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Assessment of the Volatility of the Main Cryptocurrencies, the Euro and the Direct Exchange Rate of the Ruble

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ABSTRACT

The development of financial technologies in modern conditions has contributed to the active use of digital financial instruments — cryptocurrencies — in international settlements. The availability of up-to-date information on digital currency volatility will help crypto market participants predict the consequences of their transactions. The **purpose** of this work is to construct a new measure of the volatility of financial assets, in particular, cryptocurrencies, the euro and the direct exchange rate of the ruble. In order to obtain this measure, an analysis of known volatility measures was carried out, requirements for the measure of volatility of a financial asset were formulated, and, as a result, the volatility of the main cryptocurrencies, the euro and the direct exchange rate of the ruble, was assessed by the levels of the time series of monthly quotations of these assets in the time interval from 1.01.2022 to 1.04.2023. The scientific novelty in the paper is a reasonable new measure of absolute volatility. The main **conclusions** of the study are: 1) the measure of absolute volatility constructed in this paper has the dimension of the asset value and measures the part of the asset value that is generated by uncertainty in the values of its profitability; 2) Bitcoin Cash is the most volatile cryptocurrency, Bitcoin has the least volatility among cryptocurrencies; 3) the volatility of the direct exchange rate of the ruble (the price of the US dollar in rubles) is about half the volatility of Bitcoin; 4) out of competition in terms of volatility is the euro quote (the euro price in dollars) — 10% in a year and a half.

Keywords: asset; asset yield; cryptocurrency; measures of volatility

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INTRODUCTION

In 2009, a qualitatively new kind of currency emerged — a digital currency, otherwise called a "cryptocurrency", which has no physical embodiment and is not controlled by any state or central bank. Cryptocurrencies have a number of features that in some situations make their use for international settlements more attractive than traditional methods. It should be noted that the use of cryptocurrency represents a certain interest for the domestic financial and economic system, as citizens and companies in the Russian Federation are having difficulties making foreign-trade payments due to an unprecedented number of Western sanctions, forcing them to transition from traditional payment mechanisms to payments using cryptocurrencies. Fairness stated indirectly confirms the appearance in Russia of the third form of the national currency— the digital ruble, which from 1 April 2023 is being tested by the Bank of Russia. True, digital ruble and cryptocurrencies are fundamentally different assets. Cryptocurrencies do not have a single issuer, and there is no single center that would hold its responsibilities.

Specialists of the Bank of Russia note several serious disadvantages of the use of cryptocurrencies in the system of international settlements, and one of the main disadvantages — high volatility of cryptocurrency rates.[1] In other words, the price of a cryptocurrency can fluctuate greatly over certain periods of time, which creates risks for investors and complicates the use of cryptocurrencies for international settlements.

Many researchers, analyzing in past periods of time the dynamics of quotes of major cryptocurrencies, came to the conclusion of the presence of bubbles and noted the high volatility of prices in the cryptocurrency markets [1–8]. Volatility of major cryptocurrencies in the second decade of the 21st century was investigated, in particular, in the papers [9] and [10]. What is the volatility of major cryptocurrencies nowadays? It is the assessment of the volatility of quotes (prices) of the main cryptocurrency in 2022 and in the first three months of 2023 is the purpose of this paper. In order to compare with the volatility of cryptocurrencies, the euro quote and the direct exchange rate of the ruble (US dollar prices in rubles) are estimated.

[1] Bank of Russia. Cryptocurrency: trends, risks, measures. Report for public consultations, Moscow, 2022. URL: https://cbr.ru/Content/Document/File/132241/Consultation_Paper_20012022.pdf (accessed on 01.06.2023).

PRINCIPAL CRYPTOCURRENCIES AND THEIR QUOTES IN 2022–2023

In this section we present the main cryptocurrencies circulating on the world's cryptocurrency exchanges, the quotes of which are the subject of our study.

Bitcoin (BTC)

We should give a brief overview of the emergence of the first cryptocurrency: "There are many methods to get money: you can earn it, find it on the street, fake it, steal it. And if you're Satoshi Nakamoto, the super-intelligent computer encoder, you can invent them. Satoshi did so on 3 January 2009, tapping the keyboard and creating a new currency called "Bitcoin". But there were only beats, and no coins. No paper, no copper, no silver — only 31 thousand lines of code and an advertisement on the Internet.[2]"

Bitcoin Cash (BCH)

Bitcoin Cash — is a cryptocurrency, one of the branches of Bitcoin, separated from it 1 August 2017. In November 2018 there was also a split of Bitcoin Cash into several branches.

Monero (XMR)

Monero — is a cryptocurrency that focuses on increased transaction privacy. The cryptocurrency appeared on 18 April 2014 as a branch of Bytecoin (not to be confused with Bitcoin).

[2] Joshua Davis. The Crypto-Currency. New Yorker. October 10, 2011. URL: https://www.newyorker.com/magazine/2011/10/10/the-crypto-currency (accessed on 01.06.2023).

Table 1

Prices of Cryptocurrencies and Euros in U.S. Dollars and the Price of the Dollar in Russian Rubles

Data	BTC	BCH	XMR	DASH	EUR	USA
01.01.2022	46 805	435	232	136	0,88	74
01.02.2022	36 471	285	147	95	0,89	77
01.03.2022	43 085	332	172	100	0,891	94
01.04.2022	45 064	376	212	127	0,903	84
01.05.2022	37 961	278	225	86	0,949	71
01.06.2022	31 898	204	197	66	0,931	62
01.07.2022	20 363	105	116	43	0,955	53
01.08.2022	23 456	142	156	52	0,979	61
01.09.2022	20 159	116	152	45	0,995	60
01.10.2022	19 420	119	148	42	1,02	57
01.11.2022	20 571	116	150	42	1,01	62
01.12.2022	17 137	113	143	43	0,953	61
01.01.2023	16 548	97	147	42	0,934	70
01.02.2023	23 110	134	176	61	0,916	72
01.03.2023	23 335	133	153	72	0,937	75
01.04.2023	28 761	126	157	59	0,923	77

Source: URL: https://www.calc.ru/ (accessed on 01.06.2023).

Dash (DASH)

Dash — is a free and anonymous cryptocurrency developed as an alternative to Bitcoin in 2014. The Dash cryptocurrency, also formerly known as Darkcoin or XCoin, is completely decentralized and independent of external regulators.

In 2017, Dash was one of the most popular altcoins and was among the top ten cryptocurrencies in terms of capitalization.

Table 1 shows quotes (prices) on the first date of each month 2022–2023 of cryptocurrencies BTC, BCH, XMR and DASH, expressed in US dollars. There are also quotes (prices) of the euro in US dollars and the price of the US dollar in rubles (direct quote of the ruble); these quotes will be required to compare their volatility with the volatility of cryptocurrencies.

LITERATURE REVIEW, ANALYSIS AND DEVELOPMENT OF VOLATILITY MEASURES OF FINANCIAL ASSETS

The p_t symbol indicates the price of an asset on the date t, where t discreetly changes with a constant step Δ in the interval $\left[t_0, t_f\right]$ between t_0 and t_f; for example, $t_0 = 01.01.2022$, $t_f = 01.04.2023$, $\Delta = 1$ month. The symbol $t-1$ represents the date preceding the date t. For example, $t_f - 1 = 01.03.2023$. The number of intervals of Δ between date t_0 and t_f indicates n_f So, the length of the interval $\left[t_0, t_f\right]$ between the dates t_0 and t_f is equal to $n_f \cdot \Delta$.

The task of estimating the measurement of the volatility of the variable p_t at the interval $\left[t_0, t_f\right]$. In the theory of finance, there are several rules for calculating the measure of asset volatility, an overview of which is presented in the paper by M. Yu. Kussyi [11, p. 61].

We first analyzed the two most common and popular measures of volatility (see expressions (1) and (3)), and at the end of this section we analyzed the third known measure of volatility (see (12)). As a result of the analysis, firstly, select a suitable measure of volatility, and secondly, formulate requirements for the asset's measure and, thirdly, construct a new measure with justification (13).

A review and analysis of volatility measures are presented further. The first measure, named "realized volatility" [11, p. 61] and adopted by many researchers [11–14], is defined by the rule:

$$RV = \sqrt{\sum_{t=t_0+1}^{t=t_f} \left(\ln \frac{p_t}{p_{t-1}} \right)^2} . \tag{1}$$

The $\ln \frac{p_t}{p_{t-1}}$ value in formula (1) is referred to [15, p. 247] as the asset's "logarithmic profit" at the time interval $[t-1, t]$. The basis of the name is the approximate equality, which is obtained when reasoning in the differential:

$$\ln \frac{p_t}{p_{t-1}} = \ln \left(\frac{p_{t-1} + (p_t - p_{t-1})}{p_{t-1}} \right) = \ln(1 + r_t) \approx r_t . \tag{2}$$

Below the r_t symbol we will indicate either the value of $\ln \frac{p_t}{p_{t-1}}$, or the yield value of $r_t = \frac{p_t - p_{t-1}}{p_{t-1}}$, which should not lead to misunderstandings.

The second measure of the asset's volatility, called "simply volatile" [11, p. 61],— is the average square deviation of r_t values in the interval $\left[t_0, t_f\right]$:

$$\hat{\sigma}_\Delta = \sqrt{\frac{1}{n_f - 1} \cdot \sum_{t=t_0+1}^{t=t_f} (r_t - \bar{r})^2} , \tag{3}$$

where n_f — the number of observed r_t values in the interval $\left[t_0, t_f\right]$; $\bar{r}$ — average arithmetic values r_t:

$$\bar{r} = \frac{1}{n_f} \cdot \sum_{t=t_0+1}^{t=t_f} r_t . \tag{4}$$

Let us analyze these measures. Considering (1) and (3), we note that both values RV and $\hat{\sigma}_\Delta$ are non-dimensional, i.e. they do not depend on the units of measurement of the values p_t. For this reason, measures (1) and (3) will be referred to as measures of **relative asset volatility**, and it is the measure of relative volatility that allows different assets to be compared. Do you asking which one of these measures to choose and, most importantly, what is their meaning? Below show that values (1) and (3) have different meanings! Specifically, value (3) measures the relative volatility of an asset over time intervals of Δ (the meaning of the symbol Δ is noted above). Measure (1) is an estimate of the relative volatility of an asset at the interval $\left[t_0, t_f\right]$, whose duration is $n_f \cdot \Delta$. On a certain assumption (see below), the relationship between RV and $\hat{\sigma}_\Delta$ is given by an approximate equation:

$$RV \approx \hat{\sigma}_\Delta \cdot \sqrt{n_f}\,. \tag{5}$$

Identification of a risk asset are required to justify equity (5). So, in the theory of finance [15, p. 247] is an asset whose yield r_t at each date $t \in [t_0, t_f]$ can be interpreted as a random variable. Let's add that the variable r_t as a function of time can most often be interpreted as a stationary time series with non-correlated levels (this assumption is tested in practice). Note also that if the return (2) is determined (in particular, constant at each value $t \in [t_0, t_f]$), then the asset (under the additional condition) is considered risk-free. For example, a deposit in a reliable bank is interpreted as a risk-free asset. In the following note, we formulate **two requirements for the relative degree of volatility of a financial asset**.

Note 1. The first requirement to measure the volatility of an asset seems obvious: the measure of the volatility of a risk-free asset must be zero even in a situation where the price of the asset p_t changes over time (e.g. the value of a deposit in a trusted bank).

The second requirement should be formulated as follows: the measure of the volatility of a risk asset should be based on the quantitative characteristics of its return r_t as a fixed time series. Note [15, p. 212], that stationary time series r_t with non-correlated levels are serves as two constants: the expected level of the series $\mu = E(r_t)$ and average square deviation σ. The constant σ — is average square spread of possible values of r_t around μ.

The following shows that the two requirements mentioned above are met by a measure (3), which measures the relative volatility of an asset (in particular, the average square fluctuation of possible values of its yield r_t) over time intervals of duration Δ. In turn, measure (1) measuring the relative volatility of an asset (namely, the average square fluctuation of possible values of its yield R_{t_f}) over a time period of duration $n_f \cdot \Delta$ meets only the first requirement. But the second requirement is satisfied by this measure only when the expected $E(r_t)$ level of return of r_t of the asset is zero, that is, when $\mu = E(r_t) = 0$.

A recognized representation of the asset's price on each $t \in [t_0 + 1, t_f]$ date is required to support the above:

$$p_t = p_0 \cdot (1 + r_1) \cdot \ldots \cdot (1 + r_{n_t}). \tag{6}$$

p_0 — is the value of the asset on the date t_0, $r_1 = \frac{p_{t_0+1} - p_0}{p_0}$ — assets yield on the first interval $[t_0, t_0 + 1]$, …, $r_{n_t} = \frac{p_t - p_{t-1}}{p_{t-1}}$ — assets yield on the interval $[t-1, t]$. The length of the interval $[t_0, t]$ is equal to $n_t \cdot \Delta$.

After logarithm, the equation (6) takes into account (2) the form:

$$\ln\frac{p_t}{p_0} = \ln\left(\frac{p_0 + (p_t - p_0)}{p_0}\right) = \ln(1 + R_t) = \sum_{i=1}^{n_t} r_i\,. \tag{7}$$

The symbol $R_t = \frac{p_t - p_0}{p_0}$ represents the yield of the asset at the time interval $[t_0, t]$; for example, $R_{t_0+1} = r_1$. Once again, the length of the interval $[t_0, t]$ is equal to $n_t \cdot \Delta$.

Considering in differentials (see (2)), equality (7) is presented as:

$$R_t = \sum_{i=1}^{n_t} r_i \,. \tag{8}$$

The r_t levels are assumed to be non-correlated and form a stationary time series with parameters (μ,σ). Consequently, rule (4) calculates the linear unbiased estimate $\bar{r}$ of the parameter μ, and formula (3) calculates a best estimate $\hat{\sigma}_\Delta$ of the parameters σ [15, p. 182]. Further from equality (8) follow two approvals. The first approval: the variable R_t **as a function of time is a non-standard time series,** specifically a random wander (with demolition) [16, p. 245]. The second approval [15, p. 111]: the value of R_t is a random variable with the expected value of $E(R_t) = \mu \cdot n_t$ and average square deviation $\sigma_{R_t} = \sqrt{Var(R_t)} = \sigma \cdot \sqrt{n_t}$. Therefore, the best estimate $\hat{\sigma}_{R_t}$ of average square deviation of the yield of R_t at the interval $[t_0, t]$ has the form

$$\hat{\sigma}_{R_t} = \hat{\sigma}_\Delta \cdot \sqrt{n_t} \,. \tag{9}$$

This is the measure of the relative "simple volatility" of an asset at the interval $[t_0, t]$. For the whole interval $[t_0, t_f]$ the measure of the relative "simple volatility" of an asset is determined by the rule:

$$\hat{\sigma}_{R_f} = \hat{\sigma}_\Delta \cdot \sqrt{n_f} \,. \tag{10}$$

It remains to explore the measure (1). Taking into account the known equation $E(r_t^2) = \mu^2 + \sigma^2$ and the indication (2) we calculate the expected value $E(RV^2)$ of the measure (1):

$$E(RV^2) = \sum_{t=t_0+1}^{t=t_f} E(r_t)^2 = n_f \cdot (\mu^2 + \sigma^2). \tag{11}$$

Comparing (11) and (10), we assume that "realized volatility" (1) satisfies the second requirement for the measure of the asset's volatility and is virtually the same as "simple volatility" (10) only in a situation where the expected level of asset yield $\mu = E(r_t) = 0$. Otherwise, the "realized volatility" is slightly higher than the asset's volatility. Near values (1) and (10) are an obvious symptom of the fairness of the hypothesis $H_0 : E(r_t) = 0$ of the equality of zero to the expected level of validity of the asset. Add that measure (10) is more flexible because it allows to estimate the volatility of an asset at different time intervals $[t_0, t] \in [t_0, t_f]$.

Note 2. Let us return to the characteristics of volatility (1) and (10). Both characteristics measure the uncertainty of the $\boldsymbol{R_f = \frac{p_f - p_0}{p_0}}$ **asset yield** and are disproportionate value. Yield appears to be a relative characteristic of an asset, and therefore the measures discussed above estimate the volatility of precisely the relative characteristics of the asset — its yield. This circumstance deprives the characteristics (1) and (10) of full visibility, and the meaning of the concept of "asset volatility" would be more clearly visible if the measure of volatility were expressed directly in the unit of measurement of the asset's price p_t. The Asset volatility measures review [11, p. 61] notes the third measure of volatility, which is also called "realized volatility", has a price dimension p_t and is determined by the rule:

$$V_n = \sqrt{\frac{1}{n-1} \cdot \sum_{t=1}^{t=n} (p_t - \bar{p})^2} \,. \tag{12}$$

If you compare formula (12) with expression (3), it may appear that formula (12) calculates an estimate of the average square deviation of the asset's price p_t However, this is not the case because, as is known [15, p. 243] in the general case, the price of an asset p_t — is a temporary unstable series and, therefore, the value of the average square deviation of the price p_t is a function of time [15, p. 245]. This means that the average square deviation p_t is not a constant. Consequently, the measure (12) has no clear meaning and is not justified. Furthermore, the measure (12) does not meet both of the volatility measure requirements set out in Note 1.

Is it possible to construct a reasonable measure of the price volatility of an asset expressed directly in its price units? It is possible, and this measure is constructed below and is called the measure of absolute volatility of an asset.

CONSTRUCTION OF A MEASURE OF THE ASSET'S ABSOLUTE VOLATILITY

The next measure of absolute volatility of an asset

$$\hat{\sigma}_{p_t} = p_0 \cdot \hat{\sigma}_\Delta \cdot \sqrt{n_t}\ . \tag{13}$$

In this formula, the value $\hat{\sigma}_{p_t}$ is expressed in the unit of measurement of the price p_t of the asset, and it is in these units that it measures the volatility at the interval $[t_0, t]$ of the portion of the value of an asset p_t, which is generated by the uncertainty in the values of the **yield of R_t** (see further). The size of $\hat{\sigma}_{p_t}$ can also be interpreted as the possible average losses of the investor over the time period $[t_0, t]$. Let us emphasize that the value of p_0 of the asset price on the start date t_0 is in expression (13) a known constant.

To justify the rule (13) let us return to equality (7). Asset yields level $R_t = \dfrac{p_t - p_0}{p_0}$ represented as a sum:

$$R_t = \bar{R}_t + \Delta R_t\ . \tag{14}$$

The symbol $\bar{R}_t$ indicates the determined portion of the yield's assets (for example, y of a deposit, this value is calculated according to the rule $\bar{R}_t = \mu \cdot n_t$). The symbol ΔR_t in equation (14) represents the portion of the yield of an asset that is generated by uncertainty in the value of R_t (for a deposit in a trusted bank $\Delta R_t = 0$). Let us emphasize that the measure (9) of the asset's volatility characterizes the volatility of the aggregate ΔR_t.

According to (14) rewrite the equation (7) in the form of:

$$p_t = p_0 \cdot (1 + R_t) = p_0 \cdot (1 + \bar{R}_t + \Delta R_t) = p_0 \cdot (1 + \bar{R}_t) + p_0 \cdot \Delta R_t\ . \tag{15}$$

The first term $p_0 \cdot (1 + \bar{R}_t) = \bar{p}_t$ — is the determined portion of the value of the asset's p_t value, and, according to note 1, the volatility of this aggregate is zero. But the second term $p_0 \cdot \Delta R_t = \Delta p_t$ — is that part of the value of an asset that is generated precisely by the uncertainty ΔR_t. Hence the rule (13) of the calculation of the absolute measure of "simple volatility" of an asset is justified with consideration (9).

Investigation. In analogy with the rationale of the formula (13), the rule for calculating the measure of the absolute "simple volatility" of an asset at any interval $[t_1, t_2] \subset [t_0, t_f]$, with a duration of $\Delta \cdot n_{t_1, t_2}$:

$$\hat{\sigma}_{p_{t_1, t_2}} = p_{t_1} \cdot \hat{\sigma}_\Delta \cdot \sqrt{n_{t_1, t_2}}\ . \tag{16}$$

For example, the measure of absolute volatility of an asset at interval $[t-1, t]$ with duration Δ is calculated by the rule

$$\hat{\sigma}_{p_{t-1,t}} = p_{t-1} \cdot \hat{\sigma}_{\Delta} . \quad (17)$$

Let us emphasize that in formula (16) the value of p_{t_1} the price of the asset on date t_1 is interpreted as a known constant. Similarly, in the formula (17), the value of p_{t-1} of the asset price on the date t — 1 is a known constant.

INTERPOLATION OF VOLATILITY MEASURES

The measures of relative and absolute volatility, (3) and (17) respectively, allow practical interpolation. Consider the length interval Δ between dates $[t-1, t]$; for example, $\Delta = 1$ month. According to the above, the measure (3) is average square deviation of the r_t yield of the asset in the interval $[t-1, t]$. Assume that need to calculate the measure of the relative volatility of the asset $\hat{\sigma}_{\delta}$ between dates $[t-1, t]$ at intervals of a shorter duration δ; for example, duration $\delta = 1$ day. The symbol m indicates the number of intervals of duration δ, the total duration of which is Δ; so, for example, with $\Delta = 1$ month and $\delta = 1$day, the value of $m = 30$. Recalling the additive structure of the asset's yields (see (8)), present r_t as the following sum:

$$r_t = r_{t,1} + r_{t,2} + \ldots + r_{t,i} + r_{t,m} . \quad (18)$$

$r_{t,1}$ — is asset yield at the first interval of duration δ between dates $\left[t-1, t_1\right]$, $r_{t,2}$ — is asset yield at the second interval of duration δ between dates $\left[t_1, t_2\right]$ etc.

The non-observed aggregates $r_{t,i}$ in the right part of the equation (18) are interpreted as non-correlated random variables with a single average square deviation σ_{δ}. From this assumption and the formula (18) follows the equality $\hat{\sigma}_{\Delta} = \hat{\sigma}_{\delta} \cdot \sqrt{m}$ or, equivalently, equality

$$\hat{\sigma}_{\delta} = \hat{\sigma}_{\Delta} \cdot \sqrt{\frac{1}{m}} . \quad (19)$$

This is the measure of the relative volatility of an asset at intervals of duration δ. In turn, the measure of the absolute volatility of an asset over the duration interval $\delta \cdot i$ between dates $\left[t-1, t_i\right]$ is calculated taking into account (16) according to the rule:

$$\hat{\sigma}_{p_{t-1-t_i}} = p_{t-1} \cdot \hat{\sigma}_{\delta} \cdot \sqrt{m_i} . \quad (20)$$

Where m_i — number of intervals of duration δ between dates $\left[t-1, t_i\right]$.

The following paragraphs examine the conditions for the correct use of the volatility measures discussed above for the above-mentioned cryptocurrencies, the euro, and the direct exchange rate of the ruble, and then calculate the values (1), (10), and (13) of these assets for the time period $[t_0 = 01.01.2022, \ \ t_f = 01.04.2023]$.

EXAMINATION OF PREREQUISITES OF THE CORRECTION OF THE CALCULATION OF VOLATILITY MEASURES

For the correct use of the previously discussed volatility measures (1), (3), (9), (13), (19) and (20) it is necessary to verify the assumption of the stability of the time series of returns $r_t = \ln \frac{p_t}{p_{t-1}}$ of each asset under investigation. *Table 2* presents the r_t yields of the assets under review at intervals of $\Delta = 1$ month, calculated according to *Table 1*.

Table 2

Values of the Yield of Cryptocurrencies, Euros and the Dollar Price in Rubles

t	*r*BTC	*r*BCH	*r*XMR	*r*DASH	*r*EUR	*r*USA
01.02.2022	−0.25	−0.42	−0.46	−0.36	0.01	0.04
01.03.2022	0.17	0.15	0.16	0.05	0.00	0.20
01.04.2022	0.04	0.12	0.21	0.24	0.01	−0.11
01.05.2022	−0.17	−0.30	0.06	−0.39	0.05	−0.17
01.06.2022	−0.17	−0.31	−0.13	−0.26	−0.02	−0.14
01.07.2022	−0.45	−0.66	−0.53	−0.43	0.03	−0.16
01.08.2022	0.14	0.30	0.30	0.19	0.02	0.14
01.09.2022	−0.15	−0.20	−0.03	−0.14	0.02	−0.02
01.10.2022	−0.04	0.03	−0.03	−0.07	0.02	−0.05
01.11.2022	0.06	−0.03	0.01	0.00	−0.01	0.08
01.12.2022	−0.18	−0.03	−0.05	0.02	−0.06	−0.02
01.01.2023	−0.03	−0.15	0.03	−0.02	−0.02	0.14
02.01.2023	0.33	0.32	0.18	0.37	−0.02	0.03
03.01.2023	0.01	−0.01	−0.14	0.17	0.02	0.04
04.01.2023	0.21	−0.05	0.03	−0.20	−0.02	0.03

Source: Compiled by the authors.

The study of the assumption of the stability of the time series $r_t = \ln\frac{p_t}{p_{t}}$, i.e., in short, the study of a statistical hypothesis $H_0 : r_t \in I(0)$ against the alternative $H_1 : r_t \in I(1)$, which means the instability of a time series r_t, will be carried out in the statistical appendix R first with the help of the Dickey-Fuller Test [16, p. 66], which is implemented in the function adf.test() and, let us emphasize, tests the hypothetical $H_1 : r_t \in I(1)$ against the $H_0 : r_t \in I(0)$. Then for the time series r_t of each asset, build an $ARIMA(p,d,q)$, model using the auto.arima() function. Note that in the model $ARIMA(p,d,q)$ in a stationary time series situation the parameter d takes the value 0 [16, c. 64]; if, moreover, the stationary timeline has non-correlated levels, that is **white noise**, then, in addition, the equation $p = q = 0$. is correct. Add that the auto.arima() function automatically tests the hypothesis of the equality of zero of the expected value of the return on the asset. The results of this study are presented in *Table 3*.

Table 3

Results of the Dickey-Fuller Test and the *ARIMA(p, d, q)* Model of Returns on the Surveyed Assets

Asset	The decisive rule of the Dickey-Fuller test of the hypothesis of nonstationarity of asset returns (significance level α = 0.1)	ARIMA (*p, d, q*) models of the return of an asset r_t and the outcome of the hypothesis test that the expected value of $E(r_t)$ is equal to zero
BTC	p-value = 0.02345. The non-stationary hypothesis is rejected	ARIMA (0, 0, 0) with zero mean
BCH	p-value = 0.01007. The non-stationary hypothesis is rejected	ARIMA (0, 0, 0) with zero mean
Monero	p-value = 0.01. The non-stationary hypothesis is rejected	ARIMA (0, 0, 0) with zero mean
DASH	p-value = 0.02236. The non-stationary hypothesis is rejected	ARIMA (0, 0, 0) with zero mean
EUR	p-value = 0.07642. The non-stationary hypothesis is rejected	ARIMA (0, 0, 0) with zero mean
USD	p-value = 0.02345. The non-stationary hypothesis is rejected	ARIMA (0, 0, 0) with zero mean

Source: Compiled by the authors based on the calculation in R.

The results presented in Table *3* of the study of the statistical properties of the yield of cryptocurrencies, the euro and the US dollar (direct exchange rate of the ruble) suggest that the values of yields of these assets (see *Table 2*) can be interpreted as fixed time lines with non-correlated levels and zero expected values. Consequently, the following calculation of the previously discussed volatility measures (1), (3), (9), (13), (19) and (20) is correct.

PRICE VOLATILITY MEASURES FOR CRYPTOCURRENCIES, EURO AND USD OVER A PERIOD OF TIME [01.01.2022, 01.04.2023]

Table 4 shows the values of volatility measures $\hat{\sigma}_\Delta$, $\hat{\sigma}_{R_f}$, RV, $\hat{\sigma}_\delta$, $\hat{\sigma}_{p_f}$, $\hat{\sigma}_{p_{01.04.23-11.04.23}}$ of the assets discussed above. The volatility measurement values $\hat{\sigma}_\Delta$, $\hat{\sigma}_{R_f}$, RV and $\hat{\sigma}_\delta$ are expressed in percentage. The values of the absolute volatility measures $\hat{\sigma}_{p_f}$ and $\hat{\sigma}_{p_{01.04.23-11.04.23}}$ are expressed in the unit price of the asset concerned. The measures of volatility, the values of which are presented in *Table 4*. 1) $\hat{\sigma}_\Delta$ — average square fluctuation in the yield of an asset at 1 month intervals, 2) $\hat{\sigma}_\delta$ — average square fluctuation of the return on an asset at intervals of 1 day, 3) $\hat{\sigma}_{R_f}$ and RV — average square fluctuation in the return on an asset over a 15-month period between dates 01.01.2022 и 01.04.2023; 4) $\hat{\sigma}_{p_f}$ — average square fluctuation of the asset price over a 15-month period between dates 01.01.2022 and 01.04.2023; 5) $\hat{\sigma}_{p_{01.04.23-11.04.23}}$ — average square fluctuation of the asset price within 10 days between dates 01.04.2022 and 11.04.2023.

Table 4

Measures of Volatility of Cryptocurrencies, the Euro and the Price of the Dollar

Мера / Measure	BTC	BCH	XMR	DASH	EUR	USA
$\hat{\sigma}_\Delta(\%)$	20	27	23	24	3	11
$\hat{\sigma}_{R_f}(\%)$	78	104	91	94	10	44
$RV(\%)$	77	105	85	94	10	42
$\hat{\sigma}_\delta(\%)$	3,7	4,9	4,3	4,4	0.5	2.0
$\hat{\sigma}_{p_{01.04.23-11.04.23}}$	3365 (dollars)	20 (dollars)	21 (dollars)	8 (dollars)	0.01 (dollars)	5 (rubles)
$\hat{\sigma}_{p_f}$	36736 (dollars)	452 (dollars)	210 (dollars)	127 (dollars)	0.09 (dollars)	32 (rubles)

Source: Compiled by the authors.

Comment on the contents of *Table 4* on the example of cryptocurrency Bitcoin (BTC). The monthly relative volatility of Bitcoin is an average of $\hat{\sigma}_\Delta = 20\%$; the relative volatility of Bitcoin at the interval $[t_0 = 01.01.2022,\ t_f = 01.04.2023]$ with a duration of 15 months is $\hat{\sigma}_{R_f} = 78\%$; the value of the measure of the absolute volatility of Bitcoin (i.e. the possible average losses of an investor at that interval when owning one bitcoin) is $\hat{\sigma}_{p_f}$ = 36 736 dollars. The average daily fluctuations in Bitcoin's yields are equal to $\hat{\sigma}_\delta$ = 3.7%. The estimate of the Bitcoin price fluctuation for 10 days between 01.04.2022 and 11.04.2023 was early $\hat{\sigma}_{p_{01.04.23-11.04.23}}$ = 3365 dollars.

For comparison, the estimate of the average fluctuation of the direct exchange rate of the ruble (the price of the dollar in rubles) for 10 days between 01.04.2022 and 11.04.2023 **is 5 rubles**.

RV and $\hat{\sigma}_{R_f}$ asset relative volatility is practically identical, which, firstly, serves as a symptom of the fairness of the assumption $H_0 : E(r_t) = 0$ about the equality of zero expected asset returns (which has been tested above), and, secondly, indicates the correctness of analysis of these measures.

CONCLUSION

1. Two well-known measures (1) and (3) of relative asset volatility are strictly justified. The meaning of the values of these measures is different, and their relationship is given by the equation (5). Volatility assets can only be compared using relative volatility measures (1) and (3). A more flexible measure of relative volatility is a measure (3).

2. The third known measure (12) of absolute asset volatility is not justified and has no clear meaning.

3. The measure (13) of absolute asset volatility constructed and substantiated in this paper has an asset value dimension, and its value measures the portion of the asset's value that (a portion) is generated by uncertainty in asset yield values.

4. The measure (3) of relative volatility and the measure (13) of absolute volatility allow practical interpolation, respectively (19) and (20).

5. The most volatility cryptocurrency is Bitcoin Cash. Bitcoin has the lowest relative volatility among cryptocurrencies. However, the high value of bitcoin generates a high measure of its absolute volatility, in other words, generates large possible average losses for the investor when holding bitcoin. For the equation, the relative volatility of the direct exchange rate of the ruble (the price of the dollar) is approximately twice that of Bitcoin. Out of competition for relative volatility is the quotation of the euro: the relative volatility of the EUR in the interval $[t_0 = 01.01.2022,$ $t_f = 01.04.2023]$ with a duration of 15 months is equal to: $\hat{\sigma}_{R_f} =$ 10%! This is **less than the relative volatility** of cryptocurrencies and **4 times less** than the relative volatility of the ruble's direct exchange currency.

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