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Game-Theoretic Model for Stimulating High Performance of Regional Civil Servants in Russia

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ABSTRACT

The priority goal of the country's top leadership is to ensure sustainable socio-economic development of all constituent entities of the Russian Federation and improve the quality of life of their population. In a challenging geopolitical situation, achieving this goal is difficult. Only under the condition of an effective system of public administration is it possible to solve the main socio-economic problems in the Russian regions. This requires linking the size of collective incentive payments to regional civil servants to the achieved level of socio-economic development of the constituent entities of the Russian Federation. The foregoing predetermined the **relevance** of the research topic. The **purpose** of the paper is to substantiate the bonus calculation mechanism for the executive branch employees of the Russian regions, depending on the assessment of their ability to work as part of a team to achieve results, i.e. to perform their duties to a high standard. This involves the use of modern **methods** of economic and mathematical modeling, designed in this case to ensure the objectivity of assessment of the collective and individual performance (efficiency) of regional civil servants. This is the main scientific **novelty** of the paper. The **practical** implementation of the mechanism of collective and individual incentives for regional civil servants will create the preconditions for increasing the wages of such employees in all regions of the Russian Federation. Thus, the coordinated actions of civil servants from different ministries and departments of the Russian subsidized regions in the future will help increase their financial security and transition to the group of donor regions, and ultimately will create the possibility of increasing budget expenditures on wages for the executive branch employees.

Keywords: efficiency and effectiveness; regional civil servants; collective and individual incentive payments; game-theoretic model

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INTRODUCTION

Further development of the civil service in Russia is possible based on the positive experience of Singapore. So, in Singapore, unlike Russia, there is a direct relationship between the level of remuneration of civil servants and its socio-economic development. In addition, in Singapore, the salaries of civil servants are high, which helps to reduce the level of corruption risks. The competence of such employees (when hiring, the academic knowledge of the applicant is first of all evaluated) is the key to their individual effectiveness. The author's approach to the possibility of applying this practice in Russia is presented in [1–3].

It should be noted that at present there are quite a lot of studies devoted to assessing the effectiveness of the national public administration system. Researchers pay much less attention to the development of the state civil service in the constituent entities of the Russian Federation. At the same time, there are no scientific articles that use the game-theoretic approach, both for collective and individual stimulation of regional civil servants to achieve results.

The foregoing confirms the relevance of the chosen research topic. At the same time, its main purpose is to determine the mechanism for the correct transition from individual to collective stimulation of regional civil servants to achieve results using a game-theoretic approach. This purpose predetermined the solution of some tasks and the logic of the presentation of the material, and ultimately the structure of the paper.

LITERATURE REVIEW

The country's top leadership, the scientific and expert community of Russia [4–7], as well as international organizations, show a consolidated position, believing that the main factor hindering the sustainable socio-economic development of the Russian Federation (in particular, improving the quality of life of the population) is the low efficiency of the national public administration systems. As part of the paper, relying on the studies of well-known Russian scientists, we focus on a critical analysis of such a system, clarifying the cause-and-effect relationships between the quality of public administration and the socio-economic development of the country.

For example, the Corresponding Member of the Russian Academy of Sciences V.A. Ilyin, who regularly assesses the effectiveness of the Russian public administration system, emphasizes that the course of national development proposed by the President of Russia V. V. Putin is opposed not only by the "fifth" but also by the "sixth" column. Developing the idea, V. A. Ilyin notes that while the country's top leadership managed to practically neutralize the negative influence of the "fifth" column, the "sixth" column "still continues to strengthen its position in the ruling elite of the country, largely due to the system of oligarchic capitalism it created" [8, p. 12].

It should be noted that a fairly large number of scientific papers are devoted to assessing the possibility of an effective response of the state to the main socio-economic challenges for Russia under the conditions of oligarchic capitalism [9–13]. At the same time, as a rule, researchers are of the opinion that it is impossible to effectively solve socio-economic problems (in particular, this concerns the fight against poverty) in the current situation. Moreover, they note a threat to the foundations of statehood and national security.

However, there is another opinion. For example, in scientific papers [14, 15] it is noted that even under the conditions of oligarchic capitalism in Russia, sustainable economic growth is possible and, as a result, the solution to the most acute social problems. But for this, it is necessary to ensure the effective functioning of the public administration system.

Table 1

Dynamics of the Number of State Civil Servants of the Russian Federation in 2014–2020

Indicator	2014	2015	2016	2017	2018	2019	2020
Number of employees in the federal state executive bodies of Russia, thousand people	1250	1226.2	1203.4	1231.5	1222.1	424.7	355
including:							
at the regional level of administration	1212.2	1188.9	1165.9	1193.9	1184.5	393.9	324.2
Number of employees in state executive bodies of the constituent entities of Russia, thousand people	205.9	204.1	203	206.5	203.5	166.1	168.1

Source: compiled by the authors Russian Statistical Yearbook 2021: Stat. book. M.: Rosstat; 2021. URL: https://rosstat.gov.ru/storage/mediabank/Ejegodnik_2021.pdf (accessed on 12.12.2022).

Note: Data at the end of each year.

At the same time, most researchers [11, 16, 17] believe that in addition to the patriotic policy implemented by the President of the Russian Federation V.V. Putin, the country is in dire need of an adequate model of socio-economic development, which, as a rule, offers its own "recipe" for the well-being of Russia. For example, the Academician of the Russian Academy of Sciences S. Yu. Glaz'ev [11] considers the possibility of sustainable economic growth in the country through the perspective of changing technological patterns.

The administrative reform being carried out in Russia does not affect the issues of changing the organization of regional executive authorities. Also, no attention was paid to such an important tool in a market economy as a tool to improve the efficiency of the public administration system, as material incentives for civil servants "based on results" [18, 19].

At the same time, it should be noted that in the scientific community, there is still a debatable question about the influence of the effectiveness of public administration on the socio-economic development of the country. Thus, in studies [20, 21], through empirical research, it was proved that not only the efficiency (effectiveness) of public administration but also the level of corruption control has a significant impact on the country's economic growth rates. A scientific article [22] asserts an inverse causal relationship between the quality of public administration and economic growth. The author, in the course of an empirical study, concluded that economic growth is a driver for increasing the efficiency (effectiveness) of the public service, and not vice versa. There are a number of papers [23–27], that focus on the fact that, along with the quality of public administration, a number of other factors also affect the rate of economic growth. At the same time, it can be significantly higher than the efficiency (effectiveness) of public administration. Finally, there are scientific articles [28], where the hypothesis about the relationship of the above categories (for a number of relevant indicators) is not confirmed.

Given the inconsistency of assessments of the relationship between the quality of public administration and socio-economic development, the authors conducted their own (thematic) empirical study as part of the study.

STATE CIVIL SERVICE IN RUSSIA: STATUS, PROBLEMS AND DEVELOPMENT PROSPECTS

The research will be limited to studying changes in such key indicators as the number of civil servants and the size of their wages. At the same time, civil servants are understood only as employees of the executive branch of the country.

Table 1 date (taking into account the change in the number of federal and regional civil servants) show that 2015–2018 can be considered a relatively stable period. In 2019, the situation changed dramatically: there was a sharp decrease (by 2.9 times) in the number of employees in the federal state executive bodies of the Russian Federation due to a threefold decrease in the number of employees at the regional level of governance. This was caused by the digitalization of the public administration system as a result of the implementation of the federal project of the same name (within the framework of the national program "Digital Economy"). Automation of public services rendered to legal entities and the population in the Russian regions has led to a significant release of labor resources from the territorial divisions of the federal executive authorities of the country. Also, significantly (by 22.5%) the number of employees in state executive bodies of the constituent entities of the Russian Federation decreased. In 2020, for the above reason, the trend of reducing the number of employees in the federal state executive bodies of the country continued. As a result, the value of this indicator decreased significantly (by 21.5%). At the same time, the number of employees in state executive bodies of the constituent entities of the Russian Federation has not practically changed.

Summarizing the above, we can conclude that in 2019–2020 there was a significant reduction in the number of predominantly federal civil servants in the country (due to the optimization of the number of employees in the territorial divisions). At the same time, the number of regional civil servants decreased significantly only in 2019. Hence, we can make a preliminary conclusion that at present there are prerequisites for increasing the level of remuneration, mainly for the federal civil servants of the country.

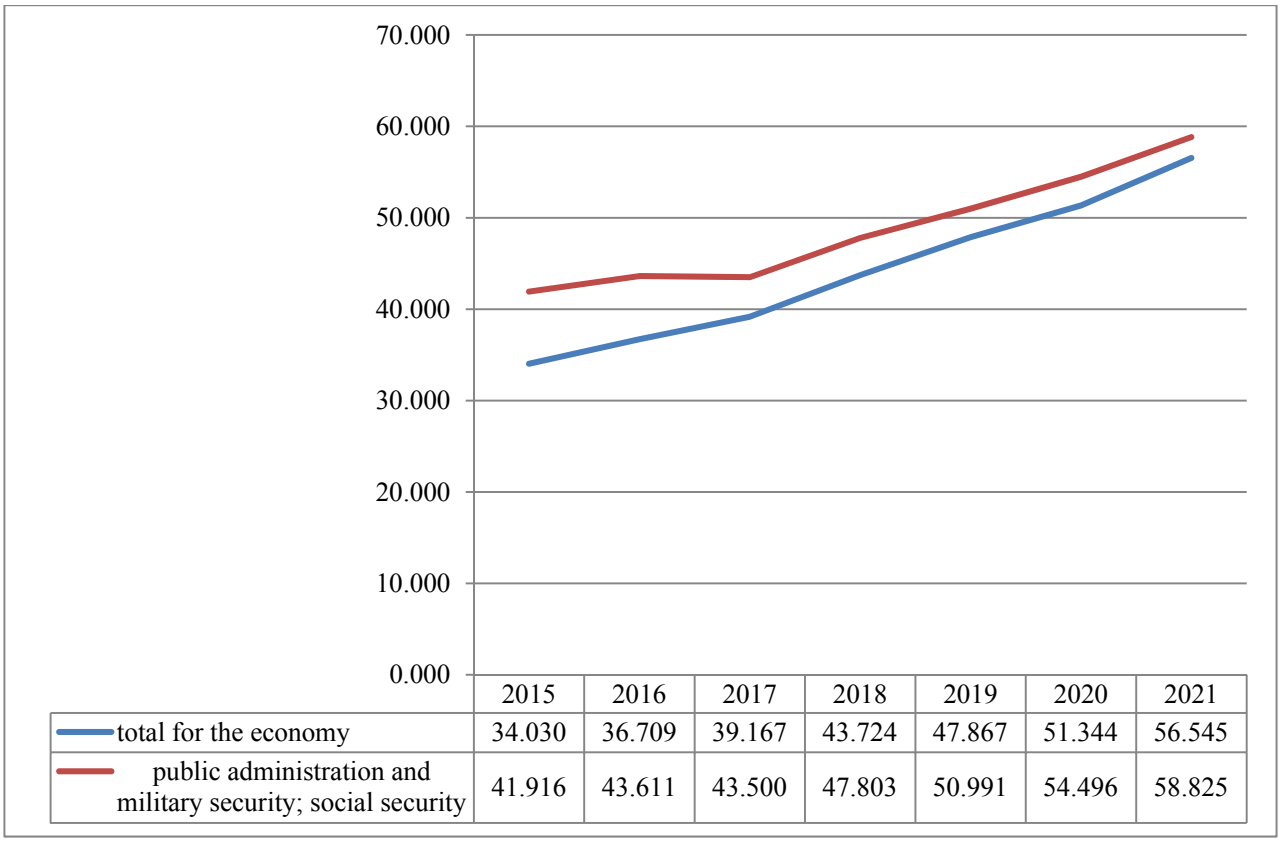
There are no statistics on the salaries of civil servants in the country. It is only possible to analyze the change in the level of wages of all those working in the country's public administration system (*Fig. 1*).

According to the data in *Fig. 1*, in general in the country's economy there was a steady upward trend in wages throughout the analyzed period, and in the public administration system growth began only in 2017. At the same time, it should be noted that the level of wages is higher than the national average. However, in 2015–2021, there was an annual decline in the ratio of indicators. So, if in 2015–2016 wages of employees of the public administration system exceeded the average Russian value of the indicator by 23.2 and 18.8%, respectively, then in 2020–2021 — only by 6.1 and 4%. In our opinion, such a relative decrease in the dynamics of the level of remuneration of workers in the public administration system of Russia is a negative trend.

Indeed, in conditions of relatively low wages, it becomes economically more profitable for a state civil servant to formally perform official duties and participate in corruption schemes.

GAME-THEORETICAL MODEL OF MATERIAL INCENTIVES OF REGIONAL CIVIL EMPLOYEES BASED ON ACHIEVED RESULTS

The main idea that should be taken into account when stimulating civil servants is that individual performance is assessed based on a system of key performance indicators (KPIs), but with the obligatory consideration of the achieved level of socio-economic development of the Russian region. At the same time, it is believed that any civil servant in the case of



	2015	2016	2017	2018	2019	2020	2021
total for the economy	34.030	36.709	39.167	43.724	47.867	51.344	56.545
public administration and military security; social security	41.916	43.611	43.500	47.803	50.991	54.496	58.825

Fig. **Dynamics of wages of employees of all organizations of the Russian Federation and those working in the public administration system, thous. rub**

Source: Compiled by the authors according to official statistics. URL: https://rosstat.gov.ru/labor_market_employment_salaries (accessed on 12.12.2022).

rational behavior seeks to increase the value of such indicators and receive a large bonus. It is also natural to assume that the task of the leadership of any region of Russia is to choose a motivation system that encourages civil servants to work not only efficiently (effectively) but also intensively (for example, they are motivated to perform to high standards and within short timeframes).

Due to the fact that civil servants carry out different types of professional service activities, for the correct application of the mechanism of individual incentives, a comprehensive assessment of their efficiency (effectiveness) is necessary.

Problem Statement

If the i-th civil servant performs b_i types of activities, then he, respectively, is characterized by the vector of indicators r_{ij}, $i \in N\{1,2,...,n\}$, $j = 1,2,...,b_i$, reflecting his competence (qualification) for each of them. Here N — a set of civil servants.

The analysis of the incentive system is proposed to be carried out on the team model, which is a two-level hierarchical system consisting of the Center (in our case, this is the top management of the region) — the upper-level and n lower-level agents (civil servants). We believe that when doing the job, the strategy of the i-th agent is to choose actions $x_i \in A_i$, $i \in N\{1,2,...,n\}$. In terms of content, the actions of an agent can be characterized by the following indicators: the number of hours worked, the number and quality of decisions made, the time for making them, etc. The agent's action x_i will be considered to belong to the set of non-negative real numbers. Hence, the indicator of the agent's activity υ — a certain function that depends on his actions.

We will also assume that the individual costs of the i-th agent when performing the j-th action are a function of the following form: $z_{ij} = f\left(x_{ij}, r_{ij}\right)$, where $i \in N$, $j = 1,2,\ldots,b_i$. Such a function decreases monotonically with increasing skill r_{ij}.

When stimulating agents, it is necessary to determine how the bonus fund should be formed. Considering that the amount of incentive payments to civil servants in Russia is not based on the achieved level of the country's socio-economic development [2], it is advisable to establish such a dependence.

Let the basic fund be F, formed for rewarding agents, I_c — an indicator of the level of socio-economic development of the country, I_{pq} — an indicator of the level of socio-economic development of the q-th region. In this case, for the bonuses for agents of the q-th region, the fund will be determined according to the following rule:

$$F_{pq} = \begin{cases} F\dfrac{I_{pq}}{I_c}, if\ I_{pq} \le I \\ F, if\ I_{pq} > I_c. \end{cases} \quad (1)$$

Accordingly, the bonus of the i-th agent of the q-th region is calculated by the formula:

$$P_{iq} = F_{pq}\frac{\upsilon_{iq}}{\sum\limits_{i=1}^{m}\upsilon_{iq}}, \quad (2)$$

where υ_{iq} — the performance indicator of the i-th agent of the q-th region, and m — the number of agents who claim a bonus from the fund F_{pq}.

The procedure for distributing the bonus fund among agents should contribute to solving the main task — increasing the efficiency of the entire team. In particular, it is designed to stimulate the quality of decisions made and reduce the time for their implementation.

Obviously, an increase in the intensity of the work of agents entails an increase in the cost of the actions performed. Without loss of generality, we will examine the actions of agents of one region considered for bonuses, based on the results of which an incentive fund was formed F_p.

In this case, we will proceed from the fact that all actions of the agent are carried out in accordance with his duties, reflected in the job description. Therefore, to determine the contribution of each agent to the final result of the team, an assessment of the performance of official duties by civil servants is used.

To distribute the bonus fund F_p the Center evaluates the performance of each agent in the performance of his duties as the ratio of the actual to the best result. We assume that in the job description of the i-th agent b_i items in order to perform the j-th item, the agent needs to perform the j-th action. The best result that can be achieved by the agent when performing the j-th action is denoted by X_{ij}, $i \in N$, $j = 1,2,\ldots,b_i$. If the action or the actual result of the agent is equal to x_{ij}, then the indicator of activity for this action is determined according to the following rule:

$$\upsilon_{ij} = \begin{cases} \dfrac{x_{ij}}{X_{ij}}, if\ X_{ij} = \max\left\{x_{ij}\right\} \\ \dfrac{X_{ij}}{x_{ij}}, if\ X_{ij} = \min\left\{x_{ij}\right\}. \end{cases} \quad (3)$$

The resulting performance indicator υ_i or a comprehensive assessment of the performance of all actions according to the job description of the i-th agent is characterized by the expression $\upsilon_i = f_i\left(\upsilon_{i1}, \upsilon_{i2}, \ldots, \upsilon_{ib_i}\right)$, where f_i — the convolution function, and the contribution of the i-th agent to the results of the activity of the entire team is defined as the ratio of the indicator of his activity to the total of the indicators of the activity of all agents.

Then we will assume that the agent's individual costs are linear and separable. When the i-th agent performs the j-th action x_{ij}, his costs are presented in the form $z_{ij} = \dfrac{x_{ij}}{r_{ij}}$, $j = 1,2,\ldots,b_i$, accordingly, the total costs of the

agent z_i in the performance of all items of duties from the job description are defined as

$$z_i = \sum_{j=1}^{b_i} \frac{x_{ij}}{r_{ij}},\ i \in N.$$

The difference between the bonus P_i and the costs of a civil servant z_i determines his target function:

$$F_i = F_p \frac{\upsilon_i}{\sum_{i=1}^{n} \upsilon_i} - \sum_{j=1}^{b_i} \frac{x_{ij}}{r_{ij}}. \quad (4)$$

Since the value of the target function of the i-th agent depends on his actions, which are estimated by the Center as υ_{ij}, $i \in N$, $j = 1,2,\ldots,b_i$, then, within the framework of the hypothesis of rational behavior, the agent will choose actions that, under the chosen incentive system, maximize his target function.

Let us assume that the efficiency of the simulation system when performing several jobs will be determined by the total of their complex assessments received by agents for each type of activity, i.e. calculated according to the formula:

$$K = \sum_{i=1}^{n} \upsilon_i, \quad (5)$$

To determine an effective incentive system, the Center can apply various options for the formation of a comprehensive assessment of the activity of the i-th agent.

We confine ourselves to considering four main procedures for determining a comprehensive assessment:

1. Comprehensive assessment — the total of all performance assessments:

$$\upsilon_i = f_i(\upsilon_{i1}, \upsilon_{i2}, \ldots, \upsilon_{ib_i}) = \sum_{j=1}^{b_i} \upsilon_{ij}. \quad (6)$$

2. Comprehensive assessment — the arithmetic mean of all assessments:

$$\upsilon_i = f_i(\upsilon_{i1}, \upsilon_{i2}, \ldots, \upsilon_{ib_i}) = \frac{1}{b_i} \sum_{j=1}^{b_i} \upsilon_{ij}. \quad (7)$$

3. Comprehensive assessment — the minimum value of all assessments:

$$\upsilon_i = f_i(\upsilon_{i1}, \upsilon_{i2}, \ldots, \upsilon_{ib_i}) = \min_j \{\upsilon_{ij}\}. \quad (8)$$

4. Comprehensive assessment — geometric mean of all assessments:

$$\upsilon_i = f_i(\upsilon_{i1}, \upsilon_{i2}, \ldots, \upsilon_{ib_i}) = \left[\prod_{j=1}^{b_i} \upsilon_{ij}\right]^{\frac{1}{b_i}}. \quad (9)$$

It should be noted that the method of constructing a complex assessment based on logical convolution matrices can also be applied to assess the activities of agents [29, 30].

Analysis of Procedures for Evaluating the Effectiveness of the Agent's Activities

The target function of the i-th agent (4) depends on both the individual and collective performance of civil servants, so the functioning of such a system is considered as a n-person game, and the effectiveness of its stimulation is determined based on the evaluation of the agents' activities obtained in the Nash equilibrium.

For the objective function (8), the Nash equilibrium is found as a result of solving the system of equations:

$$\begin{cases} \dfrac{\partial F_i}{\partial x_{ij}} = F_p \dfrac{\partial \upsilon_i}{\partial x_{ij}} \dfrac{\sum_{q=1}^{n} \upsilon_q - \upsilon_i}{\left[\sum_{q=1}^{n} \upsilon_q\right]^2} - \dfrac{1}{r_{ij}} = 0, \\ i \in N, \\ j = 1,2,\ldots,b_i \end{cases} \quad (10)$$

Without loss of generality, we will assume that $\upsilon_{ij} = \frac{x_{ij}}{X_{ij}}$. Let us consider the case when υ_i is determined in accordance with (6), i.e. $\upsilon_i = \sum_{j=1}^{b_i} \frac{x_{ij}}{X_{ij}}$, then

$\frac{\partial \upsilon_i}{\partial x_{ij}} = \frac{\partial \sum_{j=1}^{b_i} \frac{x_{ij}}{X_{ij}}}{\partial x_{ij}} = \frac{1}{X_{ij}}$, and the expression (10) can be rewritten as:

$$\begin{cases} F_p \dfrac{1}{X_{ij}} \dfrac{\sum_{q=1}^{n} \upsilon_p - \upsilon_i}{\left[\sum_{q=1}^{n} \upsilon_q\right]^2} = \dfrac{1}{r_{ij}}, \\ i \in N, \\ j = 1,2,\ldots,b_i. \end{cases} \quad (11)$$

or

$$\begin{cases} \dfrac{\sum_{q=1}^{n} \upsilon_q - \upsilon_i}{\left[\sum_{q=1}^{n} \upsilon_q\right]^2} = \dfrac{X_{ij}}{F_p r_{ij}}, \\ i \in N, \\ j = 1,2,\ldots,b_i. \end{cases} \quad (12)$$

Denoting $y_{ij} = \frac{X_{ij}}{r_{ij}}$, it is clear that these are the costs of the i-th agent when the agent obtains the best result in the course of performing the j-th action. In this case, the assessment of the activity of agents is found from the solution of the system of equations:

$$\begin{cases} \dfrac{\sum_{q=1}^{n} \upsilon_q - \upsilon_i}{\left[\sum_{q=1}^{n} \upsilon_q\right]^2} = \dfrac{y_{ij}}{F_p}, \\ \upsilon_{ij} \le 1, \\ i \in N, \\ j = 1,2,\ldots,b_i. \end{cases} \quad (13)$$

It is easy to see that when determining their actions in a Nash equilibrium, agents will first perform those actions that require the least cost to obtain the best result. In this case, it is beneficial for the agent to perform not all actions but only some of them.

A similar situation arises when the arithmetic means of all estimates (7) is used to build a comprehensive performance assessment. It follows that even if the efficiency of the simulation system $K^{(6)} = K^{(7)} = \sum_{i=1}^{n} \upsilon_i$ takes on a sufficiently high value, the agents do not perform all activities.

Let us further consider the case when υ_i is determined in accordance with (8), i.e.

$$\upsilon_i = f_i\left(\upsilon_{i1}, \upsilon_{i2}, \ldots, \upsilon_{ib_i}\right) = \min_j \left\{\upsilon_{ij}\right\}.$$

Obviously, in this case, the agent chooses such actions in order to provide the same ratings for all items of the job description, i.e. $x_{i1} = x_{i2} = \ldots = x_{ib_i}$. Let us denote $\hat{\upsilon}_i = \min_j \upsilon_{ij}$, $j = 1,2,\ldots,b_i$.

Hence, the objective function of the agent can be represented as:

$$\begin{cases} F_{\dot{p}} = F \dfrac{\hat{\upsilon}_i}{\sum_{q=1}^{n} \hat{\upsilon}_q^i} - \hat{\upsilon}_{ij} \sum_{j=1}^{b_i} y, \\ i \in N. \end{cases} \quad (14)$$

The values $\hat{\upsilon}_i$, $i \in N$ in the Nash equilibrium are found based on the solution of the system of equations:

$$\begin{cases} F_p \dfrac{\sum_{q=1}^{n} \hat{\upsilon}_q - \hat{\upsilon}_i}{\left(\sum_{q=1}^{n} \hat{\upsilon}_q\right)^2} - \sum_{j=1}^{b_i} y_{ij} = 0, \\ i \in N. \end{cases} \quad (15)$$

The solution to system (15) is written as:

$$\begin{cases} \hat{\upsilon}_{\dot{p}} = F \dfrac{n-1}{\sum_{q=1}^{n} \sum_{j=1}^{b_i} y_{ij}} \left(1 - \dfrac{n-1}{\sum_{q=1}^{n} \sum_{j=1}^{b_i} y_{ij}} \sum_{j=1}^{b_i} y\right), \\ i \in N. \end{cases} \quad (16)$$

We denote $A_i = \frac{1}{b_i}\sum_{j=1}^{b_i} y_{ij}$ (the arithmetic mean of the costs when agents obtain the best result), then

$$\begin{cases} \hat{\upsilon}_{\dot{p}} = F\ \dfrac{n-1}{\sum\limits_{q=1}^{n} b_q A_q}\left(1-\dfrac{n-1}{\sum\limits_{q=1}^{n} b_q A_q}\right). \\ i \in N. \end{cases}$$

Given the above, the following equality is logical:

$$\upsilon_{\dot{p}1} = \upsilon_{i2} = \ldots = \upsilon_{ib_i} = F\ \frac{n-1}{\sum\limits_{q=1}^{n} b_q A_q}\left(1-\frac{n-1}{\sum\limits_{q=1}^{n} b_q A_q} b\,A\right). \quad (17)$$

In this case, it is obvious that the condition must be satisfied $\hat{\upsilon}_i \le 1$, which allows us to determine the restriction on the fund F_p:

$$F_p \le \frac{\left(\sum\limits_{q=1}^{n} b_q A_q\right)^2}{(n-1)\left(\sum\limits_{q=1}^{n} b_q A_q - (n-1)\min\limits_{i}\{b_i A_i\}\right)}. \quad (18)$$

Hence, the effectiveness of the incentive system is calculated as follows:

$$K^{(8)} = F_p\ \frac{n-1}{\sum\limits_{i=1}^{n} b_i A_i}. \quad (19)$$

Let us now consider the case when the performance assessment υ_i is determined in accordance with formula (9), we obtain

$$\frac{\partial \upsilon_i}{\partial x_{ij}} = \frac{1}{x_{ij} b_i}\left[\prod_{q=1}^{n} \upsilon_{iq}\right]^{\frac{1}{b_i}},$$

and expression (10) in this case can be represented as:

$$\frac{\partial F_i}{\partial x_{ij}} = F_p\ \frac{1}{b_i}\left[\prod_{q=1}^{b_i} \upsilon_{iq}\right]^{\frac{1}{b_i}} \frac{\sum\limits_{q=1}^{n} v_q - \upsilon_i}{\left[\sum\limits_{q=1}^{n} \upsilon_q\right]^2} - \frac{x_{ij}}{r_{ij}} = 0 \quad (20)$$

or

$$F_p\ \frac{1}{b_i}\left[\prod_{q=1}^{b_i} \upsilon_{iq}\right]^{\frac{1}{b_i}} \frac{\sum\limits_{q=1}^{n} \upsilon_q - \upsilon_i}{\left[\sum\limits_{q=1}^{n} \upsilon_q\right]^2} = \upsilon_{ij} y_{ij}. \quad (21)$$

The solution of system (21) is written as follows:

$$\tilde{\upsilon}_{\dot{p}} = F\ \frac{n-1}{\sum\limits_{q=1}^{n} b_q G_q}\left(i\,1 - \bar{i}\,b\,G\ \frac{n-1}{\sum\limits_{q=1}^{n} b_q G_q}\right). \quad (22)$$

Here $G_i = \left(\prod\limits_{j=1}^{b_q} y_{iq}\right)^{\frac{1}{b_i}}$.

In this case, the condition must also be satisfied $\tilde{\upsilon}_i \le 1$, which also allows us to determine the restriction on the fund F_p:

$$F_p \le \frac{\left(\sum\limits_{q=1}^{n} b_q G_q\right)^2}{(n-1)\left(\sum\limits_{q=1}^{n} b_q G_q - (n-1)\min\limits_{i}\{b_i G_i\}\right)}. \quad (23)$$

In this case, the effectiveness of the incentive system is determined as follows:

$$K^{(5)} = F_p\ \frac{n-1}{\sum\limits_{i=1}^{n} b_i G_i}. \quad (24)$$

Values $\{\upsilon_{ij}\}$, $i \in N$, $j = 1,2,\ldots,b_i$ can be calculated by solving the problem:

$$\begin{cases} \sum\limits_{j=1}^{b_i} \dfrac{x_{ij}}{r_{ij}} = \sum\limits_{j=1}^{b_i} \dfrac{x_{ij}}{X_{ij}} \dfrac{X_{ij}}{r_{ij}} = \sum\limits_{j=1}^{b_i} \upsilon_{ij} y_{ij} \to \min, \\ \prod\limits_{j=1}^{b_i} \upsilon_{ij} = \upsilon_i^{b_i}. \end{cases} \quad (25)$$

Table 2

Evaluation of the Effectiveness of the Agent Incentive System

Comprehensive assessment	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	υ_{11}	υ_{12}	υ_{13}	υ_{21}	υ_{22}	υ_1	υ_2	K
Comprehensive assessment – the total of all performance assessments	0	6.2	12	0	13.9	0	0.62	1	0	0.93	1.62	0.93	2.55
Comprehensive assessment – the minimum value of all assessments	4.56	5.7	6.85	4.98	7.47	0.57	0.57	0.57	0.5	0.5	0.57	0.498	1.068
Comprehensive assessment – geometric mean of all assessments	4.15	5.81	7.46	4.98	7.46	0.52	0.58	0.62	0.5	0.5	0.57	0.5	1.07

Source: Compiled by the authors.

In turn, the solution to this problem can be written as follows:

$$\upsilon_{ij} = \frac{G_i}{y_{ij}}\upsilon_i . \qquad (26)$$

Assertion. The effectiveness of the incentive system for integrated assessment (9) is higher than for integrated assessment (8).

Proof. Comparing $K^{(9)}$ and $K^{(8)}$, we obtain the following inequality:

$$F_p \frac{n-1}{\sum_{i=1}^{n} b_i G_i} > F_p \frac{n-1}{\sum_{i=1}^{n} b_i A_i} . \qquad (27)$$

In this case, the value F_p satisfies the condition:

$$F_p \le \min\left\{ \frac{\left(\sum_{q=1}^{n} b_q G_q\right)^2}{(n-1)\left(\sum_{q=1}^{n} b_q G_q - (n-1)\min\limits_i \{b_i G_i\}\right)} \right\};$$

$$\frac{\left(\sum_{q=1}^{n} b_q A_q\right)^2}{(n-1)\left(\sum_{q=1}^{n} b_q A_q - (n-1)\min\limits_i \{b_i A_i\}\right)}$$

From (27) follows the fulfillment of the inequality:

$$\sum_{i=1}^{n} b_i A_i > \sum_{i=1}^{n} b_i G_i . \qquad (28)$$

Cauchy's mean inequality allows us to state that $K^{(9)} > K^{(8)}$. This means that the incentive system when constructing a comprehensive assessment in the form of a geometric mean of the agents' activity (9) gives a greater effect than the incentive system when constructing a comprehensive assessment in the form of the minimum value of all obtained assessments of the agents' activity (8). We will support the previously given theoretical calculations with calculations.

Example. Let $n=2$, $b_1=3$, $b_2=2$, $F_p=100$, $r_{11}=0,5$, $r_{12}=0,7$, $r_{13}=0,9$, $r_{21}=0,4$, $r_{22}=0,6$, $X_{11}=8$, $X_{12}=10$, $X_{13}=12$, $X_{21}=10$ and $X_{22}=15$.

Table 2 shows the values of agents' actions (x_{ij}), their performance assessment (υ_{ij}), a comprehensive assessment (υ_i) and the indicator of the effectiveness of the incentive system (K) in a Nash equilibrium for various methods of forming a comprehensive performance assessment.

It should be emphasized that the choice of various convolution functions in the formation of a comprehensive assessment of the activities of agents allows the Center to influence their strategy, i.e. if necessary, adjust, and regulate the situation development. Thus, the game-theoretic approach makes it possible to correctly establish the relationship between the size of the collective-individual bonuses for regional civil servants and the achieved level of socio-economic development of the constituent entity of the Russian Federation.

CONCLUSION

A literature review allows us to make an unambiguous conclusion about the relevance for modern Russia of improving the efficiency of the management system, and above all at the regional level.

At present, most of the constituent entities of the Russian Federation are subsidized, and this does not allow their top management to solve the problem of raising the wages of regional civil servants. This is possible through the introduction of a new (mixed or hybrid) wage system based on performance. At the same time, bonus payments make it possible to interest regional civil servants in ensuring sustainable socio-economic development of the constituent entities of the Russian Federation and ultimately contribute to their transition from the subsidized group to donor regions. In the framework of this study, in contrast to the previous studies of the author, attention is focused on ensuring the correct determination of the amount of collective-individual incentive payments to civil regional employees by applying the game-theoretic approach.

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***Authors' declared contribution*:**
R.V. Gubarev — literature review.
E.I. Dzyuba — an empirical study on the development of the state civil service in the Russian regions.
F.S. Fayzullin — abstract, introduction, conclusions, general version of the article.
A.G. Chkhartishvili — problem statement (economic and mathematical modeling).
A.V. Shchepkin — evaluation of the performance of civil servants (agents) based on a game-theoretic approach.

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