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Interest Rate Risk of Bonds in the Condition of a Changing Key Rate

N.V. Popova
Plekhanov Russian University of Economics, Moscow, Russia

ABSTRACT

The article is devoted to the analysis of the behavior of the interest rate risk of bonds in the conditions of a changing key interest rate. As known, the key rate is an instrument of monetary regulation of the Central Bank of the Russian Federation. During periods of instability, the key rate may change, which leads to changes in yields in the bond market. The latter, in turn, means that bonds on the Russian market are exposed to interest rate risk. When investing in federal loan bonds (OFZ), interest rate risk becomes the main type of risk, since there is no credit risk in such bonds. **The aim** of the paper is to obtain proof of the dependence of the interest rate risk of bonds on the term to maturity. The author applies **methods** of differential calculus to obtain the proof. **The novelty** of the research is that there is no similar proof in the literature. The instability of interest rates in the market persists at the present time, which allows us to speak about the **relevance** of this work. **Results:** it is established that with fixed values of the coupon rate, initial yield and the amount of interest rate change, the interest risk of bonds increases with an increase in the term to maturity. For long-term bonds sold at a discount, there is a term of a maximum interest rate risk. The formula for the approximate value of the term of maximum is obtained. Proven statements are confirmed by calculations, are consistent with previously performed studies, and are in line with market observations. The author comes to the **conclusion** that the proof obtained in the article of the dependence of the interest rate risk of bonds on the term to maturity can be used to analyze the behavior of the interest rate risk of bonds in the conditions of a changing key interest rate. **The practical significance**: the results of the research can be useful to the issuer and investor, as well as in theory when studying the investment properties of bonds.

Keywords: bonds; interest rate risk; mathematical methods; term to maturity; key interest rate

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INTRODUCTION

During periods of economic crises or economic difficulties, the risks of investing are exacerbated. The types of risk of investing in bonds is given in the book by F.J. Fabozzi [1]. There are two main types of bond risk: interest rate risk and credit risk [2]. Credit risk, or default risk, is the risk that an issuer will default on its obligation to pay a bond. In fixed income markets without credit risk, the main type of risk is interest rate risk [3]. Similarly, the statement of F.J. Fabozzi [4, p. 1]: "The major risk faced by participants in the bond market is interest rate risk".

F.J. Fabozzi [1, p. 22] defines interest rate risk: "interest rate risk is the risk that an increase in interest rates will lead to lower bond prices". The literature also gives a broader definition of interest rate risk as "the possibility of changing the price of a bond under the influence of changes in interest rates" [3, p. 208]. Publications [2, 5–12] are devoted to the study of various types of investment risk in bonds, including the analysis of factors that contribute to changes in market interest rates. The main macroeconomic factors are inflation, interest rate, political and economic risks. According to A.K. Isaev and V.N. Dem'yanov [5], inflation of the currency in which a given issue is nominated is a fundamental factor determining the yield of corporate bonds in a particular country. According to L.J. Gitman and M.D. Joehnk [13, p. 467], "In fact, investors are most worried about inflation. This not only undermines the purchasing power of the principal amount of the loan, but also greatly affects the dynamics of interest rates." In Russia, the Bank of Russia uses the key interest rate to regulate inflation [14, 15]. The value of the key rate affects the yield of bonds. I.M. Balkoev [16] considered the mechanism of change in the market profitability under the influence of changes in the key rate. As a result, according to [16, p. 35], "the influence of the Central Bank rate on bonds lies in the fact that when it increases, the yield on the bond market also grows, and when it decreases, the yield on bonds proportionally decreases." The impact of the key rate on the growth of market interest rates could be observed in the Russian bond market in 2021. In March 2021, the Bank of Russia began raising the key rate in order to reduce inflation and increased this indicator by 4.25 p.p.— up to 8.5% in December. According to the Moscow Exchange[1] for the period from March to December 2021, the increase in the key rate was accompanied by an increase in yields on the bond market and, as a result, an increase in the number of bonds sold at a discount. At the same time, bonds purchased on the Russian market prior to March 2021 were subject to significant interest rate risk.

A number of studies have noted that bond parameters such as coupon rate, term to maturity, and yield to maturity affect the interest rate risk of a bond. According to F.J. Fabozzi [1, p. 22] "the sensitivity of the price of each particular issue to changes in market interest rates depends on the parameters of the bond, namely the coupon rate and term to maturity". The influence of yield to maturity or, which is the same, the level of market interest rates on the percentage change in the price of a bond was proved in [17] and is formulated as follows: for a given term to maturity and coupon rate, the higher the initial yield level, the smaller the relative (percentage) price change bonds when its yield changes by a given value. Thus, "with a given change in returns, price volatility is higher in a market where returns are low, and vice versa: with high returns, volatility is low" [1, p. 95]. The effect of the coupon rate on the interest rate risk of a bond was proved in [18] and is formulated as follows: for a given term to maturity and initial yield, the relative (percentage) change in the price of a bond when its yield changes by a given value is greater, the lower the coupon rate. Studies of the effect of the coupon rate in [6, 10]

[1] Moscow Exchange. URL: http://www.moex.com/ (accessed on 10.05.2022).

confirm this statement. For example, [6, p. 254]: "Compared to coupon bonds, zero-coupon bonds are more susceptible to price volatility due to fluctuations in the market interest rate". Based on the results of studies in [6], it was concluded that the coupon rate is an important characteristic of the issue, but its value is not as great as the term to maturity of the bond.

According to [1], term to maturity is a key characteristic of a bond and is of paramount importance when evaluating any bond. The authors of papers [2, 5, 6, 10, 11] consider term to maturity to be a significant risk factor when investing in bonds, primarily due to the exposure of bonds to interest rate risk. According to [10, p. 626], bonds with longer maturities directly indicate that these bonds are subject to risks. First of all, the risks associated with interest rates, reflecting the dynamics of the economy and business. According to [6, p. 248] the probability of large risks may arise with long-term investments. This higher potential risk is related to the bonds' exposure to interest rate and price risk. As a result, bonds with longer maturities require guarantees, which typically have high yield-to-maturity requirements. According to [5, p. 139], "as a rule, investors are interested in a large risk premium when buying "long" bonds, since the uncertainty is higher with a long maturity". The authors of a number of papers note the relationship between the amount of interest rate risk and the term to maturity. According to [10, p. 621] as a result of changes in interest rates, "greater price changes will occur in bonds with a longer maturity". This conclusion is consistent with the statement in [6, p. 255, 256]: "bonds with a longer maturity tend to be subject to greater price volatility in the bond market", as well as in [2, p. 18]: "Longer bonds should provide the investor with an additional risk premium associated with higher duration and volatility". It should be noted that all of the above statements about the relationship between the amount of interest rate risk and maturity are formulated on the basis of market observations.

The interest rate risk of a bond is estimated by the value of the relative (percentage) change in the price of the bond when the market interest rate changes by a given value $\Delta P/P$. In practice, the interest rate risk of bonds is assessed using duration and convexity. According to F. J. Fabozzi [4, p. 1] "a typical way to measure interest rate risk is to approximate the effect of changes in interest rates on a bond or portfolio of bonds using duration and convexity". This statement is confirmed by works devoted to assessing the interest rate risk of bonds [19–24].

Due to the accepted method of assessing interest rate risk, the effect of the term to maturity on the interest rate risk of a bond is established on the basis of the dependence of the duration of the bond on the term to maturity. There are mathematical proofs in the literature of dependence bond duration on the term to maturity [20, 25–27]. At the same time, in the literature much less attention has been paid to the problem of proof of dependence on the term to maturity of the directly interest rate risk of the bond, and therefore the theory of investing in financial instruments with fixed income seems to be incomplete. In [28], to solve the problem of the effect of term to maturity on the percentage change in the price of a bond when the yield changes by a given value, numerical sequences of the form $\{\Delta P_n/P_n\}$, were used, where the number of the sequence member n coincided with the number of coupon periods remaining until the bond is redeemed. In this article, methods of differential calculus were used to solve the problem.

According to F. J. Fabozzi [1, p. 95] the formulation of the dependence of the interest rate risk of a bond on the term to maturity is as follows: "For a given coupon rate and initial yield, the longer the term to maturity, the higher the price volatility". This statement is clarified in this paper.

MATERIALS AND METHODS

The problem of the dependence of the interest rate risk of a coupon bond on the term to maturity is considered. As noted, the interest rate risk of a bond is estimated by the value of the relative (percentage) change in the bond price $\Delta P/P$ when the market interest rate changes by a given value. Let r and $P(r)$ — be the annual yield and the bond price at the initial time. We will consider a bond price change for an instantaneous change in the market interest rate, similarly to F.J. Fabozzi [1, p. 93]. Let $\tilde{r}$ — be the yield on a bond as a result of an instantaneous change in the interest rate by $\Delta r > 0$. If $\tilde{r} = r - \Delta r$, then $\tilde{r} < r$ — the rate instantly decreased; if $\tilde{r} = r + \Delta r$, then $\tilde{r} > r$ — the rate instantly increased. The bond price will be equal to $P(\tilde{r})$.

Since the yield and the bond price change in opposite directions, then $P(\tilde{r}) > P(r)$, if $\tilde{r} < r$ and $P(\tilde{r}) < P(r)$, if $\tilde{r} > r$. The value

$$\frac{P(\tilde{r}) - P(r)}{P(r)}, \text{ where } \tilde{r} < r, \quad (1)$$

is called relative (percentage) growth, and the value

$$\frac{P(r) - P(\tilde{r})}{P(r)}, \text{ where } \tilde{r} > r, \quad (2)$$

— relative (percentage) decrease in the bond price.[2] These are positive values.

To establish the dependence of the relative change in the bond price $\Delta P/P$ on the term to maturity n, it is sufficient to consider bonds for which coupon payments are paid once a year. In expressions (1) and (2), we will consider prices that do not contain accumulated coupon income, i.e. the quoted bond price immediately after the coupon payment, when n coupon periods remain to maturity.

Let us study the effect of term to maturity n on the value of the relative change in the bond price by differentiating with respect to the variable n function (1) for $\tilde{r} < r$ and function (2) for $\tilde{r} > r$.[3] For $\tilde{r} < r$ we get:

$$\left(\frac{P(\tilde{r}) - P(r)}{P(r)}\right)'_n = \frac{P(\tilde{r})}{P(r)}\left(\frac{P'_n(\tilde{r})}{P(\tilde{r})} - \frac{P'_n(r)}{P(r)}\right). \quad (3)$$

If $\tilde{r} > r$ we get:

$$\left(\frac{P(r) - P(\tilde{r})}{P(r)}\right)'_n = \frac{P(\tilde{r})}{P(r)}\left(\frac{P'_n(r)}{P(r)} - \frac{P'_n(\tilde{r})}{P(\tilde{r})}\right). \quad (4)$$

To set the signs of derivatives in expressions (3) and (4), it is necessary to set the sign of the difference

$\left(\frac{P'_n(\tilde{r})}{P(\tilde{r})} - \frac{P'_n(r)}{P(r)}\right)$ in expression (3) and the sign of the difference

$\left(\frac{P'_n(r)}{P(r)} - \frac{P'_n(\tilde{r})}{P(\tilde{r})}\right)$ in expression (4). To do this, the function

$\frac{P'_n(r)}{P(r)}$ must be examined for monotonicity with respect to the variable r.

RESULTS AND DISCUSSIONS

Theorem. For given values of the coupon rate f, the initial yield to maturity r, and the amount of change in the interest rate $\Delta r > 0$ the following statements are true:

1) $\lim\limits_{n\to\infty} \frac{\Delta P}{P} = \frac{\Delta r}{\tilde{r}}$;

2) the interest rate risk of a bond sold at par or at a premium increases with maturity;

3) for bonds sold at a discount, there is a term of maximum interest rate risk.

Proof. By convention, r is the bond initial yield to maturity. Then the bond price at the initial moment is equal to:

[2] Encyclopedia of financial risk management. Ed. A.A. Lobanov and A.V. Chugunov. 4th ed. M.: Alpina Business. Books. 2005. 53 p.

[3] Differentiation with respect to an integer variable is used in studying the investment properties of bonds. For example, in the works of G.A. Hawawini [25], and B. Malkiel [29].

$$P(r) = A(1+r)^{-n}\left(1-\frac{f}{r}\right)+A\frac{f}{r}, \quad (5)$$

where A — the face value of the bond; n — the term to maturity.

If $\tilde{r}$ — the bond yield as a result of an instantaneous change in the interest rate by an amount $\Delta r > 0$, then the bond price will be equal to:

$$P(\tilde{r}) = A(1+\tilde{r})^{-n}\left(1-\frac{f}{\tilde{r}}\right)+A\frac{f}{\tilde{r}}. \quad (6)$$

1. Using formulas (5) and (6), we find the limit of expressions (1) and (2) at $n \to \infty$. We get:

$$\lim_{n\to\infty}\frac{\Delta P}{P} = \frac{\Delta r}{\tilde{r}}, \text{ where } \tilde{r} = r \pm \Delta r.$$

To prove the following statements of the theorem, it is necessary to investigate the monotonicity of the function $\frac{P_n'(r)}{P(r)}$ with respect to the variable r. This function looks like:

$$\frac{P_n'(r)}{P(r)} = \frac{(f-r)\ln(1+r)}{r+f\left((1+r)^n-1\right)}. \quad (7)$$

We differentiate this function with respect to the variable r. Then we use approximate equalities:

$$(1+r)^n \approx 1+rn, \ \ln(1+r) \approx r.$$

We get:

$$\left(\frac{P_n'(r)}{P(r)}\right)_r' \approx$$

$$\approx \frac{r^2}{B^2(1+r)}\left[(r-f)fn^2 - f(1+r)n-(1+f)\right], \quad (8)$$

where B^2 — the square of the denominator of the right side of equality (7).

Let us prove the second and third statements of the theorem.

2. Under condition $f \geq r$, i.e. for bonds sold at par or at a premium, expression (8) is negative. Hence, for these bonds $\left(\frac{P_n'(r)}{P(r)}\right)_r' < 0$ — the ratio $\frac{P_n'(r)}{P(r)}$ is a decreasing function of the variable r. If $\tilde{r} < r$, then $\frac{P_n'(\tilde{r})}{P(\tilde{r})} > \frac{P_n'(r)}{P(r)}$ and expression (3) is positive, which means $\left(\frac{P(\tilde{r})-P(r)}{P(r)}\right)_n' > 0$. If $\tilde{r} > r$, then $\frac{P_n'(\tilde{r})}{P(\tilde{r})} < \frac{P_n'(r)}{P(r)}$ and then expression (4) is also positive, which means $\left(\frac{P(r)-P(\tilde{r})}{P(r)}\right)_n' > 0$. Thus, for any change in interest rates, the interest rate risk of bonds sold at par or at a premium increases with maturity. The second statement of the theorem is proved.

3. Under the condition $f < r$, i.e. for bonds sold at a discount, expression (8) with an increase in $n > 0$ changes sign at a certain value of the term n_0 from minus to plus. This follows from the expression in square brackets, where there is a square trinomial with respect to n with a positive coefficient at n^2 and a negative value of one of the roots.

If $n < n_0$, then $\left(\frac{P_n'(r)}{P(r)}\right)_r' < 0$ — the ratio $\frac{P_n'(r)}{P(r)}$ is a decreasing function of the variable r.

Then, for any change in interest rates, expressions (3) and (4) are positive, which means

$$\left(\frac{P(\tilde{r})-P(r)}{P(r)}\right)_n' > 0 \text{ at } \tilde{r} < r$$

and $\left(\frac{P(r)-P(\tilde{r})}{P(r)}\right)_n' > 0$ at $\tilde{r} > r$. Therefore,

Table 1

Dependence $\Delta P/P$ on *n* (*f* > *r*)
***f* = 10%, *r* = 8%, Δ*r* = 0,1%, $\tilde{r}=8{,}1\%$**

n	$\Delta P/P$
1	0.000925
2	0.001767
3	0.002535
4	0.003237
5	0.003882
8	0.005525
10	0.006421
15	0.008152
20	0.009360
25	0.010215
30	0.010823
35	0.011258
40	0.011570
50	0.011952
60	0.012147
$\lim\limits_{n\to\infty}\frac{\Delta P}{P}=\frac{\Delta r}{\tilde{r}}$	0.012346

Source: compiled by the author.

Table 2

Dependence $\Delta P/P$ on *n* (*f* < *r*)
***f* = 10%, *r* = 13%, Δ*r* = 0,1%, $\tilde{r}=13{,}1\%$**

n	$\Delta P/P$
1	0.00088417
3	0.00240759
5	0.00363531
10	0.00568087
20	0.00725052
30	0.00758621
40	0.00763938
43	0.00764171
44	0.00764198
45	0.00764208
46	0.00764205
47	0.00764192
50	0.00764109
55	0.00763923
60	0.00763749
$\lim\limits_{n\to\infty}\frac{\Delta P}{P}=\frac{\Delta r}{\tilde{r}}$	0.00763359

at maturity $n<n_0$ the interest rate risk of bonds sold at a discount increases with maturity.

If $n>n_0$, then $\left(\frac{P_n'(r)}{P(r)}\right)_r' > 0$ — the ratio $\frac{P_n'(r)}{P(r)}$ is an increasing function of the variable *r*. Then, for any change in interest rates, expressions (3) and (4) are negative, which means

$$\left(\frac{P(\tilde{r})-P(r)}{P(r)}\right)_n' < 0 \text{ at } \tilde{r}<r$$

and $\left(\frac{P(r)-P(\tilde{r})}{P(r)}\right)_n' < 0$ at $\tilde{r}>r$. Therefore, at maturity $n>n_0$ the interest rate risk of bonds sold at a discount decreases as the maturity increases.

From the conditions $\left(\frac{\Delta P}{P}\right)_n' > 0$ at $n<n_0$ and $\left(\frac{\Delta P}{P}\right)_n' < 0$ at $n>n_0$ it follows that n_0 — is the term of maximum interest rate risk of bonds sold at a discount. Equating expression (8) to zero, we find the approximate value of the term n_0:

Table 3

Exact and approximate values of the term according to formulas (9), (10)
f = 10%, Δr = 0,1% ($f < r$)

r %	11	12	13	15	20	25
n_0 exact, $\tilde{r} < r$, (1)	127	66	47	30	18	13
n_0 exact, $\tilde{r} > r$, (2)	115	63	45	30	18	13
n_0 (9)	120	64.5	45.7	30.3	18.1	13.7
n_0 (10)	121.1	65.3	46.4	30.7	18.0	13.3

Source: compiled by the author.

$$n_0 \approx \frac{f(1+r)+\sqrt{f^2(1+r)^2+4(r-f)f(1+f)}}{2(r-f)f}. \quad (9)$$

The theorem has been proven.

The statements about the behavior of interest rate risk proved in the theorem are confirmed by calculations using formulas (1) and (2) of the relative changes in the prices of bonds sold at a premium or at a discount for various maturities. Examples of such calculations are shown in *Table 1* and *2*. It should be noted that in these tables, as n increases, the values $\Delta P/P$ approach their limiting values obtained by the formula from statement 1 of the theorem. *Table 2* shows that the term of the maximum value $\Delta P/P$ with an increase in yield by Δr = 0.1% of a bond with a coupon rate f = 10% and an initial yield r = 13% is $n_0 = 45$ years.

In [28], for bonds sold at a discount, the following expression was obtained for the approximate value of the term of the maximum value $\Delta P/P$:

$$n_0 \approx 1+\frac{(r+1)(2r-f)+\sqrt{f^2(r-1)^2+4r(r-f+rf)}}{2r(r-f)}. \quad (10)$$

Table 3 shows the exact and approximate values of the term n_0 for given values of the coupon rate f and the amount of change in the interest rate $\Delta r > 0$ for various values of the initial yield r. The exact values of n_0 are obtained from direct calculations using formulas (1) and (2) of the relative price changes of bonds sold at a discount for various maturities n. Approximate values of the term n_0 are obtained by formulas (9) and (10).

As we can see, the calculation results agree with each other. As the difference $(r-f)$ increases the exact and approximate values of the term n_0 converge. As follows from formulas (9), (10), and calculations, the the term of maximum of interest rate risk increases with decreasing difference $(r-f)$ and is practically absent at close values of the coupon rate and yield.

CONCLUSIONS

Macroeconomic interest rate risk factors such as inflation and the key interest rate are considered. The role of bond parameters in the level of interest risk is noted. Maturity is a significant risk factor for bond investments, primarily because of the bond's exposure to interest rate risk. The proof obtained in the article of the dependence of the interest rate risk of bonds on term to maturity is confirmed by calculations and can be used to analyze the behavior of the interest rate risk of bonds in the context of a changing key interest rate.

The results are obtained under the condition that the market yield curve is horizontal and its movements are parallel. In reality, the yield curve is not horizontal, and its shifts are not necessarily parallel. However, it is known that the investment properties of bonds are studied under given conditions. The results of the study can be useful to the issuer and investor, as well as in theory when studying the investment properties of bonds.

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ABOUT THE AUTHOR

Natal'ya V. Popova — Cand. Sci. (Phys.-Math.), Assoc. Prof., Department of Higher Mathematics in Plekhanov Russian University of Economics, Moscow, Russia
http://orcid.org/0000-0002-3700-5249
nat_popova_@mail.ru

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