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Method for Determining the Risk Profile of Investors Based on the Relationship of Two Stock Investing Problems

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ABSTRACT

The subject of research in this paper is the investor's risk profile as a characteristic of his behavior in the stock market. **The purpose** of the study is to assess the investor's risk profile in the form of a risk ratio in a model with a linear convolution of expected return and variance. A financial consultant can use this information to create a portfolio of financial instruments that corresponds with an investor's acceptance of risk. This makes the study **relevant** because it addresses the problem of minimizing potential risks in the management of an investment portfolio, which is related to the investor's attitude toward risk. **The scientific novelty** lies in the development of a mathematical approach to solving the problem of determining the risk profile based on the relationship between the solutions of two problems of choosing an investment portfolio, expressed as conditions on the parameters under which the solutions of these problems exist and coincide. Wherein, mathematical programming **methods** were used, as well as the Python programming language. **As a result**, the risk coefficient is expressed in terms of the model parameter with a constraint on profitability; a classification of the risk profile according to the acceptable value of the risk coefficient is proposed; the method is implemented as a set of programs and demonstrated on the example of the Russian stock market. **The conclusion** is made about the possibilities of trust managers using this approach when making decisions on choosing the best portfolio.

Keywords: risk coefficient; risk profile; expected return; criteria convolution; investment portfolio

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INTRODUCTION

The development of the Russian economy largely depends on the stock market, which plays an important role in the redistribution of financial resources. The economic situation in the Russian Federation contributes to an increase in investment volumes in the securities of Russian enterprises. Economic progress is closely related to the results of investment activity. Attracting investors is one of the key issues related to the Russian stock market. Investing in shares of Russian companies carries a high risk of losing investment.

The main function of the financial market is associated with the transformation of risks. The market mechanism analyzes many types of risks and forms the so-called risk premium. It is important to note that even under conditions of market equilibrium, when all risks are fairly assessed, securities will not be equally attractive to all investors. Factors influencing investor preferences include their financial condition, individual attitude to risk, composition of assets and liabilities, current market conditions and much more. It is important to note that attempts to completely avoid risk result in a portfolio return approaching the risk-free rate, which may not be in the interest of the investor. Identifying the specific types of risk, that need to be addressed allows for a controlled increase in portfolio performance.

To achieve positive results in investment activities, investors form portfolios of securities that reduce the risk of losses and maximize profits [1–3]. To reduce the risks associated with managing investment portfolios, various equity portfolio management strategies are used. One approach to making such investment decisions is to determine the investor's risk profile. The risk profile of an investor determines the level of a person's willingness to take the risk associated with the loss of an investment. Each investor has a different attitude to market volatility or risk, and this attitude depends, for example, on factors such as available funds, age, etc. Risk profiling allows both the investor and the financial advisor to create a portfolio of financial instruments that corresponds to the risk investor profile [4]. The investor's trustee can take proactive or reactive measures to minimize and, in certain cases, even prevent potential losses after the risk profile is determined.

The investor's risk profile is divided into three types: conservative (low risk propensity), moderate (moderate risk propensity), and aggressive (the greatest willingness to withstand market volatility) [5, 6]. In normal practice, testing is carried out to determine the risk profile [7, 8]. A selection of different data collection instruments was used in [9]: several structured online questionnaires, designed to provide an understanding of risk profiles and personalities, and a software package for simulating investments, used to track investors decisions when managing a portfolio. The article [10] proposes the experimental method, with the application of structured questionnaires, and computer simulation of investments with Expecon software utilizing data on real financial instruments that are available on the market. The articles [11–13] are devoted to the development of platforms for robotic consultants for risk analysis and investor profiling. An empirical method for studying investor risk tolerance is discussed in [14].

This paper proposes to determine the investor's risk profile using quantitative methods, i.e., the problem of finding the optimal portfolio is solved, taking into account the individual attitude of the investor to risk, expressed as a risk coefficient. Thus, the main role of the manager is to determine the goals, restrictions, the choice of appropriate types of securities, the selection of acceptable returns and risks, as well as the formulation of the optimization problem.

A portfolio manager must deal with multicriteria tasks and the problem of lack of information when choosing methods for assessing future results. The choice of an approach to multicriteria, uncertainty and risk leads to a formulated mathematical programming problem that can be solved using available optimization methods [15–19], and using machine-learning methods [20]. When choosing a portfolio, an investor is primarily interested in the expected return and the risk of loss. Really, the investor is interested in profitability, but one cannot determine it. Therefore, when optimizing a portfolio, this uncertainty is formalized by averaging real return values for the previous period and introducing a risk assessment as a deviation from the average. Accordingly, in this paper, the accounting for these two indicators is formalized in the form of two optimization problems. In the first problem, the linear convolution of the mathematical expectation and the return variance is maximized. In the second problem, the variance is minimized, and the mathematical expectation of the return must be equal to the given value. Conditions on the parameters under which the solutions of these problems exist and coincide are given.

To implement this analytical approach, a software package was developed using the Python programming language version 3.8.5 and the Google Colaboratory environment. It was tested on a practical example with real data from the Russian stock market.

THEORETICAL BASIS OF THE METHOD

The models under consideration are based on the assumption that there is a set of assets, which is described by the vector $\bar{r} = (\bar{r}_1, \ldots, \bar{r}_i, \ldots, \bar{r}_n)$, where $\bar{r}_i$ — the expected return of the i-th financial instrument, and the covariance matrix $V = (\sigma_{ij})_{n \times n}$.

The investor's strategy consists in the distribution of funds between assets and is described by the vector $x=(x_1,\dots,x_i,\dots,x_n)$, where x_i — share of funds invested in the i-th financial instrument.

The formulation of the first problem of determining the optimal portfolio:

$$\max_x\left[\bar{r}x-\alpha(xVx)\right],\ \ xe=1, \qquad (1)$$

where $\alpha>0$ — weighting factor that determines the investor's attitude to risk (risk coefficient), e = (1,…, 1).

The optimal composition of the portfolio x^* and the corresponding value of the Lagrange multiplier λ^* are found from the system of linear algebraic equations:

$$\bar{r}-2\alpha Vx^*=\lambda^* e, x^* e=1. \qquad (2)$$

Assume that the covariance matrix V is nondegenerate and introduce the notation for scalar quantities $a=eV^{-1}e$, $b=\bar{r}V^{-1}e$, $c=\bar{r}V^{-1}\bar{r}$ and vectors $h=V^{-1}e$ and $g=V^{-1}\bar{r}$. Solving system (2), we obtain the composition of the optimal portfolio

$$x^*(\alpha)=\frac{h}{a}+\left(g-\frac{b}{a}h\right)\frac{1}{2\alpha}. \qquad (3)$$

The formulation of the second problem of determining the optimal portfolio:

$$\min_x xVx,\ \ \bar{r}x=r_p,\ \ xe=1, \qquad (4)$$

where r_p — the expected return of the portfolio, given by the investor.

Optimal portfolio composition x^* and the corresponding values of the Lagrange multipliers λ_1^* and λ_2^* are found from the system of linear algebraic equations:

$$2Vx^*=\lambda_1^*\bar{r}+\lambda_2^* e,\ \ \bar{r}x^*=r_p, x^* e=1. \qquad (5)$$

Find x^* from the first vector equation of the system (5):

$$x^*=\frac{1}{2}\left(\lambda_1^* g+\lambda_2^* h\right). \qquad (6)$$

Substituting (6) into the second and third equations of system (5), we obtain a system for finding λ_1^* and λ_2^*. It is shown that if all $\bar{r}_i$ are distinct, then $ca-b^2>0$. Then we have the values of the Lagrange multipliers

$$\lambda_1^*=\frac{2(ar_p-b)}{ca-b^2}, \lambda_2^*=\frac{2(c-br_p)}{ca-b^2}.$$

It is proved that problem (4) has a solution if the following conditions are satisfied

$$\max\left\{\frac{b}{ar_p},\frac{b^2}{ac}\right\}1,<\min\left\{\frac{b}{ar_p},\frac{b^2}{ac}\right\}>1. \qquad (7)$$

If the risk coefficient $\alpha=\dfrac{ca-b^2}{2(ar_p-b)}>0$, then the solutions of problems (1) and (4) coincide.

To classify the investor's risk profile according to the type of conservative, moderate, aggressive it is convenient to map range values [0, ∝) of the coefficient α into the segment [0, 1] using the function

$$\beta(\alpha)=1-\frac{1}{1+\alpha}. \qquad (8)$$

Let's connect the investor's risk profile with the value of the coefficient β, namely,

- aggressive, if $\beta\in[0,0.25]$,
- moderate, if $\beta\in(0.25,0.75]$,
- conservative, if $\beta\in(0.75,1]$.

Note that this classification of the risk profile does not exclude the possibility of its adjustment by financial consultants.

TOOLS FOR IMPLEMENTING THE PROPOSED METHOD

The developed software package selects the period and the initial list of shares, data processing, entering the defining parameters and checking the conditions, calculating the risk profile of the investor and the composition of the optimal portfolio (the programs were written with the participation of a student at the Financial University A. V. Karasev).

0	2008-01-09	0.018400	0.011200	0.028600	0.005800	0.051200	0.060600	0.023100	-0.017100	-0.001500	-0.033400
1	2008-01-10	0.019013	0.007866	-0.000822	-0.008177	0.003824	0.005102	0.046834	0.012558	0.016495	0.000822
2	2008-01-11	0.091167	0.017187	0.009960	-0.012609	0.000000	0.045685	0.000829	-0.031310	0.011301	0.000000
3	2008-01-14	-0.031385	0.023276	0.021888	-0.015717	-0.009524	0.029029	0.038079	0.002776	0.002865	-0.006568
4	2008-01-15	-0.003352	0.019377	-0.009513	-0.006487	0.000000	-0.009388	0.018979	-0.009018	0.006762	-0.015702
...	...	...	...	...	...	...	...	...	...	...	...
3742	2022-12-26	-0.002492	0.004952	0.007645	-0.000125	0.005933	0.004198	-0.001853	0.026507	0.021821	0.010337
3743	2022-12-27	-0.001665	0.016125	0.012028	0.004613	0.005551	-0.000380	0.006787	0.008655	-0.007449	-0.006139
3744	2022-12-28	-0.007506	-0.020498	-0.013714	-0.003227	-0.003278	-0.000760	-0.011702	-0.000985	-0.002788	-0.030882
3745	2022-12-29	0.000000	-0.000675	0.006736	0.001619	-0.002769	0.012367	-0.004633	0.012672	0.010394	0.008194
3746	2022-12-30	0.003361	0.017789	-0.002148	0.011810	0.021867	0.013343	-0.005431	0.016129	0.001348	-0.013245

3747 rows x 11 columns

Fig. 1. **Content of Dataframe Profit_DF**

Source: Compiled by the author.

For the specific implementation of the practical part, stock quotes of ten companies of the Russian stock market were selected:

- AFK Sistema (AFKS)
- Gazprom (GAZP)
- Lukoil (LKOH)
- NLMK (NLMK)
- NOVATEK (NVTK)
- Pole (PLZL)
- Rosneft (ROSN)
- Sberbank (SBER)
- VTB (VTBR)
- Severstal (CHMF)

As the data under consideration, we will take the daily closing prices of the corresponding shares for the period from 2008 to 2022, inclusive. The choice of this period is determined by the completeness of data on closing prices for the selected shares.

Export of quotes of company shares was made from the site "Investing.com".[1] On the main page of "Investing.com" in the "Quotes" tab, select "Shares", "Russia", then select the type of shares, for example, shares of VTB (VTBR). Then you need to open the tabs "Overview", "Past data" and select "Time period" — "Day". Set the upload boundaries from "01/01/2008" to "12/31/2022" and sort the data in ascending order for the convenience of further manipulations, click on "Download data".

A unified data frame profit_df containing the date and returns for the specified date for the corresponding tickers is shown in *Fig. 1*.

Further, the possibility of limiting the considered period is implemented by entering the start and end dates from the keyboard, as well as selecting certain securities from the specified list.

Then, in the "Select stocks" block, we will enter the tickers of the stocks we are interested in, from which we want to make an investment portfolio.

A program has been developed that calculates the vector of mathematical expectations of returns mean_vec and the covariance matrix of returns cov_matrix for selected stocks for a limited period (*Fig. 2*).

For the selected set of stocks, we find the vector of mathematical expectations of returns and the covariance matrix (*Fig. 3*).

We use the inverse covariance matrix for calculations (*Fig. 4*).

Fig. 5 shows the fragment program for calculating mathematical expressions from conditions (7).

Fig. 6 shows a program that allows, at a given level of profitability, to check the

[1] Investing.com. URL: https://ru.investing.com/equities/russia (accessed on 05.02.2023).

```
[25] mean_vec = profit_df.iloc[:, 1:].mean()
     mean_vec
```

```
[27] cov_matrix = profit_df.iloc[:, 1:].cov()
     cov_matrix
```

Fig. 2. **Formation of Mathematical Expectations of Stock Returns and Covariance Matrix**

Source: Compiled by the author.

```
vtbr    -0.000526
gazp     0.000516
nlmk     0.000038
plzl     0.000698
sber    -0.000042
dtype: float64
```

	vtbr	gazp	nlmk	plzl	sber
vtbr	0.000590	0.000342	0.000252	0.000178	0.000445
gazp	0.000342	0.000655	0.000215	0.000182	0.000368
nlmk	0.000252	0.000215	0.000427	0.000158	0.000264
plzl	0.000178	0.000182	0.000158	0.000560	0.000184
sber	0.000445	0.000368	0.000264	0.000184	0.000621

Fig. 3. **Calculation of Mathematical Expectations of Stock Returns and Covariance Matrix**

Source: Author calculations.

```
vtbr     1.0
gazp     1.0
nlmk     1.0
plzl     1.0
sber     1.0
dtype: float64
```

	vtbr	gazp	nlmk	plzl	sber
vtbr	4037.797394	-609.049459	-647.816870	-181.060099	-2202.751357
gazp	-609.049459	2475.869965	-275.337073	-255.346884	-838.026319
nlmk	-647.816870	-275.337073	3474.592059	-450.628332	-714.762674
plzl	-181.060099	-255.346884	-450.628332	2105.606688	-152.918728
sber	-2202.751357	-838.026319	-714.762674	-152.918728	4034.890055

Fig. 4. **Calculation Inverse Covariance Matrix**

Source: Author calculations.

```
ratio1 = (mean_vec @ inv_cov_matrix @ e.T) / (r_p * (e @ inv_cov_matrix @ e.T))

ratio2 = ((e @ inv_cov_matrix @ mean_vec.T) ** 2) / ((mean_vec @ inv_cov_matrix @ mean_vec.T) * (e @ inv_cov_matrix @ e.T))
```

Fig. 5. **Matrix operations**

Source: Author calculations.

conditions that ensure the equivalence of problems (1) and (4). We introduce the required value of the mathematical expectation of the daily return of the portfolio r_p and check the conditions (7).

For a given level of return $r_p = 0.0005$ got the value of the risk coefficient $\alpha = 5.4799$.

The program for calculating optimal portfolios uses formulas (3) and (6), as well as numerical optimization methods of the CVXPY

```
condition = (max(ratio1, ratio2) < 1) or (min(ratio1, ratio2) > 1)
print(condition)

True

term1 = mean_vec @ inv_cov_matrix @ mean_vec.T
term2 = e @ inv_cov_matrix @ e.T
term3 = (e @ inv_cov_matrix @ mean_vec.T) ** 2

alpha = (term1 * term2 - term3) / (2 * (r_p * term2 - (mean_vec @ inv_cov_matrix @ e.T)))

True    alpha = 5.47985274407859
```

Fig. 6. **Checking conditions and calculating the risk profile for a given level of return**

Source: Author calculations.

```
alpha: 5.47985274407859          r_p =  0.0005
 vtbr   -0.122357                 vtbr   -0.122357
 gazp    0.264364                 gazp    0.264364
 nlmk    0.372643                 nlmk    0.372643
 plzl    0.413066                 plzl    0.413066
 sber    0.072284                 sber    0.072284
 dtype: float64                   dtype: float64
```

Fig. 7. **Finding optimal portfolios**

Source: Author calculations.

library. *Fig. 7* shows the result of calculating the optimal portfolios for the selected set of financial instruments.

As can be seen from *Fig. 5*, in this example, there is a negative value of one of the components of the portfolio composition vector. A negative value of the share, as you know, means a short sale. If it is required to build a portfolio without short sales, then the process of finding a solution in problems (1), (4) is reduced to enumeration of square submatrices of the original covariance matrix. When using tools, one can build a portfolio without short sales, for example, by adding a non-negativity condition on the variables and using a numerical method for convex programming problems (1), (4).

For the found value $\alpha = 5.4799$, we get the value $\beta = 0.8457$ by formula (8). This means that, in accordance with the above classification, the constructed portfolio is suitable for a conservative investor.

CONCLUSION

Thus, the investor's risk profile, which is a certain characteristic of the investor's behavior in the stock market, helps the trustee to choose the right investment strategy. At the same time, for each type of investor, the return on his portfolio is related to, for example, the inflation rate or the rate on deposits.

The purpose of a conservative investor is to protect against inflation and preserve his capital. It is reasonable for him to get paid for investing 70% of the money at the rate of return on deposits or the rate at which inflation is indicated.

The moderate investor is focused on stable accumulation. He only invests 50% in bonds and the rest in stocks. The return on investment should be well above the rate of inflation.

An aggressive investor is prepared to accept chances in order to generate high profits. He tries to maximize return on investment by investing about 80% of the assets in highly volatile financial instruments.

If, when making decisions on choosing the best portfolio, convolution of sum-type criteria with a risk coefficient for dispersion is used, then the problem of setting the risk coefficient (investor's risk profile) arises. In this paper, a mathematical approach to solving such a problem is developed.

The procedure for determining the relationship between the coefficient α and the expected level of return can help financial advisers in making informed decisions on compiling investment portfolios for clients with different risk profiles.

The technical implementation of the proposed method makes it possible to automate the process of determining the investor's risk profile. The use of the Python 3.8.5 programming language and the user-friendly Google Colaboratory environment, which does not require the installation of additional software, allow multiple users to work together.

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