0.00989	0.57556	0.07616
$\widehat{\alpha}_1$	0.15650	0.01949	0.23909	0.09160
$\widehat{\beta}_1$	0.69698	0.01449	0.41665	0.28679
$\widehat{\omega}_1$	0.00422	0.00775	0.42178	0.42301
$\widehat{\gamma}_1$	−0.38740	0.01844	–	–
$\widehat{\mu}_2$	0.30147	0.01049	0.34372	0.04506
$\widehat{\alpha}_2$	0.24293	0.02145	0.19880	0.05745
$\widehat{\beta}_2$	0.49663	0.02966	0.27136	0.24751
$\widehat{\omega}_2$	0.00680	0.00894	0.58725	0.58823
$\widehat{\gamma}_2$	−0.29459	0.01483	–	–
$\hat{a}$	0.50096	0.01049	0.46280	0.03975
$\hat{b}$	0.20832	0.01703	0.24197	0.04593
AIC	54 513.010	–	56 382.13	–
BIC	54 599.534	–	56 454.239	–

Out-of-sample forecast quality						
	DCC-EGARCH			DCC-GARCH		
Period	$RMSE_{\sigma_1}$	$RMSE_{\sigma_2}$	$RMSE_{\rho}$	$RMSE_{\sigma_1}$	$RMSE_{\sigma_2}$	$RMSE_{\rho}$
h = 1	0.03634 (0.88)	0.02700 (0.78)	0.01979 (0.74)	0.18460 (0.12)	0.12051 (0.22)	0.05066 (0.26)
h = 5	0.22242 (0.56)	0.17189 (0.43)	0.26866 (0.52)	0.24088 (0.44)	0.17434 (0.57)	0.30764 (0.48)

Note: in parentheses are the proportions of simulations in which the model was characterized by a lower value of the RMSE criterion relative to the alternative model.

Source: compiled by the authors.

obtained using both the proposed DCC-EGARCH model and the symmetric DCC-GARCH model, and the average values of the information criteria AIC and BIC for each of the assumed technical characteristics. Simulation averaged RMSE values are also presented separately to assess the quality of the out-of-sample forecast of conditional volatilities and correlations in each of the models. In addition, simulation averages of RMSE are also given for all coefficient estimates.

Based on the *Table 2*, it can be concluded that the asymmetric DCC-EGARCH model has a significant advantage over the symmetric specification. The leverage effect captured by the presented method provides a significantly higher accuracy of the coefficient estimates compared to the DCC-GARCH specification, which shows a significant bias of the estimates from the true parameters. In addition, the significant advantage of the DCC-EGARCH model is also evidenced by the values of the information criteria AIC and BIC, which are significantly lower compared to the DCC-GARCH method.

Based on the average values of the RMSE criterion calculated from coefficient estimates, we note that it is significantly lower in the case of considering an asymmetric model, which indicates in favor of obtaining more efficient estimators by the DCC-EGARCH method compared to a symmetrical specification. It is also important to note the significantly higher predictive power of the asymmetric model.

In particular, the analysis considered out-of-sample forecasts of conditional volatility and correlation for 1 and 5 steps ahead. The RMSE criterion was used as an indicator of the quality of forecasts. Additionally, in parentheses are the proportions of simulations for which the RMSE value of the considered model turned out to be less than the alternative specification. *Table 2* shows that the DCC-EGARCH model has a higher predictive power, as evidenced by the lower average value of the RMSE criterion relative to the DCC-GARCH model for each of the periods. In particular, it is worth highlighting the significant superiority of the quality of the asymmetric model in the case of considering one-period forecasts. Thus, based on the RMSE, the symmetric model is several times inferior to DCC-EGARCH in the case of considering a one-period forecast of conditional volatilities for each series. The advantage of the DCC-EGARCH model is also evidenced by the proportion of simulations for which the RMSE value turned out to be lower in the asymmetric model compared to the DCC-GARCH specification. Thus, for the asymmetric model, this share is in most cases much higher, which testifies in favor of the greater accuracy of the out-of-sample forecast of conditional volatilities. It is important to note that the presented method also has a higher quality of conditional correlation prediction, as evidenced by the lower corresponding values of the RMSE criterion, and the proportion of simulations for which this indicator turned out to be lower in the case of considering an asymmetric specification.

Based on the results of the analysis, it can be concluded that the asymmetric multivariate DCC-EGARCH model has a significant advantage over the DCC-GARCH model when considering data characterized by the presence of a leverage effect. By providing less biased and more efficient estimators, the developed method is characterized by both a higher quality of predictive power and the quality of the model itself, as evidenced by lower values of information criteria. Based on this, the development and application of the presented asymmetric multivariate DCC-EGARCH model is justified, since the presence of the leverage effect introduces significant changes to the data generation process, which causes the lack of stability of the symmetric DCC-EGARCH model estimates for the presence of the effect of asymmetry.

4. APPLICATION TO REAL DATA

After examining the properties of estimators and comparing methods using simulated data, this section applies the proposed DCC-EGARCH model to real data analysis. The main task of the analysis is to study time series for the presence of the effect of an asymmetric response of variance to shocks in returns, and to evaluate the conditional correlation between them.

4.1. Data Description

As a sample, the stock returns of Rosneft Oil Company PJSC are used, and time series of changes in prices for Brent crude oil.[9] The considered time period covers the interval from 05.01.2016 to 13.03.2018.[10] The prices under consideration are the daily closing prices, due to which the sample size is 550 observations. The database source for each time series is Investing.com.[11]

Based on *Fig. 3*, we can conclude that the dynamics of changes in the studied series are similar. For this reason, it can be assumed that the considered time series are correlated with each other, which requires the use of multivariate GARCH models to assess the joint dynamics of their conditional volatility. The assumption of a correlation between the considered time series is due to the fact that the financial results of oil companies are highly dependent on oil prices. At the same time, the correlation structure may change over time due to changes in various market conditions.

4.2. Econometric analysis

To study the considered series of returns for the presence of leverage effect, and estimates of

[9] The oil prices are the prices of futures contracts with the nearest expiration period.

[10] The considered interval was chosen due to the lack of structural breaks on it.

[11] Investing.com. URL: https://www.investing.com/commodities/brent-oil-historical-data.

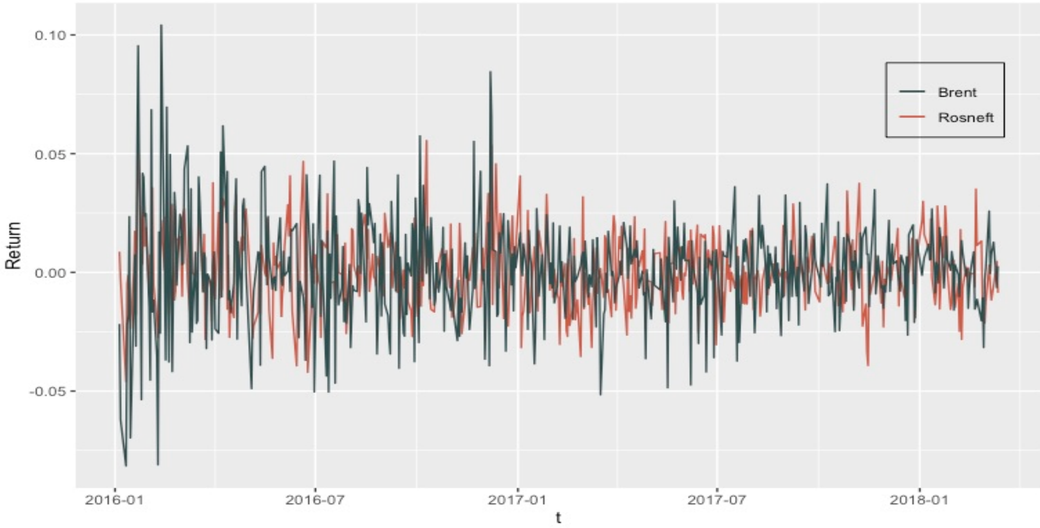

Fig. 3. **Dynamics of Rosneft stock returns and changes in Brent oil prices**

Source: compiled by the authors.

the conditional correlation between assets, in this study two different models were evaluated: the asymmetric DCC-EGARCH model with the possibility of considering the leverage effect, and its limited symmetrical version, hereinafter referred to as the DCC-EGARCH-R model. Note that the limited version differs from the presented DCC-EGARCH model by the lack of a coefficient reflecting the contribution of the leverage effect to the conditional variance equation for each of the assets, while the functional form remains similar to the DCC-EGARCH. Thus, the conditional variance equation in the DCC-EGARCH-R specification takes the following form:

$$\sigma_t = \sqrt{\exp\left(\omega + \alpha\,|z_{t-1}| + \gamma z_{t-1} + \beta \ln\left(\sigma_{t-1}^2\right)\right)}.$$

Note that DCC-EGARCH-R is nested within the DCC-EGARCH model, which allows directly evaluating the contribution of the leverage effect when comparing these two specifications.[12] The results of the evaluation of each of the models are presented in *Table 3*.

First of all, we note that, according to the results of the evaluation of the DCC-EGARCH model, a significant negative estimate of the coefficient $\hat{\gamma}_{Brent}$, responsible for the leverage effect was obtained. This result argues for the presence of an asymmetric effect of dispersion on shocks in returns in the case of considering the time series of oil prices. Note that the estimate of the coefficient $\hat{\gamma}_{Brent}$ is negative, which agrees with the ideas of [5–8]. This observation means that the variance reacts more inertially to negative shocks in returns than to positive ones since financial market participants tend to perceive negative shocks as more critical [5]. At the same time, we note that in the case of considering the time series of the Rosneft stock returns, no statistical evidence was found in favor of the asymmetry effect. This conclusion is due to the insignificance of the coefficient estimate $\hat{\gamma}_{Rosneft}$, obtained by applying the DCC-EGARCH model, which indicates in favor of the same change in conditional volatility under positive and negative shocks. In other words, financial market participants tend to equally perceive multidirectional shocks in returns of the asset under consideration. Thus, according to the results of the evaluation of the asymmetric DCC-EGARCH model, the presence of the leverage effect was statistically revealed when considering the time series of prices for Brent oil and its absence in the data of Rosneft.

It is also interesting to compare the evaluation results of the symmetric DCC-EGARCH-R model with the presented asymmetric DCC-EGARCH specification. Despite the fact that a significant

[12] The DCC-EGARCH-R specification was chosen instead of the DCC-GARCH model to highlight the leverage effect on real data since the DCC-EGARCH-R model has the same functional form as the proposed DCC-EGARCH method.

Table 3

Real data model estimation results

	DCC-EGARCH	DCC-EGARCH-R
$\hat{\mu}_{Rosneft}$	0.00073 (0.00065)	0.00070 (0.00062)
$\hat{\alpha}_{Rosneft}$	0.50200*** (0.13114)	0.50951*** (0.13285)
$\hat{\beta}_{Rosneft}$	0.79592*** (0.09335)	0.79121*** (0.09612)
$\hat{\omega}_{Rosneft}$	−1.67902** (0.76465)	−1.71758** (0.78732)
$\hat{\gamma}_{Rosneft}$	−0.01354 (0.05421)	–
$\hat{\mu}_{Brent}$	0.00038 (0.00077)	0.00077 (0.00075)
$\hat{\alpha}_{Brent}$	0.36179*** (0.05603)	0.38123*** (0.05822)
$\hat{\beta}_{Brent}$	0.91751*** (0.02163)	0.91282*** (0.02289)
$\hat{\omega}_{Brent}$	−0.64092*** (0.16855)	−0.67710*** (0.17811)
$\hat{\gamma}_{Brent}$	−0.07114* (0.04075)	–
$\hat{a}$	0.01929 (0.01253)	0.01875 (0.01193)
$\hat{b}$	0.96243*** (0.02802)	0.96389*** (0.02593)
AIC	– 5,707.344	– 5,708.454
BIC	– 5,655.625	– 5,665.355

Note: *** – p < 0.01, ** – p < 0.05, *– p < 0.1, estimates of standard errors are in parentheses.

Source: compiled by the authors.

leverage effect was obtained, the symmetric DCC-EGARCH-R model turned out to be better than the asymmetric counterpart, as evidenced by slightly lower values of the AIC and BIC information criteria in the case of considering the symmetric model. This result is unexpected due to the presence of a significant estimate of the coefficient $\hat{\gamma}_{Brent}$, which is responsible for the leverage effect in the DCC-EGARCH model. However, it is important to note the rather small absolute value of the obtained coefficient estimate compared to the contributions of the ARCH and GARCH parts.[13] Other things being equal, a relatively small value of this coefficient can lead to a negligible influence of the leverage effect on the dynamics of the conditional variance compared to the parts of ARCH and GARCH.

The next important step in the analysis is to consider the estimates of the parameters responsible for the change in the conditional correlation matrix over time between the studied time series. Table 3 shows that in both the asymmetric DCC-EGARCH model and the DCC-EGARCH-R specification, the parameter estimate b is statistically significant at any reasonable level, while the coefficient estimate a is negligible. To test the hypothesis about the presence of a dynamic correlation between the time series under consideration, a likelihood ratio (LR) test was carried out for the joint significance of the

[13] Estimates of the coefficients α and β.

parameters that determine the dynamics of the change in the conditional correlation. As a result of the verification, the null hypothesis was rejected at a significance level of 5%, which indicates in favor of the presence of a correlation structure dynamic in time between the assets under study. This conclusion justifies the feasibility of using the multivariate DCC-EGARCH specification to be able to take into account the dynamic conditional correlation between assets.

To test the hypothesis of a normal distribution of shocks, following [25, 26], the Kolmogorov-Smirnov test and the Shapiro-Wilk test were applied to estimates of standardized shocks. For both series, in both models under consideration, the hypothesis of a normal distribution was rejected at a significance level of 1%. Evidence that the marginal distributions of shocks (separately for each series) deviate from normal also suggests that the assumption of a multivariate normal distribution of shocks is likely not to hold. However, there is evidence in the literature in favor of the stability of the GARCH model estimates against the violation of the normal distribution assumption [27]. To test the stability of the results obtained in the study, each of the two series was evaluated using a univariate EGARCH model under different assumptions about the distribution of random shocks. Student's *t*-distribution, non-centered Student's *t*-distribution and generalized normal distribution (GED) were used. The values and signs of the coefficients remained the same, which indicates the stability of the result obtained against the violation of the assumption of normality. Checking the stability of the estimates a and b requires weakening the assumption not only about the marginal normality of shocks but also about the fact that the relationship between shocks is described using a Gaussian copula. The implementation of the corresponding model is potentially interesting, but technically difficult, which remains for further research.

CONCLUSIONS

In this study, a multivariate asymmetric DCC-EGARCH model was proposed. The developed method makes it possible to evaluate the joint dynamics of conditional volatility and correlation of several assets with the possibility of taking into account the influence of leverage effect in financial markets. The proposed method is implemented by generalizing the asymmetric EGARCH model to the multivariate case using the multivariate DCC-GARCH specification as the basis. The advantages of this approach lie in the weakening of the assumption about the invariance of the correlation matrix with respect to time, and in the significant simplification of the optimization problem due to the use of a two-step estimation procedure. These features justify the development of the considered model since the previous multivariate asymmetric BEKK-GARCH models [12, 13] were characterized by the "curse of dimensionality" phenomenon, and the existing adaptations of the EGARCH process to the multivariate case [14, 15] assumed that the correlation matrix is constant in time.

It is important to note that the properties of the estimators of the proposed method were studied using the simulated data analysis. As a result, statistical data were found in favor of the DCC-EGARCH method, which provides more efficient estimators compared to the symmetric DCC-GARCH model when considering the data generation process with leverage effect. In addition, the proposed method was able to provide a higher quality of out-of-sample forecasts for 1 and 5 periods ahead.

After analyzing the simulated data using the DCC-EGARCH method, we studied the joint dynamics of conditional volatility and the correlation of the Rosneft stocks and Brent oil prices. As a result of the analysis, statistical evidence was found in favor of the asymmetry effect in the data presented by oil prices, which justifies the use of the asymmetric DCC-EGARCH specification. However, despite the significance of the leverage effect, the use of a symmetrical analog showed slightly lower values of information criteria compared to the presented method, which may be due to the weak influence of asymmetric perception of shocks on the volatility of the assets under consideration. Finally, it is important to note that statistical evidence was found in favor of a significant dynamic correlation structure between the considered time series, which justifies the use of multivariate specifications with a dynamic correlation matrix to model the joint dynamics of the considered assets.

In conclusion, we note that based on the analysis of simulated data, the proposed DCC-EGARCH model has a significant advantage over the classical DCC-GARCH specification due to the possibility of taking into account the leverage effect. However, in further studies, it is of interest to apply the developed specification to data characterized by the presence of a more pronounced leverage effect, due to which the asymmetric multivariate DCC-EGARCH model can demonstrate a serious advantage over symmetrical counterparts.

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