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On the Yield to Maturity of a Coupon Bond

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ABSTRACT

The article is devoted to one of the main characteristics of the bond-the yield to maturity. **The subject** of the study is the type of yield to maturity indicator. It is known, that there are two approaches to determining the yield to maturity of a bond: the nominal interest rate and the effective interest rate method. **The relevance** of the study is due to the fact that, as preliminary comparison has shown, these two approaches to determining the yield to maturity may be unequal in research. **The purpose** of this paper is to conduct a study of the dependence of the research results on the type of yield to maturity indicator. For this purpose, the problem of the dependence of the interest rate risk of a bond on the number of coupon payments per year was chosen. The literature contains reports on the dependence on the frequency of coupon payments of the duration of a bond that evaluates interest rate risk. The problem of the dependence directly of the interest rate risk of a bond on the number of coupon payments per year has not been considered in the literature. The task was set to determine which of the two approaches to determining the yield to maturity allows us to obtain results for interest rate risk that are consistent with the dependence of the duration of the bond on the number of coupon payments per year. **Methods** of differential calculus are used to solve the problem. As a **result,** it was proved that the use of the yield to maturity determined by the effective interest rate method allows us to obtain results consistent with the dependence of the duration of the bond on the number of coupon payments per year. The results obtained by using the yield to maturity determined by the nominal interest rate method do not agree with the dependence of the duration of the bond on the number of coupon payments per year. It is **concluded** that the yield to maturity determined by the nominal interest rate method in researches may lead to incorrect results, in contrast to the yield to maturity in the form of an effective interest rate. Results of the work can be useful to both the bond issuer and the investor, as well as in theoretical studies of investments in bonds.

Keywords: yield to maturity; mathematical methods; effective interest rate; nominal interest rate; interest rate risk of bonds; number of coupon payments per year

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INTRODUCTION

By definition, annual yield to maturity (YTM) — is a compound interest rate at which the current (present) value of the expected bond payment flow is equal to its current price. The type of discount rate depends on the approach to determining the YTM indicator. There are two approaches to determining annual yield to maturity bonds[1] — the nominal interest rate and the effective interest rate methods. According to the first approach, if coupon payments on bond, the price of which P, are m times a year, then the annual nominal rate for yield to maturity $r^{(m)}$, corresponding to compound interest m times a year is applied for discounting the cash flow:

$$P=\sum_{i=1}^{n}\frac{q}{\left(1+r^{(m)}/m\right)^{m\,t_i}}+\frac{A}{\left(1+r^{(m)}/m\right)^{m\,T}},$$

where A — bonds' nominal value; q — amount of each coupon payment; t_i years (i = 1, 2,..., n) — payment terms for coupons; T years — maturity period. YTM as annual nominal interest rate $r^{(m)}$ is used under the Fair Credit Reporting Act[2] (the USA, 1969). According to this Act the annual rate of yield to maturity is determined as follows: 1) yield to maturity is calculated for the period equal to the minimum interval between coupon payments, i.e. for the coupon period equal to $1/m$ year; 2) the interest rate received is multiplied by the number of coupon periods per year m. This rule of calculating the annual yield to maturity is called the market agreement,[3] adopted "to reduce the problems" of market participants. On the basis of this Act in the markets it is accepted to consider the yield to maturity is the annual nominal rate of return [1, p. 65].

Another approach, the effective interest rate method uses an annual rate r to discount cash flow, corresponding to compound interest once a year:

$$P=\sum_{i=1}^{n}\frac{q}{\left(1+r\right)^{t_i}}+\frac{A}{\left(1+r\right)^{T}}.$$

As follows from the papers [2–8], the calculation of YTM by the method of effective interest rate is used in the Russian securities market, in particular, on the Moscow Stock Exchange. According to [7, 8] the method of calculating the yield to maturity of the coupon system of public debt in the form of an effective interest rate is regulated by the Bank of Russia (OFZ-AD, OFZ-PK2 etc.).[4]

F. J. Fabozzi refers to the rate $r^{(m)}$ as approximation [1, p. 62], which may be explained by the rate $r^{(m)}$ origin. As we have seen, according to the Fair Credit Reporting Act, the rate $r^{(m)}$ value is given formally — by simply multiplying the rate over the period by the number of periods per year, which does not guarantee the exact value of the annual rate.

THE ROLE AND IMPORTANCE OF YIELD TO MATURITY

According to L. J. Gitman and M. D. Joehnk, the authors of the famous investment guide [9], yield to maturity — is the most important and widely used measure of bond valuation. The following are the primary comments concerning the YTM indicator that have been made in the financial literature: 1) YTM rate of a fairly priced bond[5] is roughly equal to that of an alternative investment with comparable risk [1]; 2) yield to maturity — is the rate of return on a bond investment, obtained

[1] Fabozzi F. J. Investment management. Moscow: Infra-M; 2000:486, 908. University textbook.

[2] Sharpe W.F., Alexander G.J., Bailey J.V. Investments. Moscow: Infra-M; 2018:127. University textbook.

[3] Fabozzi F. J. Investment management. Moscow: Infra-M; 2000:486, 908. University textbook.

[4] Letter from the Bank of Russia No. 28–1–2/39 from 19.01.1998. URL: https://gkrfkod.ru/zakonodatelstvo/pismo-banka-rossii-ot-19011998-n-28–1–239/ (accessed on 10.08.2021).

[5] Sharpe W.F., Alexander G.J., Bailey J.V. Investments. Moscow: Infra-M; 2018:421University textbook.

by an investor under two conditions: the investor owns the bonds until maturity, and all bond payments are reinvested at a rate equal to the yield to maturity at the time of purchase.[6] In this case, a significant part of the return on the bonds during its validity period is derived from reinvestment of coupons [9].

The second statement implies that if the investor follows this strategy, yield to maturity is a measure of return on the investment in the bond [9, p. 473], at the same time, YTM representing the minimum return on the investment in the bond expected by the investor [9, p. 469].

Because of the YTM indicator's significance role in bond valuation, in the literature factors, influencing this indicator, receive a lot of attention. According to [1] the YTM value of the bond is the sum of the basic risk-free interest rate and risk premium. Base, or benchmark, the interest rate is the yield to the treasury securities of the same duration. Thus, the YTM value is directly related to the risk of investing in this bond. One of the main types of risk — is credit risk, i.e. risk that the note issuer may default on payment on bonds. In this regard, the literature first of all investigates the link between bond yield and indicators characterizing the state of the issuing company: bond rating, debt to equity ratio (DER), return on assets (ROA), firm size, as well as factors that create investor risks: inflation, interest risk bonds, interest rate, bond parameters. Detailed lists of factors that influence the yield to maturity are given in monograph [10]. Let us now discuss the major findings of the research on the effect of factors on the YTM indicator that were conducted in the papers [11–22]. The [11–20] research obtained findings by sampling several dozen bond-issuing corporations over a period of time. Data were analysed using statistical methods such as correlation, regression, determination coefficient and variance analysis. For example, the [12] sample consisted of 104 corporate bonds from 40 companies traded on the Indonesian Stock Exchange (IDX) in 2017–2018. Panel data regression was used for analysis. In the paper [14] results are based on data collected on 67 companies and 138 bonds of the Indonesian bond market for the period January 2015 — July 2016. Multiple linear regression analysis was used for data analysis and interpretation. Main results of works [11–22] are as follows.

According to [11, 12, 14, 16, 18, 21] bonds ratings are negatively correlated with YTM. The higher the bond credit rating, the lower the bond yield rate. Companies with low bond ratings will offer high yield bonds to attract investor interest and provide greater YTM as compensation for higher risk. High-grade bonds are usually issued by companies with good financial performance, so the risk is lower. Bond ratings are considered by investors as a guide in decision making, as well as to determine the risk level and expected value of YTM.

Investors can assess the state of the company by comparing the company's equity and debt. If net worth is more than borrowed, then the company is healthy and it is not easy to bankrupt [17]. Debt-to-equity ratio gives an idea of the company's capital structure and assesses the risk of default on the company's bonds. The DER coefficient equal to the company's debt-to-equity ratio, also called solvency,— is one way to measure the company's ability to meet its long-term obligations. The lower the DER, the higher the company's ability to meet its obligations. The higher the debt (DER), the higher the expected yield [13, 21].

According to [10–12], ROA (return on assets) demonstrates the efficacy of the issuer's asset management. The higher the return on assets ratio, the lower the investment risk and hence the yield of corporate bonds.

[6] Fabozzi F.J. Investment management. Moscow: Infra-M; 2000:494. University textbook.

According to [11, 12, 15–17, 20] company size (total assets) has a significant negative correlation with yield to maturity. The larger the company size, the smaller the YTM. Although on the results of [19] firm size does not affect the bond yield.

Interest rate — basic risk-free interest rate, for example, the yield of State certificates. According to the author [13], the interest rate — is the most likely measure for investors to use bonds. Based on [13], which presents the research results of companies whose bonds were traded on the Indonesian Stock Exchange (IDX) from 2009 to 2013, the optimal rate of yield was considered the interest rate of certificates of the Bank of Indonesia (SBI). This is due to the fact that SBI is supported and fully guaranteed by the government, in this case by the Bank of Indonesia (BI), which forces securities market participants to consider SBI as expensive and risk-free certificates. According to the paper [15] 7-day repo rate of Bank of Indonesia (BI) is used as base interest rate. In line with [13, 15, 18–20], an increase in interest rates raises bond yields, whereas a fall in interest rates lowers bond yields.

Based on [21], inflation in the currency in which a particular issue is denominated is the fundamental factor determining the yield of corporate bonds. According to the authors [9, p. 467], investors are most concerned about inflation. It depreciates the principal value of the loan, forcing the issuer to compensate for inflation losses. According to the research [21], inflation and bond yields are in direct correlation: the more inflation, the more YTM.

According to [18, 21], investors are usually interested in a large risk premium when buying long-term bonds, as the uncertainty is higher for the long-term of circulation period. The more "long" bonds should provide the investor with an additional premium for the risk associated with higher duration and interest rate risk. In line with [14, 16, 18, 21], period to maturity has a significant positive correlation with yield to maturity. Coupon rate also has a significant positive impact on bond yields [14].

Based on [18, 21], bonds that have repayment options have a lower yield rate. Secured bonds have lower yields, while unsecured bonds have higher yields.

In paper [21] the influence of such a factor as the share of state participation in the company is considered. Investments in companies with a large share of state participation are considered less risky as there is a guarantee of state assistance in difficult economic conditions. In this regard, corporate bonds of private companies provide higher yields than those of government-affiliated companies.

According to [22] the value of YTM is influenced by the quality of the non-financial information disclosure about the company. According to [22] companies that provide more quality information on corporate social responsibility, it gets higher ratings and lower yields on bond issues.

The value of YTM, as evidenced, reflects almost all the information that an investor needs to make a decision to bond purchases. According to [9, p. 467] the yield to maturity is the single most important criterion in the bond market. This criterion is intended to monitor market performance as well as to determine the return on invested capital. This paper is devoted to the adequate use of indicator YTM in research.

PURPOSE OF THE STUDY

The [23] researches of the coupon bond price dependence on the number of coupon payments per year revealed that using the nominal interest rate in the bond price formulas produced results that made no economic sense, in contrast to the results obtained in the paper [24] when using the yield to maturity method of effective interest rate. In this regard, the purpose was to study the impact of the type of yield to maturity indicator on research results. The problem of the impact of the number of coupon payments per year on the interest rate risk of the bond was chosen.

The task selection is explained as follows. It was necessary to consider bonds of the same type as in the works of [23, 24], i.e. bonds without credit risk, such as federal loan bonds in the Russian market. As highlighted, for instance, in the paper of O. V. Buklemishev [25, p. 208], in fixed income securities markets without credit risk the main risk factor is interest risk — the possibility of bond price change due to market interest rate change. Interest rate risk of a bond is estimated by the relative (interest) change in the price of the $\Delta P/P$ bond when the market interest rate changes by a given amount. According to [1, p. 87] the essence of $\Delta P/P$ — is the reaction of the bond price to the change in the market interest rate.

The literature provides reports of the dependence of $\Delta P/P$ on the main parameters of the bond: coupon rate, period to maturity, and yield to maturity [1, p. 91]. In the paper [26], the proof of the dependence of the bond duration, which estimates the value of $\Delta P/P$, on the secondary parameter, the number of coupon payments in the year m, was obtained. However, the problem of the influence of this parameter directly on the value of $\Delta P/P$ has not been considered previously. Thus, a task was chosen to achieve the purpose of the paper, which, on the one hand, was not considered earlier, on the other hand, the results of this task is predictable based on earlier studies. In this paper, the solution to the problem is obtained for two approaches to the determination of yield to maturity bonds: method of nominal interest rate and method of effective interest rate. It was necessary to determine which of the two types of yield to maturity would produce results, consistent with the dependence of the bond duration on the number of coupon payments per year established in [26].

METHODS

Methods of differential calculus was used to solve the problem. Suppose, at the moment, there is a bond in the market with a YTM of y, where $y = r^{(m)}$ or $y = r$ — the initial annual nominal or effective rate of return. Bond price is equal $P(y)$. The change in the price of the bond we will consider for an instantaneous change of the market interest rate, similar to F. J. Fabozzi [1, p. 89]. Let $\tilde{y}$ — bond yield, that the nominal $\tilde{y} = \tilde{r}^{(m)}$ or effective $\tilde{y} = \tilde{r}$, as a result of an instant change in the market interest rate. Bond price will be equal $P(\tilde{y})$.

The relative (percentage) change in the price of the bonds due to the change of the market interest rate by the value $\Delta y = \tilde{y} - y$ is equal by definition[7]:

$$\frac{\Delta P(y)}{P(y)} = \frac{P(\tilde{y}) - P(y)}{P(y)}. \qquad (1)$$

Since the price of the bonds is a decreasing function of the yield, then $P(\tilde{y}) > P(y)$ at $\tilde{y} < y$ and $P(\tilde{y}) < P(y)$ at $\tilde{y} > y$. Then it follows from (1) that $\Delta P(y)/P(y) > 0$ when the interest rate is lowered and $\Delta P(y)/P(y) < 0$ when the interest rate is increased. As already noted, $\Delta P(y)/P(y)$ value estimates the interest risk of the bonds. Since the $\Delta P(y)/P(y)$ may be positive or negative, we will consider a module of this value $|\Delta P(y)/P(y)|$. By definition, this value is non-negative. Therefore, $|\Delta P(y)/P(y)|$ — is a percentage change in the price of a bond when the yield to maturity is changed by Δy, without a sign. Sign $\Delta P(y)/P(y)$ means interest growth or percentage decline in bond price. Thus, we have a task regarding the value dependency of $|\Delta P(y)/P(y)|$ on parameter m.

Result Criterion

The solution of the problem of the dependence of the value $|\Delta P(y)/P(y)|$ on

[7] Encyclopedia of financial risk management. Lobanov A.A., Chugunov A.V., ed. Moscow: Alpina Business Books; 2005:59.

parameter m is obtained for two approaches to determination of yield to maturity of the bond: the method of nominal interest rate and the method of effective interest rate. To obtain the criterion of choice of results was used from the work [26] dependence of the duration bond, estimating the value $\Delta P(y)/P(y)$, on the number of coupon payments in the year m. According to [26], at fixed values of basic parameters the bond duration decreases with increase of parameter m:

$$D_{m=m_2} < D_{m=m_1}, \quad (2)$$

where $m_1 < m_2$. As it is known [27, p. 751], Macaulay duration D is related to the percentage risk of bond under the formula:

$$\Delta P(y)/P(y) \approx -D\frac{\Delta y}{1+y}.$$

Hence

$$|\Delta P(y)/P(y)| \approx D\frac{|\Delta y|}{1+y}. \quad (3)$$

Then on the basis of formulas (2) and (3) it is possible to formulate the select result criterion:

$$|\Delta P(y)/P(y)|_{m=m_2} < |\Delta P(y)/P(y)|_{m=m_1}, \quad (4)$$

where $m_1 < m_2$, $y = r^{(m)}$ или $y = r$.

Ratio (4) means that as the parameter m increases, the interest risk of the bond should decrease. The purpose of the paper — is to determine which of the two types of yield to maturity will allow to obtain results that meet the criterion (4).

Algorithm to solve the problem

To study the influence of parameter m, where m = 1, 2, ..., on the value $|\Delta P(y)/P(y)|$ we consider the auxiliary function $\varphi(x,y)$, where ($x \ge 1$, $0 < y < 1$). The variable y makes sense yield to maturity of bond, $y = r^{(m)}$ or $y = r$. The expressions for the function $\varphi(x,y)$ we get from the corresponding expressions for the bond price by replacing the discrete variable m on continuous variable $x \ge 1$. The function $\varphi(x,y)$ and price of bonds at y and $\tilde{y}$ yields are related by the ratios:

$$\varphi(m,y) = P(y), \ \varphi(m,\tilde{y}) = P(\tilde{y}),$$

where m — positive integer, $\tilde{y} = \tilde{r}^{(m)}$ or $\tilde{y} = \tilde{r}$.

Then

$$|\Delta P(y)/P(y)| = |\Delta\varphi(m,y)/\varphi(m,y)|. \quad (5)$$

The function $\varphi(x,y)$ is differentiable by variables x and y. The effect of a variable x on the function $|\Delta\varphi(x,y)/\varphi(x,y)|$ will be studied by differentiating this function by variable x.

If $\tilde{y} < y$ we get:

$$\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|'_x = \left(\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right)'_x = \left(\frac{\varphi(x,\tilde{y})}{\varphi(x,y)} - 1\right)'_x = \frac{\varphi(x,\tilde{y})}{\varphi(x,y)}\left(\frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})} - \frac{\varphi'_x(x,y)}{\varphi(x,y)}\right). \quad (6)$$

If $\tilde{y} > y$ we get:

$$\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|'_x = \left(-\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right)'_x = \left(1 - \frac{\varphi(x,\tilde{y})}{\varphi(x,y)}\right)'_x = \frac{\varphi(x,\tilde{y})}{\varphi(x,y)}\left(\frac{\varphi'_x(x,y)}{\varphi(x,y)} - \frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})}\right). \quad (7)$$

To establish the derivative sign $\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|'_x$ in expressions (6) and (7), need to set the difference sign $\left(\frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})} - \frac{\varphi'_x(x,y)}{\varphi(x,y)}\right)$ in (6) and difference sign $\left(\frac{\varphi'_x(x,y)}{\varphi(x,y)} - \frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})}\right)$ in (7). In turn, to set the signs of these differences, it is necessary to study the monotony by the variable y function $\frac{\varphi'_x(x,y)}{\varphi(x,y)}$.

This action algorithm is used in each of the two problem solutions. The solutions are obtained at the given values of the basic parameters of the bond: term to maturity of T years, where $T > 1$ (otherwise at $m = 1$ bond is not coupon), coupon rate f and initial yield to maturity $r^{(m)}$ or r. Relative price changes of bonds that do not contain accumulated coupon income are considered.

RESULTS AND DISCUSSION

Theorem 1. With a specified term to maturity, coupon rate and initial yield to maturity $r^{(m)}$, determined by the method of nominal interest rate, the percentage change in the bond price when the market interest rate changes by a given amount increases with an increase in the number of coupon payments per year.

A proof of the theorem 1. According to the condition, $r^{(m)}$ — initial yield to maturity of the bond, determined by the method of nominal interest rate. Then the initial bond price is equal:

$$P\left(r^{(m)}\right)=\sum_{i=1}^{n}\frac{q}{\left(1+r^{(m)}/m\right)^{i}}+\frac{A}{\left(1+r^{(m)}/m\right)^{Tm}}, \quad (8)$$

where $q=(1/m)Af$ — amount of each coupon payment. If $\tilde{r}^{(m)}$ — the yield to maturity of the bond as a result of an instant change in the market interest rate at a given value, then price of the bond will become equal:

$$P\left(\tilde{r}^{m}\right)=\sum_{i=1}^{n}\frac{q}{\left(1+\frac{\tilde{r}^{(m)}}{m}\right)^{i}}+\frac{A}{\left(1+\frac{\tilde{r}^{(m)}}{m}\right)^{Tm}}. \quad (9)$$

Formula (8) is converted to form:

$$P\left(r^{(m)}\right)=\frac{A}{\left(1+r^{(m)}/m\right)^{Tm}}\left(1-\frac{f}{r^{(m)}}\right)+A\frac{f}{r^{(m)}}.$$

Auxiliary function in this case is:

$$\varphi(x,y)=\frac{A}{\alpha(x,y)}\left(1-\frac{f}{y}\right)+A\frac{f}{y},$$

where $\alpha(x,y)=(1+y/x)^{Tx}$, $x\geq 1$, $y=r^{(m)}$. Then

$$\varphi_x'(x,y)=-\frac{A}{\alpha^2(x,y)}\alpha_x'(x,y)\left(1-\frac{f}{y}\right),$$

$$\frac{\varphi_x'(x,y)}{\varphi(x,y)}=\frac{\frac{\alpha_x'(x,y)}{\alpha(x,y)}\left(\frac{f}{y}-1\right)}{\frac{f}{y}\left(\alpha(x,y)-1\right)+1}.$$

We use decomposition of functions $\alpha(x,y)$, $\frac{\alpha_x'(x,y)}{\alpha(x,y)}$ into power series:

$$\alpha(x,y)\approx 1+Ty+\frac{1}{2}T^2y^2-\frac{Ty^2}{2x},$$

$$\frac{\alpha_x'(x,y)}{\alpha(x,y)}\approx\frac{1}{2}T\left(\frac{y}{x}\right)^2,$$

where $0<y<1$. We get:

$$\frac{\varphi_x'(x,y)}{\varphi(x,y)}\approx\frac{1}{2x^2}T\frac{\left(fy-y^2\right)}{1+fT+\frac{fT}{2}\left(T-\frac{1}{x}\right)y}.$$

We get the sign of derivative $\left(\frac{\varphi_x'(x,y)}{\varphi(x,y)}\right)_y'<0$ differentiating function $\frac{\varphi_x'(x,y)}{\varphi(x,y)}$ by variable y. Then, $\frac{\varphi_x'(x,y)}{\varphi(x,y)}$ — decreasing function of variable y. If $\tilde{y}<y$, then $\frac{\varphi_x'(x,\tilde{y})}{\varphi(x,\tilde{y})}>\frac{\varphi_x'(x,y)}{\varphi(x,y)}$ and expression (6) has a sign:

$$\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|_x'=\frac{\varphi(x,\tilde{y})}{\varphi(x,y)}\left(\frac{\varphi_x'(x,\tilde{y})}{\varphi(x,\tilde{y})}-\frac{\varphi_x'(x,y)}{\varphi(x,y)}\right)>0.$$

Table1

Dependence $\left|\Delta P(r^{(m)})/P(r^{(m)})\right|$ on the Parameter '*m*'

	$\left\|\Delta P(r^{(m)})/P(r^{(m)})\right\|$	
m / $\tilde{r}^{(m)}$	5%	7%
1	0.04329	0.04100
2	0.04376	0.04158
3	0.04392	0.04178
4	0.04400	0.04188
5	0.04405	0.04194
6	0.04408	0.04198
7	0.04410	0.04201
8	0.04412	0.04203
9	0.04413	0.04205
10	0.04414	0.04206
15	0.04418	0.04211
20	0.04419	0.04213
$\lim\limits_{m\to\infty}\left\|\frac{\Delta P(r^{(m)})}{P(r^{(m)})}\right\|$	0.04424	0.04219

Source: Compiled by the author.

If $\tilde{y} > y$, then $\frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})} < \frac{\varphi'_x(x,y)}{\varphi(x,y)}$

and expression (7) has a sign:

$$\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|'_x = \frac{\varphi(x,\tilde{y})}{\varphi(x,y)}\left(\frac{\varphi'_x(x,y)}{\varphi(x,y)} - \frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})}\right) > 0.$$

Thus, at any rate the derivative is $\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|'_x > 0$. This means that the function $\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|$ is increasing by variable x. If $1 \le m_1 < m_2$, then

$$\left|\Delta\varphi(m_1,y)/\varphi(m_1,y)\right| < \left|\Delta\varphi(m_2,y)/\varphi(m_2,y)\right|.$$

Given the relation (5), we get:

$$\left|\Delta P(r^{(m)})/P(r^{(m)})\right|_{m=m_1} < \left|\Delta P(r^{(m)})/P(r^{(m)})\right|_{m=m_2},$$

where $m_1 < m_2$.

The more often coupons are paid, the greater the percentage change in the price of a bond when the market rate changes by a given amount, i.e. the greater the interest rate risk of the bond. The limit value:

$$\lim_{m\to\infty}\left|\frac{\Delta P\left(r^{(m)}\right)}{P\left(r^{(m)}\right)}\right| = \left|\frac{\left(1-\frac{f}{\tilde{r}^{(m)}}\right)e^{-T\tilde{r}^{(m)}} + \frac{f}{\tilde{r}^{(m)}}}{\left(1-\frac{f}{r^{(m)}}\right)e^{-Tr^{(m)}} + \frac{f}{r^{(m)}}} - 1\right|. \quad (10)$$

Theorem is proved.

Calculations. In *Table 1*, the calculations of the $\left|\Delta P(r^{(m)})/P(r^{(m)})\right|$ are given for the values of the yield $\tilde{r}^{(m)} < r^{(m)}$ or $\tilde{r}^{(m)} > r^{(m)}$ when T = 5 years, f = 6%, $r^{(m)}$ = 6%. The prices are calculated by formulas (8) and (9), the limit values are calculated by formula (10).

As we can see, the calculation results confirm the assertion of Theorem 1. The proof of Theorem 1 showed that the result of using the yield to maturity in the form of a nominal interest rate turned out to be unsatisfactory due to its inconsistency with criterion (4).

Consider another solution to the task.

Theorem 2. With a specified term to maturity, coupon rate and initial yield to maturity r, determined by the method of effective interest rate, the percentage change in the price of the bond when the market interest rate changes by a given amount decreases with an increase in the number of coupon payments per year.

A proof of the theorem 2. According to the condition, r — initial yield to maturity of the bond, determined by the method of effective interest rate. Then the price of the bond at the initial moment is calculated by the formula:

$$P(r)=\sum_{i=1}^{n}\frac{q}{(1+r)^{i/m}}+\frac{A}{(1+r)^{T}}. \quad (11)$$

If $\tilde{r}$ — yield to maturity of the bond as a result of an instant change in the market interest rate by a given value, then the price of the bond will be equal:

$$P(\tilde{r})=\sum_{i=1}^{n}\frac{q}{(1+\tilde{r})^{\frac{i}{m}}}+\frac{A}{(1+\tilde{r})^{T}}. \quad (12)$$

Formula (11) is converted to form:

$$P(r)=Af\left(1-\frac{1}{(1+r)^{T}}\right)\frac{1}{m\left((1+r)^{\frac{1}{m}}-1\right)}+\frac{A}{(1+r)^{T}}.$$

Auxiliary function in this case is:

$$\varphi(x,y)=Af\left(1-\frac{1}{(1+y)^{T}}\right)\beta(x,y)+\frac{A}{(1+y)^{T}},$$

where $\beta(x,y)=\dfrac{1}{x\left((1+y)^{\frac{1}{x}}-1\right)}$, $x\geq 1$, $y=r$.

Then

$$\varphi'_x(x,y)=Af\left(1-\frac{1}{(1+y)^{T}}\right)\beta'_x(x,y),$$

where $\beta'_x(x,y)=-\beta^2(x,y)\left(x\left((1+y)^{\frac{1}{x}}-1\right)\right)'_x$.

Hence

$$\frac{\varphi'_x(x,y)}{\varphi(x,y)}=\frac{f\left((1+y)^{T}-1\right)\dfrac{\beta'_x(x,y)}{\beta(x,y)}}{f\left((1+y)^{T}-1\right)+\dfrac{1}{\beta(x,y)}}.$$

We use approximate equalities:

$$\frac{1}{\beta(x,y)}=x\left((1+y)^{\frac{1}{x}}-1\right)\approx y+\frac{1}{2}\left(\frac{1}{x}-1\right)y^2,$$

Table 2

Dependence $|\Delta P(r)/P(r)|$ on the Parameter 'm'

	$\lvert\Delta P(r)/P(r)\rvert$	
m / $\tilde{r}$	5%	7%
1	0.04935	0.04100
2	0.04545	0.04035
3	0.04414	0.04014
4	0.04349	0.04003
5	0.04310	0.03996
6	0.04284	0.03992
7	0.04266	0.03989
8	0.04252	0.03986
9	0.04241	0.03985
10	0.04232	0.03983
15	0.04206	0.03979
20	0.04193	0.03977
$\lim_{m\to\infty}\left\lvert\frac{\Delta P(r)}{P(r)}\right\rvert$	0.04191	0.03970

Source: Compiled by the author.

$$\left(x\left((1+y)^{\frac{1}{x}}-1\right)\right)'_x\approx-\frac{y^2}{2x^2},\quad (1+y)^{T}-1\approx yT,$$

where $0<y<1$. We get:

$$\frac{\beta'_x(x,y)}{\beta(x,y)}\approx\frac{\dfrac{y}{2x^2}}{1+\dfrac{1}{2}\left(\dfrac{1}{x}-1\right)y},$$

$$\frac{\varphi'_x(x,y)}{\varphi(x,y)}\approx\frac{\dfrac{fT}{2x^2}y}{fT+1+\dfrac{y}{2}\left(\dfrac{1}{x}-1\right)(fT+2)}.$$

We get the sign of derivative $\left(\dfrac{\varphi'_x(x,y)}{\varphi(x,y)}\right)'_y>0$ differentiating function $\dfrac{\varphi'_x(x,y)}{\varphi(x,y)}$ by variable y.

Then, $\frac{\varphi'_x(x,y)}{\varphi(x,y)}$ — increasing function of variable y. If $\tilde{y} < y$, then $\frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})} < \frac{\varphi'_x(x,y)}{\varphi(x,y)}$ and expression (6) has a sign:

$$\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|'_x = \frac{\varphi(x,\tilde{y})}{\varphi(x,y)}\left(\frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})} - \frac{\varphi'_x(x,y)}{\varphi(x,y)}\right) < 0.$$

If $\tilde{y} > y$, then $\frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})} > \frac{\varphi'_x(x,y)}{\varphi(x,y)}$ and expression (7) has a sign:

$$\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|'_x = \frac{\varphi(x,\tilde{y})}{\varphi(x,y)}\left(\frac{\varphi'_x(x,y)}{\varphi(x,y)} - \frac{\varphi'_x(x,\tilde{y})}{\varphi(x,\tilde{y})}\right) < 0.$$

Thus, at any rate $\tilde{y} = \tilde{r}$ the derivative is $\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|'_x < 0$. This means that the function $\left|\frac{\Delta\varphi(x,y)}{\varphi(x,y)}\right|$ is decreasing by variable x. If $1 \le m_1 < m_2$, then

$$\left|\Delta\varphi(m_2,y)/\varphi(m_2,y)\right| < \left|\Delta\varphi(m_1,y)/\varphi(m_1,y)\right|.$$

Given the relation (5), we get:

$$\left|\Delta P(r)/P(r)\right|_{m=m_2} < \left|\Delta P(r)/P(r)\right|_{m=m_1},$$

where $m_1 < m_2$.

The more often coupons are paid, the smaller the percentage change in the price of a bond when the market interest rate changes by a given amount, i.e. the lower the interest rate risk of the bond. The limit value:

$$\lim_{m\to\infty}\left|\frac{\Delta P(r)}{P(r)}\right| = \left|\frac{f\left(1-\frac{1}{(1+\tilde{r})^T}\right)\frac{1}{\ln(1+\tilde{r})}+\frac{1}{(1+\tilde{r})^T}}{f\left(1-\frac{1}{(1+r)^T}\right)\frac{1}{\ln(1+r)}+\frac{1}{(1+r)^T}}-1\right|. \quad (13)$$

Theorem is proved.

Calculations. In *Table 2*, the calculations of the $|\Delta P(r)/P(r)|$ are given for the values of the yield $\tilde{r} < r$ и $\tilde{r} > r$ when T = 5 years, f = 6%, r = 6%. The prices are calculated by formulas (11) and (12), the limit values are calculated by formula (13).

As you can see, the results of the calculations confirm the statement of theorem 2 and correspond to criterion (4).

CONCLUSION

Dependence of the results of studies of influence of coupon payments frequency on interest risk of bonds on the type of yield to maturity indicator is established. The use of yield to maturity, determined by the method of effective interest rate, these gave the results for interest risk bonds, consistent with the dependence of the duration bond on the number of coupon payments per year. Based on these consistent dependencies on the parameter m, it is possible to formulate the dependence of interest risk of the bond on the number of coupon payments per year: with the specified term to maturity, coupon rate and initial yield to maturity, the more often coupons are paid, the lower the interest rate risk of the bond.

The use of yield to maturity, determined by the method of nominal interest rate, did not yield results that can be given an economic explanation, which is similar to the result of using this indicator in the task about the price of the bond [23]. It can be concluded that the market agreement on the yield to maturity in the form of a nominal interest rate, although it has a predominant use in the market, in researches can lead to incorrect results. This indicator, which was formally developed for the convenience of market participants, is the approximate value of the bond yield to maturity, which may be considered as the major reason for the divergence of results.

The paper's results may be useful both to the bond issuer when constructing bond parameters, and to the investor when making investment decisions.

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