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Approbation of the Averaged Method of Chain Substitutions for Three- and Four- Multiples and Multiplicative-Multiples Factor Models

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ABSTRACT

The **aim** of the present study is to present the results of the approbation of the methodology of the averaged method of chain substitutions for three and four-multiple and multiplicative-multiple factor models and to systematize in tabular form all mathematical expressions developed so far to determine the individual factor influences by types of factor models. The **relevance** of the research is caused by the disadvantages and the limited applicability of the methods of deterministic factor analysis developed so far, which is one of the areas of financial and economic analysis. The **scientific novelty** of the research is the new mathematical expressions developed by the author for determining the individual factor influences according to the methodology of the averaged method of chain substitutions for three and four multiple and multiplicative-multiple factor models. Previous and new mathematical expressions according to the averaged method of chain substitutions are systematized by types of factor models in tabular form. The **main conclusion** is that the averaged method of chain substitutions has complete universality of application for all types of factor models and is characterized by accuracy and unambiguity of the results obtained for quantification of individual factor influences.

Keywords: deterministic factor analysis; mathematical methods; averaged method of chain substitutions; economic analysis

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INTRODUCTION

Deterministic factor analysis (DFA) is one of the directions of financial and economic analysis. DFA is aimed at accurate and unambiguous determination of quantitative influences that affect changes of participating factor variables in mathematically deterministic (determinable) factor models on absolute change of the result indicator.

The type of factor model is determined by the type of mathematical dependence describing the relationship between the resulting indicator (P) and participating factor variables ($a,b,c,\ldots$), by many authors factors for brevity.

The following types of factor models are identified in DFA practice:

- additive — $P = a + b + \ldots$;
- multiplicative — $P = a * b * \ldots$;
- multiple (relative) — $P = \frac{a}{b}$,

$$P = \frac{a}{\frac{b}{c}},\quad P = \frac{\frac{a}{b}}{c},\quad P = \frac{\frac{a}{b}}{\frac{c}{d}};$$

- mixed (combination) models — are a combination of additive, multiplicative and multiple models and can be: multiplicative-multiple, additive- multiple or additive-multiplicative-multiple models.

The distribution of absolute change of the resulting indicator (ΔP) by factor variables is based on the work of a number of Russian and foreign authors, namely: S. M. Yugenburg [1], A. Humal [2], A. D. Sheremet [3], A. D. Sheremet, G. G. Dei and V. N. Shapovalov [4], V. E. Adamov [5], V. Fedorova and Yu. Egorov [6], M. I. Bakanov and A. D. Sheremet [7], S. V. Chebotarev [8], N. P. Lyubushin [9], N. Sh. Kremer [10], K. N. Lebedev [11], V. A. Prokofiev, V. V. Nosov, T. V. Salomatina [12], G. V. Savitskaya [13], S. A. Ross, R. W. Westerfield and J. F. Jaffe [14], G. Foster [15], D. R. Emery, J. D. Finnerty and J. D. Stowe [16], J. J. Wild, L. A. Bernstein, K. R. Subramanyam [17], R. Brealey, S. Myers, F. Allen [18], V. Mitev [19] and others.

In DFA, the following methods are most used to quantify the influence of individual factors in a mathematically deterministic factor model: differential; coefficients; chain substitution; absolute differences; relative differences; equity participation; simple addition of an indelible balance; weighted finite differences; logarithmic; factor splitting; integral; index.

Each of the DFA methods has developed methodology, specific applicability, opportunities, advantages and disadvantages. All are described in detail in the scientific and educational literature in the field of DFA. Unfortunately, the above methods do not solve the accurate and unambiguous distribution of the so-called “indelible balance” between the influence of factor variables.

Integrated and chain substitution methods are most commonly used in DFA. The essence, methodology, applicability, accuracy, advantages and disadvantages of both methods are presented in detail in the scientific and educational literature.

Chain substitution method has absolute versatility of application for all possible types of factor models, but does not provide accurate and unambiguous results, since the influence of individual factors depends on the sequence of substitutions of factor variables in the construction of factor chains. This is the only and insurmountable disadvantage of the chain substitution method, namely — the ambiguous results for individual factor influences when changing the order of the factor variables substitution. This disadvantage leads to the need to rank the factor variables, namely: it is necessary to accurately determine which of the factors involved in the factor model is primary, which is secondary, which is third in order, etc., which creates considerable difficulties for managers and financial analysts.

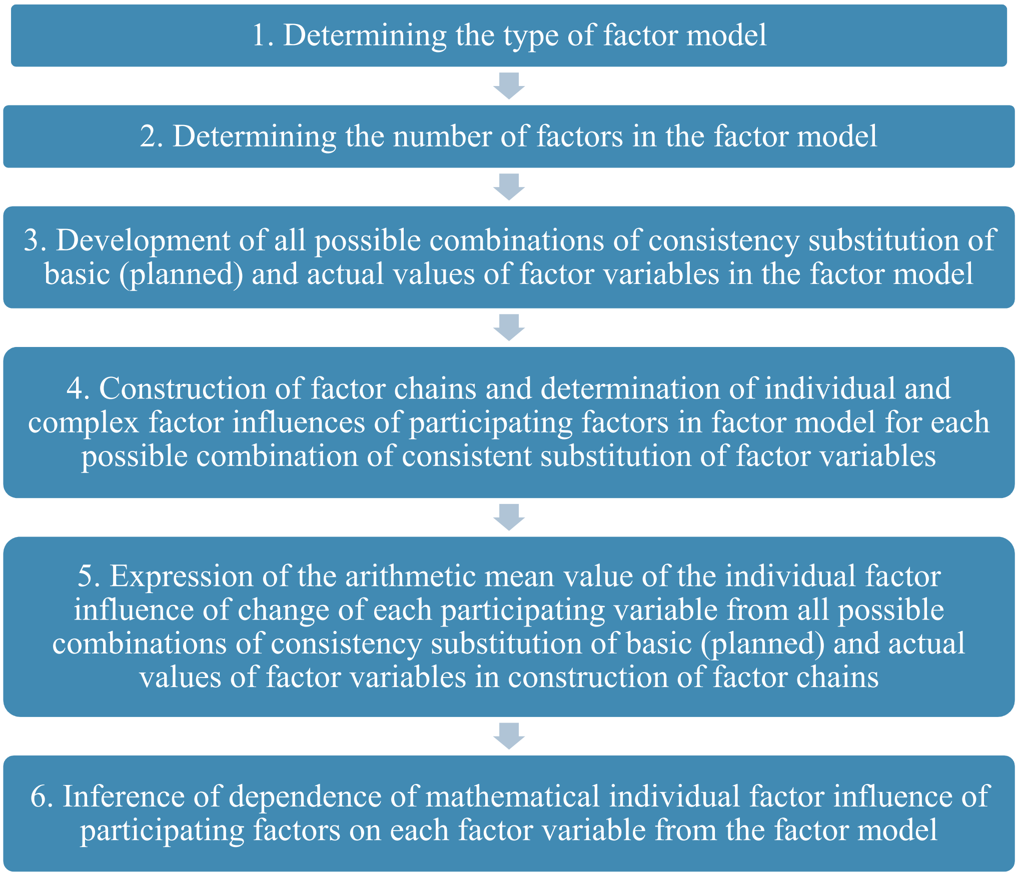

Fig. **Stages of the averaged method of chain substitutions**

Source: Mitev V. [20].

Integrated method developed by a group of Russian scientists — A.D. Sheremet, G.G. Dei, V.N. Shapovalov in 1971. It was developed for a limited number of types of factor models, namely: for all multiplicative ($P=a*b*\ldots$) and for a limited number of multiple and additive-multiple: $P=\frac{a}{b}$; $P=\frac{a}{b+c+\ldots}$, where: P — resulting indicator in factor model; a,b,c etc. are the participating factor variables in the factor model.

As described in [20, p. 97]: “ In multiplicative factor models, the integral method gives accurate and unambiguous results, but for a limited number of multiples and additive-multiples models, the accurate of the results is compromised by using the function of natural logarithm in mathematical expressions to determine the influence of the factor a , i.e. factor in the numerator of the factor model, and then determine the influence of other factors in the factor models ($b,c,\ldots$), as they are a function of no longer very clearly defined influence of the factor a ».

Two preceding articles in Bulgarian [20, 21] present the methodology, essence, advantages, disadvantages and results of the developed new DFA method, namely: “average method of chain substitution”. It has absolute versatility of application for all types of factor models, accurate and unambiguous results obtained to quantify the individual factor influence of factors involved in factor models.

The aims of this research — are present the results of testing the methodology of the average method of chain substitution for the three-, four- multiples and multiplicative factor models and systematize in tabular form all mathematical expressions

Table 1

New formulas for determining the individual factor influences by the averaged method of chain substitutions

Factor model	Influence of the factor a, $\Delta P_{(a)}$	Influence of the factor b, $\Delta P_{(b)}$	Influence of the factor c, $\Delta P_{(c)}$	Influence of the factor d, $\Delta P_{(d)}$
Multiple (relative) factor models				
$P=\frac{a}{\frac{b}{c}}=\frac{a*c}{b}$	$\frac{\Delta a}{6}\left(\frac{2c_0+c_1}{b_0}+\frac{2c_1+c_0}{b_1}\right)$	$\frac{1}{6}\left(\frac{2(a_1c_1+a_0c_0)+a_1c_0+a_0c_1}{b_1}-\frac{2(a_1c_1+a_0c_0)+a_1c_0+a_0c_1}{b_0}\right)$	$\frac{\Delta c}{6}\left(\frac{2a_0+a_1}{b_0}+\frac{2a_1+a_0}{b_1}\right)$	-
$P=\frac{\frac{a}{b}}{c}=\frac{a}{b*c}$	$\frac{1}{6}\left(\frac{2\Delta a}{b_0c_0}+\frac{2\Delta a}{b_1c_1}+\frac{\Delta a}{b_1c_0}+\frac{\Delta a}{b_0c_1}\right)$	$\frac{1}{6}\left(\frac{2a_1+a_0}{b_1c_1}-\frac{2a_0+a_1}{b_0c_0}+\frac{2a_0+a_1}{b_1c_0}-\frac{2a_1+a_0}{b_0c_1}\right)$	$\frac{1}{6}\left(\frac{2a_1+a_0}{b_1c_1}-\frac{2a_0+a_1}{b_0c_0}+\frac{2a_0+a_1}{b_0c_1}-\frac{2a_1+a_0}{b_1c_0}\right)$	-
$P=\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a*d}{b*c}$	$\frac{\Delta a}{12}\left(\frac{3d_0+d_1}{b_0c_0}+\frac{3d_1+d_0}{b_1c_1}+\frac{d_0+d_1}{b_0c_1}+\frac{d_0+d_1}{b_1c_0}\right)$	$\frac{1}{12}\left(\frac{3a_1d_1+a_0d_0+a_1d_0+a_0d_1}{b_1c_1}-\frac{3a_0d_0+a_1d_1+a_1d_0+a_0d_1}{b_0c_0}+\frac{3a_0d_0+a_1d_1+a_1d_0+a_0d_1}{b_1c_0}-\frac{3a_1d_1+a_0d_0+a_1d_0+a_0d_1}{b_0c_1}\right)$	$\frac{1}{12}\left(\frac{3a_1d_1+a_0d_0+a_1d_0+a_0d_1}{b_1c_1}-\frac{3a_0d_0+a_1d_1+a_1d_0+a_0d_1}{b_0c_0}+\frac{3a_0d_0+a_1d_1+a_1d_0+a_0d_1}{b_0c_1}-\frac{3a_1d_1+a_0d_0+a_1d_0+a_0d_1}{b_1c_0}\right)$	$\frac{\Delta d}{12}\left(\frac{3a_0+a_1}{b_0c_0}+\frac{3a_1+a_0}{b_1c_1}+\frac{a_0+a_1}{b_0c_1}+\frac{a_0+a_1}{b_1c_0}\right)$
Multiplicative-multiple models				
$P=\frac{a*b}{c}$	$\frac{\Delta a}{6}\left(\frac{2b_0+b_1}{c_0}+\frac{2b_1+b_0}{c_1}\right)$	$\frac{\Delta b}{6}\left(\frac{2a_0+a_1}{c_0}+\frac{2a_1+a_0}{c_1}\right)$	$\frac{1}{6}\left(\frac{2(a_1b_1+a_0b_0)+a_1b_0+a_0b_1}{c_1}-\frac{2(a_1b_1+a_0b_0)+a_1b_0+a_0b_1}{c_0}\right)$	-
$P=\frac{a*b*c}{d}$	$\frac{\Delta a}{12}\left(\frac{3b_0c_0+b_1c_0+b_0c_1+b_1c_1}{d_0}+\frac{3b_1c_1+b_1c_0+b_0c_1+b_0c_0}{d_1}\right)$	$\frac{\Delta b}{12}\left(\frac{3a_0c_0+a_1c_0+a_0c_1+a_1c_1}{d_0}+\frac{3a_1c_1+a_1c_0+a_0c_1+a_0c_0}{d_1}\right)$	$\frac{\Delta c}{12}\left(\frac{3a_0b_0+a_1b_0+a_0b_1+a_1b_1}{d_0}+\frac{3a_1b_1+a_1b_0+a_0b_1+a_0b_0}{d_1}\right)$	$\frac{1}{12}\left(\frac{3(a_0b_0c_0+a_1b_1c_1)+a_1b_1c_0+a_1b_0c_1+a_1b_0c_0+a_0b_1c_1+a_0b_1c_0+a_0b_0c_1}{d_1}+\frac{3(a_0b_0c_0+a_1b_1c_1)+a_1b_1c_0+a_1b_0c_1+a_1b_0c_0+a_0b_1c_1+a_0b_1c_0+a_0b_0c_1}{d_0}\right)$

Source: author's development.

developed so far to determine the influence of individual factors in different types of factor models.

RESEARCH METHODS AND METHODOLOGY OF THE AVERAGE METHOD OF CHAIN SUBSTITUTION

The following methods were used: critical analysis; synthesis; dialectic method; combinatorics; averaging method; average method of chain substitution.

The main steps of the methodology of the average method of chain substitution are presented on *Figure*.

The essence of the methodology of the average method of chain substitution is based on the derivation of all mathematical expressions to determine the influence of individual factors by the method of chain substitution for each possible combination of the sequence of basic (planned) substitution and actual values of factor variables in the analytic factor model. The number of possible combinations is $N=n!$, where n — number of participating factor variables in the factor model. The resulting mathematical expressions for the influence of individual factors are averaged as they are summed and divided by the number of possible combinations of the sequence of substitution of factor variables ($N=n!$). The resulting mathematical expression for the influence of an individual factor is subjected to mathematical transformations and reduction by inferring simplified analytical dependencies for quantifying the influence of a variable factor on the absolute change of a resulting indicator. This procedure applies to each factor variable of the analytic factor model.

Averaging received mathematical expressions to determine the influence of individual factors by chain substitution for each possible combination of order of substitution of factor variables in the

Table 2

Systematization of factor models and formulas for determining the individual factor influences by the averaged method of chain substitutions

Factor model	Influence of the factor a, $\Delta P_{(a)}$	Influence of the factor b, $\Delta P_{(b)}$	Influence of the factor c, $\Delta P_{(c)}$	Influence of the factor d, $\Delta P_{(d)}$
Multiplicative factor models				
$P=a*b$	$\frac{\Delta a}{2}(b_0+b_1)$	$\frac{\Delta b}{2}(a_0+a_1)$	–	–
$P=a*b*c$	$\frac{\Delta a}{3}\left(b_0.c_0+b_1.c_1+\frac{b_1.c_0+b_0.c_1}{2}\right)$	$\frac{\Delta b}{3}\left(a_0.c_0+a_1.c_1+\frac{a_1c_0+a_0.c_1}{2}\right)$	$\frac{\Delta c}{3}\left(a_0.b_0+a_1.b_1+\frac{a_1.b_0+a_0.b_1}{2}\right)$	–
$P=a*b*c*d$	$\frac{\Delta a}{4}(b_0.c_0.d_0+b_1.c_1.d_1)+\frac{\Delta a}{12}\begin{pmatrix}b_1.c_0.d_0+b_0.c_1.d_0+\\ b_0.c_0.d_1+b_1.c_1.d_0+\\ b_1.c_0.d_1+b_0.c_1.d_1\end{pmatrix}$	$\frac{\Delta b}{4}(a_0.c_0.d_0+a_1.c_1.d_1)+\frac{\Delta b}{12}\begin{pmatrix}a_1.c_0.d_0+a_0.c_1.d_0+\\ a_0.c_0.d_1+a_1.c_1.d_0+\\ a_1.c_0.d_1+a_0.c_1.d_1\end{pmatrix}$	$\frac{\Delta c}{4}(a_0.b_0.d_0+a_1.b_1.d_1)+\frac{\Delta c}{12}\begin{pmatrix}a_1.b_0.d_0+a_0.b_1.d_0+\\ a_0.b_0.d_1+a_1.b_1.d_0+\\ a_1.b_0.d_1+a_0.b_1.d_1\end{pmatrix}$	$\frac{\Delta d}{4}(a_0.b_0.c_0+a_1.b_1.c_1)+\frac{\Delta d}{12}\begin{pmatrix}a_1.b_0.c_0+a_0.b_1.c_0+\\ a_0.b_0.c_1+a_1.b_1.c_0+\\ a_1.b_0.c_1+a_0.b_1.c_1\end{pmatrix}$
Multiple (relative) factor models				
$P=\frac{a}{b}$	$\frac{1}{2}\left(\frac{\Delta a}{b_0}+\frac{\Delta a}{b_1}\right)$	$\frac{1}{2}\left(\frac{a_1+a_0}{b_1}-\frac{a_1+a_0}{b_0}\right)$	–	–
$P=\frac{a}{\frac{b}{c}}=\frac{a*c}{b}$	$\frac{\Delta a}{6}\left(\frac{2c_0+c_1}{b_0}+\frac{2c_1+c_0}{b_1}\right)$	$\frac{1}{6}\begin{pmatrix}\frac{2(a_1c_1+a_0c_0)+a_1c_0+a_0c_1}{b_1}\\ -\frac{2(a_1c_1+a_0c_0)+a_1c_0+a_0c_1}{b_0}\end{pmatrix}$	$\frac{\Delta c}{6}\left(\frac{2a_0+a_1}{b_0}+\frac{2a_1+a_0}{b_1}\right)$	-
$P=\frac{\frac{a}{b}}{c}=\frac{a}{b*c}$	$\frac{1}{6}\left(\frac{2\Delta a}{b_0c_0}+\frac{2\Delta a}{b_1c_1}+\frac{\Delta a}{b_1c_0}+\frac{\Delta a}{b_0c_1}\right)$	$\frac{1}{6}\begin{pmatrix}\frac{2a_1+a_0}{b_1c_1}-\frac{2a_0+a_1}{b_0c_0}\\ +\frac{2a_0+a_1}{b_1c_0}-\frac{2a_1+a_0}{b_0c_1}\end{pmatrix}$	$\frac{1}{6}\begin{pmatrix}\frac{2a_1+a_0}{b_1c_1}-\frac{2a_0+a_1}{b_0c_0}\\ +\frac{2a_0+a_1}{b_0c_1}-\frac{2a_1+a_0}{b_1c_0}\end{pmatrix}$	-
$P=\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a*d}{b*c}$	$\frac{\Delta a}{12}\begin{pmatrix}\frac{3d_0+d_1}{b_0c_0}+\frac{3d_1+d_0}{b_1c_1}\\ +\frac{d_0+d_1}{b_0c_1}+\frac{d_0+d_1}{b_1c_0}\end{pmatrix}$	$\frac{1}{12}\begin{pmatrix}\frac{3a_1d_1+a_0d_0+a_1d_0+a_0d_1}{b_1c_1}\\ -\frac{3a_0d_0+a_1d_1+a_1d_0+a_0d_1}{b_0c_0}\\ +\frac{3a_0d_0+a_1d_1+a_1d_0+a_0d_1}{b_1c_0}\\ -\frac{3a_1d_1+a_0d_0+a_1d_0+a_0d_1}{b_0c_1}\end{pmatrix}$	$\frac{1}{12}\begin{pmatrix}\frac{3a_1d_1+a_0d_0+a_1d_0+a_0d_1}{b_1c_1}\\ -\frac{3a_0d_0+a_1d_1+a_1d_0+a_0d_1}{b_0c_0}\\ +\frac{3a_0d_0+a_1d_1+a_1d_0+a_0d_1}{b_0c_1}\\ -\frac{3a_1d_1+a_0d_0+a_1d_0+a_0d_1}{b_1c_0}\end{pmatrix}$	$\frac{\Delta d}{12}\begin{pmatrix}\frac{3a_0+a_1}{b_0c_0}+\frac{3a_1+a_0}{b_1c_1}\\ +\frac{a_0+a_1}{b_0c_1}+\frac{a_0+a_1}{b_1c_0}\end{pmatrix}$
Multiplicative-multiple models				
$P=\frac{a}{b*c}$	$\frac{1}{6}\left(\frac{2\Delta a}{b_0c_0}+\frac{2\Delta a}{b_1c_1}+\frac{\Delta a}{b_1c_0}+\frac{\Delta a}{b_0c_1}\right)$	$\frac{1}{6}\begin{pmatrix}\frac{2a_1+a_0}{b_1c_1}-\frac{2a_0+a_1}{b_0c_0}\\ +\frac{2a_0+a_1}{b_1c_0}-\frac{2a_1+a_0}{b_0c_1}\end{pmatrix}$	$\frac{1}{6}\begin{pmatrix}\frac{2a_1+a_0}{b_1c_1}-\frac{2a_0+a_1}{b_0c_0}\\ +\frac{2a_0+a_1}{b_0c_1}-\frac{2a_1+a_0}{b_1c_0}\end{pmatrix}$	–
$P=\frac{a*b}{c}$	$\frac{\Delta a}{6}\left(\frac{2b_0+b_1}{c_0}+\frac{2b_1+b_0}{c_1}\right)$	$\frac{\Delta b}{6}\left(\frac{2a_0+a_1}{c_0}+\frac{2a_1+a_0}{c_1}\right)$	$\frac{1}{6}\begin{pmatrix}\frac{2(a_1b_1+a_0b_0)+a_1b_0+a_0b_1}{c_1}\\ -\frac{2(a_1b_1+a_0b_0)+a_1b_0+a_0b_1}{c_0}\end{pmatrix}$	-
$P=\frac{a}{b*c*d}$	$\frac{\Delta a}{12}\begin{pmatrix}\frac{3}{b_0.c_0.d_0}+\frac{3}{b_1.c_1.d_1}\\ +\frac{1}{b_1.c_0.d_0}+\frac{1}{b_1.c_1.d_0}\\ +\frac{1}{b_0.c_1.d_1}+\frac{1}{b_0.c_1.d_0}\\ +\frac{1}{b_1.c_0.d_1}+\frac{1}{b_0.c_0.d_1}\end{pmatrix}$	$\frac{1}{12}\begin{pmatrix}\frac{3.a_1+a_0}{b_1.c_1.d_1}-\frac{3.a_1+a_0}{b_0.c_1.d_1}\\ +\frac{3.a_0+a_1}{b_1.c_0.d_0}-\frac{3.a_0+a_1}{b_0.c_0.d_0}\\ +\frac{a_0+a_1}{b_1.c_1.d_0}-\frac{a_0+a_1}{b_0.c_1.d_0}\\ +\frac{a_0+a_1}{b_1.c_0.d_1}-\frac{a_0+a_1}{b_0.c_0.d_1}\end{pmatrix}$	$\frac{1}{12}\begin{pmatrix}\frac{3.a_1+a_0}{b_1.c_1.d_1}-\frac{3.a_0+a_1}{b_0.c_0.d_0}\\ +\frac{3.a_0+a_1}{b_0.c_1.d_0}-\frac{3.a_1+a_0}{b_1.c_0.d_1}\\ +\frac{a_0+a_1}{b_1.c_1.d_0}-\frac{a_0+a_1}{b_1.c_0.d_0}\\ +\frac{a_0+a_1}{b_0.c_1.d_1}-\frac{a_0+a_1}{b_0.c_0.d_1}\end{pmatrix}$	$\frac{1}{12}\begin{pmatrix}\frac{3.a_1+a_0}{b_1.c_1.d_1}-\frac{3.a_0+a_1}{b_0.c_0.d_0}\\ +\frac{3.a_0+a_1}{b_0.c_0.d_1}-\frac{3.a_1+a_0}{b_1.c_1.d_0}\\ +\frac{a_0+a_1}{b_0.c_1.d_1}-\frac{a_0+a_1}{b_0.c_1.d_0}\\ +\frac{a_0+a_1}{b_1.c_0.d_1}-\frac{a_0+a_1}{b_1.c_0.d_0}\end{pmatrix}$
$P=\frac{a*b*c}{d}$	$\frac{\Delta a}{12}\begin{pmatrix}\frac{3.b_0c_0+b_1c_0+b_0c_1+b_1c_1}{d_0}+\\ \frac{3.b_1c_1+b_1c_0+b_0c_1+b_0c_0}{d_1}\end{pmatrix}$	$\frac{\Delta b}{12}\begin{pmatrix}\frac{3.a_0c_0+a_1c_0+a_0c_1+a_1c_1}{d_0}+\\ \frac{3.a_1c_1+a_1c_0+a_0c_1+a_0c_0}{d_1}\end{pmatrix}$	$\frac{\Delta c}{12}\begin{pmatrix}\frac{3.a_0b_0+a_1b_0+a_0b_1+a_1b_1}{d_0}+\\ \frac{3.a_1b_1+a_1b_0+a_0b_1+a_0b_0}{d_1}\end{pmatrix}$	$\frac{1}{12}\begin{pmatrix}\frac{3(a_0b_0c_0+a_1b_1c_1)+a_1b_1c_0+a_1b_0c_1+a_1b_0c_0+a_0b_1c_1+a_0b_1c_0+a_0b_0c_1}{d_1}+\\ +\frac{3(a_0b_0c_0+a_1b_1c_1)+a_1b_1c_0+a_1b_0c_1+a_1b_0c_0+a_0b_1c_1+a_0b_1c_0+a_0b_0c_1}{d_0}\end{pmatrix}$
$P=\frac{a*b}{c*d}$	$\frac{\Delta a}{12}\begin{pmatrix}\frac{3.b_0+b_1}{c_0d_0}+\frac{3.b_1+b_0}{c_1d_1}\\ +\frac{b_0+b_1}{c_0d_1}+\frac{b_0+b_1}{c_1d_0}\end{pmatrix}$	$\frac{\Delta b}{12}\begin{pmatrix}\frac{3.a_0+a_1}{c_0d_0}+\frac{3.a_1+a_0}{c_1d_1}\\ +\frac{a_0+a_1}{c_0d_1}+\frac{a_0+a_1}{c_1d_0}\end{pmatrix}$	$\frac{1}{12}\begin{pmatrix}\frac{3.a_1b_1+a_0b_0+a_1b_0+a_0b_1}{c_1d_1}\\ -\frac{3.a_0b_0+a_1b_1+a_1b_0+a_0b_1}{c_0d_0}\\ +\frac{3.a_0b_0+a_1b_1+a_1b_0+a_0b_1}{c_1d_0}\\ -\frac{3.a_1b_1+a_0b_0+a_1b_0+a_0b_1}{c_0d_1}\end{pmatrix}$	$\frac{1}{12}\begin{pmatrix}\frac{3.a_1b_1+a_0b_0+a_1b_0+a_0b_1}{c_1d_1}\\ -\frac{3.a_0b_0+a_1b_1+a_1b_0+a_0b_1}{c_0d_0}\\ +\frac{3,a_0b_0+a_1b_1+a_1b_0+a_0b_1}{c_0d_1}\\ -\frac{3,a_1b_1+a_0b_0+a_1b_0+a_0b_1}{c_1d_0}\end{pmatrix}$

factor model means, that the probability of occurrence of each possible consistency of substitutions of factor variables — is the same. Here we get a result that allows the same probability of occurrence of each possible combination of substitution consistency of factors in construction of factor chains. There is no need to rank the factors involved in the factor model, resulting in unambiguous results for factor influences.

Table 2 (continued)

Factor model	Influence of the factor a, $\Delta P_{(a)}$	Influence of the factor b, $\Delta P_{(b)}$	Influence of the factor c, $\Delta P_{(c)}$	Influence of the factor d, $\Delta P_{(d)}$
Multiplicative-multiple models				
$P=\frac{a}{b+c}$	$\frac{1}{6}\left(\frac{2\Delta a}{b_0+c_0}+\frac{2\Delta a}{b_1+c_1}+\frac{\Delta a}{b_1+c_0}+\frac{\Delta a}{b_0+c_1}\right)$	$\frac{1}{6}\left(\frac{2a_1+a_0}{b_1+c_1}+\frac{2a_0+a_1}{b_1+c_0}-\frac{2a_1+a_0}{b_0+c_1}-\frac{2a_0+a_1}{b_0+c_0}\right)$	$\frac{1}{6}\left(\frac{2a_1+a_0}{b_1+c_1}+\frac{2a_0+a_1}{b_0+c_1}-\frac{2a_1+a_0}{b_1+c_0}-\frac{2a_0+a_1}{b_0+c_0}\right)$	–
$P=\frac{a}{b-c}$	$\frac{1}{6}\left(\frac{2\Delta a}{b_0-c_0}+\frac{2\Delta a}{b_1-c_1}+\frac{\Delta a}{b_1-c_0}+\frac{\Delta a}{b_0-c_1}\right)$	$\frac{1}{6}\left(\frac{2a_1+a_0}{b_1-c_1}+\frac{2a_0+a_1}{b_1-c_0}-\frac{2a_1+a_0}{b_0-c_1}-\frac{2a_0+a_1}{b_0-c_0}\right)$	$\frac{1}{6}\left(\frac{2a_1+a_0}{b_1-c_1}+\frac{2a_0+a_1}{b_0-c_1}-\frac{2a_1+a_0}{b_1-c_0}-\frac{2a_0+a_1}{b_0-c_0}\right)$	–
$P=\frac{a+b}{c}$	$\frac{1}{2}\left(\frac{\Delta a}{c_0}+\frac{\Delta a}{c_1}\right)$	$\frac{1}{2}\left(\frac{\Delta b}{c_0}+\frac{\Delta b}{c_1}\right)$	$\frac{1}{2}\left(\frac{a_1+a_0+b_1+b_0}{c_1}-\frac{a_1+a_0+b_1+b_0}{c_0}\right)$	–
$P=\frac{a-b}{c}$	$\frac{1}{2}\left(\frac{\Delta a}{c_0}+\frac{\Delta a}{c_1}\right)$	$\frac{1}{2}\left(\frac{b_0-b_1}{c_0}+\frac{b_0-b_1}{c_1}\right)$	$\frac{1}{2}\left(\frac{a_1+a_0-b_1-b_0}{c_1}+\frac{b_1+b_0-a_1-a_0}{c_0}\right)$	–
$P=\frac{a+b}{c+d}$	$\frac{1}{6}\left(\frac{2\Delta a}{c_0+d_0}+\frac{2\Delta a}{c_1+d_1}+\frac{\Delta a}{c_0+d_1}+\frac{\Delta a}{c_1+d_0}\right)$	$\frac{1}{6}\left(\frac{2\Delta b}{c_0+d_0}+\frac{2\Delta b}{c_1+d_1}+\frac{\Delta b}{c_0+d_1}+\frac{\Delta b}{c_1+d_0}\right)$	$\frac{1}{6}\left[\frac{2(a_1+b_1)+(a_0+b_0)}{c_1+d_1}-\frac{2(a_0+b_0)+(a_1+b_1)}{c_0+d_0}+\frac{2(a_0+b_0)+(a_1+b_1)}{c_1+d_0}-\frac{2(a_1+b_1)+(a_0+b_0)}{c_0+d_1}\right]$	$\frac{1}{6}\left[\frac{2(a_1+b_1)+(a_0+b_0)}{c_1+d_1}-\frac{2(a_0+b_0)+(a_1+b_1)}{c_0+d_0}+\frac{2(a_0+b_0)+(a_1+b_1)}{c_0+d_1}-\frac{2(a_1+b_1)+(a_0+b_0)}{c_1+d_0}\right]$
$P=\frac{a-b}{c-d}$	$\frac{1}{6}\left(\frac{2\Delta a}{c_0-d_0}+\frac{2\Delta a}{c_1-d_1}+\frac{\Delta a}{c_0-d_1}+\frac{\Delta a}{c_1-d_0}\right)$	$-\frac{1}{6}\left(\frac{2\Delta b}{c_0-d_0}+\frac{2\Delta b}{c_1-d_1}+\frac{\Delta b}{c_0-d_1}+\frac{\Delta b}{c_1-d_0}\right)$	$\frac{1}{6}\left[\frac{2(a_1-b_1)+(a_0-b_0)}{c_1-d_1}-\frac{2(a_0-b_0)+(a_1-b_1)}{c_0-d_0}+\frac{2(a_0-b_0)+(a_1-b_1)}{c_1-d_0}-\frac{2(a_1-b_1)+(a_0-b_0)}{c_0-d_1}\right]$	$\frac{1}{6}\left[\frac{2(a_1-b_1)+(a_0-b_0)}{c_1-d_1}-\frac{2(a_0-b_0)+(a_1-b_1)}{c_0-d_0}+\frac{2(a_0-b_0)+(a_1-b_1)}{c_0-d_1}-\frac{2(a_1-b_1)+(a_0-b_0)}{c_1-d_0}\right]$
$P=\frac{a+b}{c-d}$	$\frac{1}{6}\left(\frac{2\Delta a}{c_0-d_0}+\frac{2\Delta a}{c_1-d_1}+\frac{\Delta a}{c_0-d_1}+\frac{\Delta a}{c_1-d_0}\right)$	$\frac{1}{6}\left(\frac{2\Delta b}{c_0-d_0}+\frac{2\Delta b}{c_1-d_1}+\frac{\Delta b}{c_0-d_1}+\frac{\Delta b}{c_1-d_0}\right)$	$\frac{1}{6}\left[\frac{2(a_1+b_1)+(a_0+b_0)}{c_1+d_1}-\frac{2(a_0+b_0)+(a_1+b_1)}{c_0+d_0}+\frac{2(a_0+b_0)+(a_1+b_1)}{c_1+d_0}-\frac{2(a_1+b_1)+(a_0+b_0)}{c_0+d_1}\right]$	$\frac{1}{6}\left[\frac{2(a_1+b_1)+(a_0+b_0)}{c_1-d_1}-\frac{2(a_0+b_0)+(a_1+b_1)}{c_0+d_0}+\frac{2(a_0+b_0)+(a_1+b_1)}{c_0-d_1}-\frac{2(a_1+b_1)+(a_0+b_0)}{c_1-d_0}\right]$
$P=\frac{a-b}{c+d}$	$\frac{1}{6}\left(\frac{2\Delta a}{c_0+d_0}+\frac{2\Delta a}{c_1+d_1}+\frac{\Delta a}{c_0+d_1}+\frac{\Delta a}{c_1+d_0}\right)$	$\frac{1}{6}\left(-\frac{2\Delta b}{c_0+d_0}-\frac{2\Delta b}{c_1+d_1}-\frac{\Delta b}{c_0+d_1}-\frac{\Delta b}{c_1+d_0}\right)$	$\frac{1}{6}\left[\frac{2(a_1-b_1)+(a_0-b_0)}{c_1-d_1}-\frac{2(a_0-b_0)+(a_1-b_1)}{c_0-d_0}+\frac{2(a_0-b_0)+(a_1-b_1)}{c_1-d_0}-\frac{2(a_1-b_1)+(a_0-b_0)}{c_0-d_1}\right]$	$\frac{1}{6}\left[\frac{2(a_1-b_1)+(a_0-b_0)}{c_1+d_1}-\frac{2(a_0-b_0)+(a_1-b_1)}{c_0+d_0}+\frac{2(a_0-b_0)+(a_1-b_1)}{c_0+d_1}-\frac{2(a_1-b_1)+(a_0-b_0)}{c_1+d_0}\right]$
$P=\frac{a}{b+c+d}$	$\frac{\Delta a}{12}\left(\frac{3}{b_0+c_0+d_0}+\frac{3}{b_1+c_1+.d_1}+\frac{1}{b_1+c_0+d_0}+\frac{1}{b_1+c_1+d_0}+\frac{1}{b_0+c_1+d_1}+\frac{1}{b_0+c_1+d_0}+\frac{1}{b_0+c_0+d_1}+\frac{1}{b_1+c_0+d_1}\right)$	$\frac{1}{12}\left(\frac{3.a_1+a_0}{b_1+c_1+d_1}-\frac{3.a_1+a_0}{b_0+c_1.d_1}+\frac{3.a_0+a_1}{b_1+c_0+d_0}-\frac{3.a_0+a_1}{b_0+c_0+d_0}+\frac{a_0+a_1}{b_1+c_1+d_0}-\frac{a_0+a_1}{b_0+c_1+d_0}+\frac{a_0+a_1}{b_1+c_0+d_1}-\frac{a_0+a_1}{b_0+c_0+d_1}\right)$	$\frac{1}{12}\left(\frac{3.a_1+a_0}{b_1+c_1+d_1}-\frac{3.a_0+a_1}{b_0+c_0+d_0}+\frac{3.a_0+a_1}{b_0+c_1+d_0}-\frac{3.a_1+a_0}{b_1+c_0+d_0}+\frac{a_0+a_1}{b_1+c_1+d_0}-\frac{a_0+a_1}{b_1+c_0+d_0}+\frac{a_0+a_1}{b_0+c_1+d_1}-\frac{a_0+a_1}{b_0+c_0+d_1}\right)$	$\frac{1}{12}\left(\frac{3.a_1+a_0}{b_1+c_1+d_1}-\frac{3.a_0+a_1}{b_0+c_0+d_0}+\frac{3.a_0+a_1}{b_0+c_0+d_1}-\frac{3.a_1+a_0}{b_1+c_1+d_0}+\frac{a_0+a_1}{b_0+c_1+d_1}-\frac{a_0+a_1}{b_0+c_1+d_0}+\frac{a_0+a_1}{b_1+c_0+d_1}-\frac{a_0+a_1}{b_1+c_0+d_0}\right)$
$P=\frac{a+b+c}{d}$	$\frac{1}{2}\left(\frac{\Delta a}{d_0}+\frac{\Delta a}{d_1}\right)$	$\frac{1}{2}\left(\frac{\Delta b}{d_0}+\frac{\Delta b}{d_1}\right)$	$\frac{1}{2}\left(\frac{\Delta c}{d_0}+\frac{\Delta c}{d_1}\right)$	$\frac{1}{2}\left(\frac{a_1+a_0+b_1+b_0+c_1+c_0}{d_1}-\frac{a_1+a_0+b_1+b_0+c_1+c_0}{d_0}\right)$

Source: author's development.

The assumption of the average method of chain substitution is as follows. The period under analysis is examined discretely, i.e. in two moments T_0 and T_1 (the beginning of the base or planning period and the end of the reporting period), and the change of factor variables during the period $T_0 - T_1$ happen at the same time, i.e. the resulting indicator (P) in the interval of variation ($\Delta P = P_1 - P_0$) changes at a constant speed, i.e. direct. This assumption is similar to the integral method, the third variant of the simple addition of an indelible balance and the weighted finite differences methods.

TESTING OF THE AVERAGE METHOD OF CHAIN SUBSTITUTION FOR THREE- AND FOUR- MULTIPLES AND MULTIPLICATIVE-MULTIPLES FACTOR MODELS

Table 1 presents the new mathematical expressions obtained to determine the influence of individual factors using the average method of chain substitution for three- and four- multiples and multiplicative models.

When determining mathematical expressions for individual factor influences in three- and four- multiples factor models, it is necessary to lead the factor model to a simplified form of multiplicative factor model, as shown in the first three rows of the first column in *Table 1*. Otherwise, direct application of the average method of chain substitution will lead to erroneous mathematical expressions about individual factor influences, participating in the factor model.

The average method of chain substitution method was tested in MS Excel by assigning quantitative values to the base (planned) and actual values of the factor variables. During testing, a number of combinations of input values of factor variables were used to confirm the correctness of the obtained results of the derived mathematical expressions to determine individual factor influences of factor models presented in *Table 1*.

SYSTEMATIZATION OF FACTOR MODELS AND RECEIVED MATHEMATICAL EXPRESSIONS ABOUT FACTOR INFLUENCES BY AN AVERAGE METHOD OF CHAIN SUBSTITUTION

Table 2 presents in table form the systematization of factor models and derived formulas by average method of chain substitution to determine the influence of individual factors. Systematization is performed by types of factor models, namely: multiplicative; multiple; multiplicative-multiple and additive- multiple.

Table 2 shows that for factor models that contain more than two factor variables, the mathematical expressions obtained by the average method of chain substitution are significantly complicated to determine the influence of individual factors, i.e. as the number of factor variables (n) increases, mathematical expressions become more complex to determine the influence of individual factors. This disadvantage of the method is easily overcome by using predefined templates in spreadsheets or MS Excel.

CONCLUSION

The average method of chain substitution has the versatility of the chain substitution method and is characterized by the accuracy achieved by the integral method in multiplicative factor models for which both methods produce the same results. The average method of chain substitution has absolute accuracy as opposed to the integral method in a limited range of multiple and additive- multiple models developed for it. Therefore, the developed method is characterized by the following advantages over other DFA methods, namely: full versatility of types of factor models, accurate and unambiguity of the obtained results.

The mathematical expressions presented in *Table 2* for determining the influence of individual factors for multiplicative, multiple, additive-multiple and multiplicative-multiple factor models composed of two-, three- and four-factor variables are characterized by accurate, unambiguity and significantly expand the practical applicability of the average method of chain substitution in the practice of financial and economic analysis.

Methodology of the average method of chain substitution can also be used to determine the influence of individual factors and more complex factor models describing the relationship between the participating of factor variables and the resulting indicator. Certainly, the increase in the number of factor variables in the factor model leads to an increase in the number of combinations of consistency substitutions basic (planned) and actual values of factor variables in the construction of factor chains and the subsequent determination of the influence of individual factors. This significantly complicates, but does not make it virtually impossible to deduce mathematical expressions for the effects of individual factors on the change of the resulting indicator in five or more factor models, but it is a very laborious process, which will lead to more complex mathematical expressions to determine the influence of individual factors. This is the only but insurmountable disadvantage of the average method of chain substitution.

The average method of chain substitution can be easily applied to obtain mathematical expressions to quantify the influence of individual factors on change resulting indicator and for other mixed factor models not presented in *Table 2*.

REFERENCES

1. Yugenburg S.M. On the expansion of absolute increments by factors. In: Scientific notes on statistics. Vol. 1. Moscow: USSR Academy of Sciences; 1955:66–83. (In Russ.).
2. Humal A. The division of the multiplication increase. In: Scientific notes on statistics. Vol. 8. Moscow: USSR Academy of Sciences; 1964:206–212. (In Russ.).
3. Sheremet A.D. Theory of economic analysis. Moscow: Infra-M; 2002. 332 p. (In Russ.).
4. Sheremet A.D., Dei G.G., Shapovalov V.N. Chain substitution method and improvement of factor analysis of economic indicators. *Vestnik Moskovskogo universiteta. Seriya 6: Ekonomika = Moscow University Economics Bulletin.* 1971;(4):62–69. (In Russ.).
5. Adamov V.E. Factor index analysis (Methodology and problems). Moscow: Politizdat; 1977. 200 p. (In Russ.).
6. Fedorova V. Egorov Yu. On the issue of decomposing increase into factors. *Vestnik statistiki.* 1977;(5):71–73. (In Russ.).
7. Bakanov M.I., Sheremet A.D. Theory of economic analysis. Moscow: Finansy i statistika; 2001. 416 p. (In Russ.).
8. Chebotarev S.B. Method of Lagrange and Budan-Fourier theorem in economic factorial analysis. *Sistemy upravleniya i informatsionnye tekhnologii.* 2003;(1–2):30–35. (In Russ.).
9. Lyubushin N.P. Analysis of the financial and economic activities of the enterprise. Moscow: Unity-Dana; 2009. 471 p. (In Russ.).
10. Kremer N. Sh., ed. Higher mathematics for economic specialties (in 2 pts.). Moscow: Vysshee obrazovanie; 2005. 893 p. (In Russ.).
11. Lebedev K.N. Problems of factor analysis based on nethods of determined factor analysis (problems of science “economic analysis”). *ETAP: ekonomicheskaya teoriya, analiz, praktika = ETAP: Economic Theory, Analysis, and Practice*. 2012;(3):4–13. (In Russ.).
12. Prokofiev V.A., Nosov V.V., Salomatina T.V. Prerequisites and conditions for the development of deterministic factor analysis (problems of the science “economic analysis”). *ETAP: ekonomicheskaya teoriya, analiz, praktika = ETAP: Economic Theory, Analysis, and Practice*. 2014;(4):134–144. (In Russ.).
13. Savitskaya G.V. Theory of business analysis. Moscow: Infra-M; 2012. 288 p. (In Russ.).
14. Ross S.A., Westerfield R.W., Jaffe J.F. Corporate finance. 2nd ed. Homewood, IL: Irwin; 1990. 833 p.
15. Foster G. Financial statement analysis. Englewood Cliffs, NJ: Prentice-Hall; 1996. 704 p.
16. Emery D.R., Finnerty J.D., Stowe J.D. Corporate financial management. 2nd ed. Englewood Cliffs, NJ: Prentice-Hall; 2004. 825 p.

17. Wild J.J., Bernstein L.A., Subramanyam K.R. Financial statement analysis. 7th ed. New York: McGraw-Hill/Irwin; 2001. 1040 p.
18. Brealey R., Myers S.C, Allen F. Corporate finance. New York: McGraw-Hill/Irwin; 2006. 969 p.
19. Mitev V. The method of chain substitutions — practical application in financial business analysis: Advantages and shortcomings. *Annual of University of Mining and Geology "St. Ivan Rilski"*. 2008;51(Pt.4):45–48. (In Bulgar.).
20. Mitev V. Averaged chain substitution method. *Ikonomiceski i Sotsialni Alternativi = Economic and Social Alternatives*. 2020;(4):90–100. (In Bulgar.). DOI: 10.37075/ISA.2020.4.09
21. Mitev V. Averaged chain substitution method — applicability, advantages, and disadvantages. *Ikonomiceski i Sotsialni Alternativi = Economic and Social Alternatives*. 2021;(2):127–138. (In Bulgar.). DOI: 10.37075/ISA.2021.2.08

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