ORIGINAL PAPER

DOI: 10.26794/2587-5671-2025-29-4-1902-01
UDC 378:50(045)
JEL C32, C49, C53, F49

Analysis of the Stability of the Model for Forecasting Mutual Volumes Russia's Trade with BRICS Partners

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ABSTRACT

The paper is devoted to the construction of models for forecasting the volume of trade between Russia and the BRICS countries under sanctions. Trade between the BRICS countries is the economic foundation of their comprehensive interaction and prosperity, therefore the problem of high-quality forecasting of the volume of this trade under unprecedented Western sanctions against Russia seems to be a **relevant task** of econometric modeling. The aim of the study is to improve the accuracy of forecasts of Russia's trade turnover with BRICS partners by ensuring the stability of the forecasting model in the context of sanctions pressure from Western countries and the pandemic. The econometric tool chosen is a **system of simultaneous equations** describing the foreign trade turnover of each country (other than Russia) using annual levels of macroeconomic factors: the GDP of the BRICS countries, Brent oil prices, the US dollar exchange rate and the pandemic indicator over the time period 2000–2022. In order to take into account structural changes in fast-growing economies such as India and China, **two-phase models** (switching models) were used to describe their behavioral equations in a system of simultaneous equations. As a test for the significance of structural changes, due to the small sample size after structural changes, the Chow forecast test was used. Taking into account significant structural changes (in the post-pandemic period) within the framework of switching models allowed us to increase the accuracy of the forecast of the volume of trade turnover of the Russian Federation by 2.5 times.

Keywords: foreign trade turnover; system of simultaneous equations; autoregressive model with distributed lags; model diagnostics; structural breaks; Chow predictive test; two-phase model

For citation: **Babeshko L.O., Byvshev V.A. Analysis of the stability of the model for forecasting mutual volumes Russia's trade with BRICS partners. *Finance: Theory and Practice.* 2025;29(4). DOI: 10.26794/2587-5671-2025-29-4-1902-01**

INTRODUCTION

Foreign trade, as a component of cooperation between various states, serves as the economic foundation of their comprehensive interaction. This fully applies to the interaction of the countries that are part of the BRICS association. Starting the discussion and addressing the tasks in the proposed work, let us recall that BRICS is a platform for developing dialogue and cooperation between Brazil, Russia, India, China, and South Africa, which together account for 30% of the world's land area and 42% of the world's population, 21% of the global gross domestic product (GDP), 17.3% of global merchandise trade, 12.7% of global services trade, and 45% of global agricultural production [1]. Despite a number of difficulties related to geographical factors, food security issues, anti-Russian sanctions, and the possibility of secondary sanctions faced by BRICS countries, their interaction in the trade and economic sphere is only increasing (the participating countries account for 18% of global exports). This is significantly facilitated by agreements between countries, including free trade agreements, tariff exemptions, tariff reductions, as well as the simplification of trade procedures in various sectors of goods and services. Besides trade relations, cooperation among BRICS countries has contributed to the increase of both domestic and foreign direct investments (FDI), making BRICS countries important players in the global economy [2]. The significance of FDI is hard to overestimate, as they contribute to the dissemination of new technologies in the manufacturing sector of the host country [3].

To effectively ensure coordination between Russia and partner countries in the implementation of trade and economic relations (transport and logistics projects, food security, etc.), the task of forecasting the volumes of mutual trade turnover arises. One of the approaches to solving this problem is the construction of econometric models that allow for the evaluation and forecasting of endogenous variable values, and based on the quantitative analysis of causal relationships, making informed economic and financial decisions. The main requirement for forecasting models is the stability of their coefficients. However, the period of study used in this work (from 2000 to 2023) is characterized by significant structural changes in the economies of the BRICS countries, caused by unprecedented sanctions from Western countries.

Linear models used for the periods 2000–2019 and 2000–2020 are unable to "capture" the emerging nonlinear patterns. To address this issue, the study employs regime-switching models — time series models characterized by different behaviors over various time intervals. Due to the insufficient number of observations in the period after structural changes (2020–2023), the Chow test is used to check their significant impact on the stability of the forecasting model for Russia's trade turnover with partner countries.

LITERATURE REVIEW

For modelling the foreign trade turnover of BRICS countries, gravitational models are used in most scientific studies. The ideas of the gravity approach started with the works [4, 5] and were one of the forms of assessing the intensity of trade turnover in urban networks, and later in the global economy. In work [6], gravity models reflecting the dependence of foreign trade turnover of BRICS countries on GDP (economic potential of the trading countries) and the distance between them (a factor determining the magnitude of costs) were used for modelling. As the model of the relationship between the variables, a polynomial model was chosen.

The paper [7] discusses options for dynamic gravity models of external trade among BRICS countries, where exports and imports depend on the indicators of both trading partners, particularly on the size of their economies, characterized by gross domestic product. The study of the dynamics of trade between

India and BRICS countries is the focus of the paper [8]. The basis of the trade analysis is the gravity model. For the quantitative assessment of the alignment of India's exports with the needs of BRICS partners, the authors have proposed a new index — the export aspiration index.

From the applications of multidimensional time series models to the study of export-import dynamics, vector auto regression models and error correction models have been widely used [9, 10]. In the work [9], a global vector auto regression (*GVAR*) model covering 33 countries was constructed and evaluated. The possibility of including financial variables such as long-term interest rates, real stock prices, real output, and inflation as regresses has been considered. As a result of the study, the authors concluded that the inclusion of long-term interest rates and real stock prices helps improve forecasts only for developed economies.

The purpose of the paper [10] is to study the impact of external and internal shocks on Pakistan's textile exports using a structural vector auto regression (SVAR) model. External shocks in the model characterize the impact of unfavorable supply conditions, negative financial results, and positive shocks — income from financial operations. Internal shocks reflect the influence of internal macroeconomic factors such as economic production, price levels, interest rates, and exchange rates. As a result of applying vector auto regression model research tools (impulse response functions and variance decomposition), the author concluded the significance of the impact of external and internal shocks on the variability of demand for textile sector exports in Pakistan.

As a tool for studying the relationship between economic growth, trade openness, and gross capital formation in BRICS countries, the article [11] employs autoregressive distributed lag models and error correction models. For most BRICS countries, the Granger causality test shows a unidirectional causal relationship from trade openness to economic growth. Bidirectional causality was found between trade openness and economic growth only for China.

In the study [12], autoregressive distributed lag models were used to analyze the causal relationships between foreign direct investment, trade openness, and gross domestic product in BRICS countries over the period 1990–2018. Empirical results showed that FDI and trade openness have a positive impact on long-term economic growth.

The application of econometric models for panel data in studying the volume of trade between several countries is discussed in the papers [13–16]. The paper [13] examines the trade turnover between the Republic of Belarus and the EU countries. As predetermined variables, the model includes: gross national product, the official exchange rate of the ruble to the US dollar, the lagged value of exports (to correct for autocorrelation of the model's random error), and the distances between capitals.

In the paper [14], as a result of the econometric analysis of the relationship of trade within the Eurasian Economic Union, the functional form of the model (log-linear) and the type of model for panel data in an industry breakdown (fixed effects model) were chosen. The authors have formulated and solved the problem of zero observations that arise when estimating gravity models in log-linear form. The fixed effects model proved to be adequate even when modelling mutual trade among the EAEU member states with goods aggregated to the level of the industrial economy.

The application of panel data models in studying the relationship between trade openness and taxation in BRICS countries is presented in the paper [15]. As a result of the study, the authors concluded that trade openness, the trade coefficient, and the average trade volume increase the tax-to-GDP ratio and tax collection.

For modelling international trade and national vegetable production in Romania, the article [16] applied the spatial regression method to panel data. To justify the competitiveness of vegetable production in the country, the authors used M. Porter's model.

In the paper [1], a system of simultaneous equations is used to assess Russia's trade turnover with BRICS countries. This approach allows for the consideration of the specific characteristics of the economic patterns of export and import in the participating countries.

DESCRIPTION OF SPECIFICATION AND DATA ECONOMETRIC MODEL

To forecast the level of Russia's foreign trade turnover with BRICS countries, this work employs an econometric model in the form of a system of simultaneous linear equations (SLE). The behavioral equations of this model describe the current levels of foreign trade turnover of each country (other than Russia) for period t, depending on a number of macroeconomic factors, which then in the identity allow determining the current value of Russia's foreign trade turnover:

$$\begin{cases} FT_{1t} = a_{10} + a_{11}FT_{1t-1} + a_{12}Y_{t-1} + a_{13}Y_{1t-1} + a_{14}P_t + a_{15}Oil_t + a_{16}D_t + u_{1t} \\ FT_{2t} = a_{20} + a_{21}FT_{2t-1} + a_{22}Y_{t-1} + a_{23}Y_{2t-1} + a_{24}P_t + a_{25}Oil_t + a_{26}D_t + u_{2t} \\ FT_{3t} = a_{30} + a_{31}FT_{3t-1} + a_{32}Y_{t-1} + a_{33}Y_{3t-1} + a_{34}P_t + a_{35}Oil_t + a_{36}D_t + u_{3t} \\ FT_{4t} = a_{40} + a_{41}FT_{4t-1} + a_{42}Y_{t-1} + a_{43}Y_{4t-1} + a_{44}P_t + a_{45}Oil_t + a_{46}D_t + u_{4t} \\ FT_t = FT_{1t} + FT_{2t} + FT_{3t} + FT_{4t} \end{cases}, \quad (1)$$

where $FT_{jt} = EX_{jt} + IM_{jt}$ — Russia's foreign trade turnover with the country $j = 1,\dots,4$ (Brazil — 1, India — 2, China — 3, South Africa — 4); EX_{jt} — export from Russia to country j, IM_{jt} — import from Russia to country j, u_{jt}, $t = 1,\dots,n$ — random disturbances, FT_t — total level of Russia's foreign trade turnover. As macroeconomic factors influencing the endogenous variables of the model, the following were selected as a result of statistical analysis: lagged values of the endogenous variables (FT_{jt-1}, $j = 1,\dots,4$), lagged level of Russia's GDP (Y_{t-1}), lagged level of country GDP $j(Y_{jt-1})$, current prices of *Brent* (Oil_t), current direct exchange rate of the US dollar (D_t) and an indicator of the presence of a pandemic in the current period (P_t). Thus, the levels of foreign trade turnover of each country are chosen as the predicted (endogenous) variables of the model:

$$\left(FT_{1t},\ FT_{2t},\ FT_{3t},\ FT_{4t},\ FT_t\right), \quad (2)$$

and the explanatory (predetermined) variables are

$$\left(FT_{1t-1},\ FT_{2t-1},\ FT_{3t-1},\ FT_{4t-1},\ Y_{t-1},\ Y_{1t-1},\ Y_{2t-1},\ Y_{3t-1},\ Y_{4t-1},\ P_t,\ Oil_t,\ D_t\right). \quad (3)$$

The system of behavioral equations in (1) is a system of independent equations, in which each endogenous variable from (2) is considered as a function of a single set of regresses from (3) (excluding those that insignificantly affect the endogenous variable in the given equation), therefore the parameters of the model (1) can be estimated in isolation for each behavioral equation separately. Taking into account the structure of the vector of predetermined variables (3), the behavioral equations of the system represent autoregressive models with distributed lags (*ARDL*) (1, 1) (*autoregressive distributed lags model*). The values of the maximum lags of the endogenous and exogenous variables of the model are indicated in parentheses. The parameters of autoregressive models can be estimated using the least squares method (LSM) if two

Table 1

GDP Levels of BRICS Countries

t	Russia Y_t	Brazil Y_{1t}	India Y_{2t}	China Y_{3t}	South Africa Y_{4t}
2000	260	655	468	1211	136
2001	307	559	485	1339	122
2002	345	508	515	1471	115
2003	430	558	608	1660	175
2004	591	669	709	1955	229
2005	764	892	820	2286	258
2006	990	1108	940	2752	272
2007	1300	1397	1217	3550	299
2008	1661	1696	1199	4594	287
2009	1223	1667	1342	5102	296
2010	1525	2209	1676	6087	375
2011	2046	2616	1823	7552	416
2012	2208	2465	1828	8532	396
2013	2292	2473	1857	9570	367
2014	2059	2456	2039	10476	351
2015	1363	1802	2104	11062	318
2016	1277	1796	2295	11233	296
2017	1574	2064	2651	12310	350
2018	1657	1917	2701	13895	368
2019	1687	1878	2871	14280	351
2020	1483	1445	2623	14723	302
2021	1840	1830	2700	15800	357
2022	2270	1920	3420	16300	360

Source: BRICS Joint Statistical Publication, 2020: Brazil, Russia, India, China, South Africa. M.: Rosstat. 2020. 226 p. BRICS Joint Statistical Publication, 2021: Brazil, Russia, India, China, South Africa. Government of India, 2021. 228 p. URL: https://www.sbs-consulting.ru/upload/iblock/837/5e1wtc39kgsdg8ewq9o0iuyhk5vwcv14.pdf?ysclid=lw4vao8de9574341910 (accessed on 10.10.2024).

conditions are met: stability (for the j-th equation, this condition means that the parameters $a_{j1} < 1$, $j = 1,\ldots,4$ are stable), and the absence of autocorrelation of disturbances.

To build forecasts of trade volumes between BRICS countries, the coefficients in model (1) must be constant over time. Therefore, the task of this study is to analyse the stability of the model of mutual trade volumes between Russia and BRICS partners and to choose a method to adjust for the impact of structural changes related to sanctions imposed by

Table 2

Russia's Foreign Trade Turnover with BRICS Members

t	Brazil, FT_{1t}	India, FT_{2t}	China, FT_{3t}	South Africa, FT_{4t}	Oil_t	D_t
2000	0.65	1.64	6.20	0.10	28.3	28.1
2001	1.11	1.67	7.24	0.08	24.4	29.2
2002	1.53	2.13	9.24	0.13	25	31.4
2003	1.74	3.32	11.57	0.12	28.9	30.7
2004	1.74	3.15	14.85	0.14	38.3	28.8
2005	2.95	3.10	20.31	0.17	54.4	28.3
2006	3.71	3.89	28.67	0.18	65.4	27.2
2007	5.24	4.34	39.57	0.28	72.7	25.6
2008	6.71	6.95	55.92	0.48	97.7	24.9
2009	4.59	7.46	39.53	0.52	61.9	31.7
2010	5.79	7.55	58.74	0.52	79.6	30.4
2011	6.48	7.43	82.73	0.58	111	29.4
2012	5.66	10.61	87.53	0.96	112	31.1
2013	5.48	10.07	88.80	1.07	108.8	31.9
2014	6.26	7.57	88.27	0.98	98.9	38.4
2015	4.85	6.81	63.53	0.86	52.4	61
2016	4.89	8.36	76.29	0.74	44.8	67
2017	5.90	10.22	98.62	0.85	55	58.4
2018	5.05	10.98	108.24	1.07	71.5	62.7
2019	4.61	11.23	111.46	1.11	64.6	64.7
2020	4.01	9.26	103.97	0.98	42	72.2
2021	7.3	12.0	145.7	1.036	70.68	73.89
2022	9.9	43.5	190.2	0.834	97.88	67.65

Source: BRICS Joint Statistical Publication, 2020: Brazil, Russia, India, China, South Africa. M.: Rosstat. 2020. 226 p. BRICS Joint Statistical Publication, 2021: Brazil, Russia, India, China, South Africa. Government of India, 2021. 228 p. URL: https://www.sbs-consulting.ru/upload/iblock/837/5e1wtc39kgsdg8ewq9o0iuyhk5vwcv14.pdf?ysclid=lw4vao8de9574341910 (accessed on 10.10.2024).

Table 3

Intervals for Forming Training and Control Samples

Interval number (N)	Number of Observations	Training sample	Control sample
1	19	2000–2019	2020
2	20	2000–2020	2021
3	21	2000–2021	2022

Source: Compiled by the authors.

Western countries, in order to enhance its predictive properties.

To construct model (1), data from the period 2000–2022 were used. The data tables presented in work [1] are supplemented with observations from 2021 and 2022. Such a period was chosen due to the lack of access to more recent data. The GDP levels characterizing the economic development of BRICS countries are presented in *Table 1* at current prices (billion USD), in *Table 2* — the foreign trade turnover of Russia with BRICS members (FT_{jt} , billion USD), Brent oil prices (Oil_t , USD per barrel), the current direct exchange rate of the US dollar (D_t , USD to rubles).

For the analysis of the stability of parameter estimates and the adequacy of models, cumulative samples were formed over the time interval of 2000–2022, as presented in *Table 3*.

EMPIRICAL RESULTS

The estimation and diagnostics of the behavioral equations of model (1) were performed in the *R* software environment using econometric packages [17]. To test the assumptions of multiple linear regression models

$$Y_t = \beta_1 X_{1t} + \ldots + \beta_i X_{it} + \ldots + \beta_k X_{kt} + \varepsilon_t ,$$

$$t = 1, 2, \ldots, n , \quad (4)$$

taking into account the characteristics of the tested behavioral equations (small sample size, inclusion of lagged endogenous variables as regresses), the following tests were used[1]: Goldfeld-Quandt test (*GQ*) [18], Breusch-Godfrey test (*LM*) [19], Ramsey test (*RESET*) [20], Jarque-Bera test (*JB*) [21]. Since the economic patterns of export and import of BRICS member countries differ significantly, only statistically significant regresses from the general set (3) included in the behavioral equations of the system are retained. *Tables 4–7* present the results of estimating the trade turnover models of Russia with each of the BRICS countries based on cumulative training samples from three-time intervals (N = 1, 2, 3), as shown in *Table 3*. The table header includes the designations for the research interval number (N), predetermined variables of the regression equation significantly affecting trade turnover with the *j-th* country (j = 1, 2, 3, 4), the adjusted coefficient of determination (R^2_{adj}), the standard error of the model (*sigma*), *F*-statistic, and the statistics of diagnostic tests. Under the parameter estimates are their standard errors, and under the test statistic values are the *p-value*.

The significant variables affecting the trade turnover between Brazil and Russia turned out to be: the trade turnover in the previous year, the current price of Brent crude oil, and the current exchange rate of the dollar (*Table 4*).

The parameter estimates of the model are statistically significant. All the assumptions

[1] The test statistics are indicated in parentheses.

Table 4

Estimates of Parameters and Testing Results of the Trade Turnover Model between Russia and Brazil

N	FT_{1t-1}	Oil_t	D_t	R^2_{adj}	*sigma*	*F*	*GQ*	*LM*	*RESET*	*JB*
1	0.362 0.157	0.035 0.008	0.016 0.009	0.978	0.719	237 0.000	4.358 0.128	0.521 0.471	1.324 0.268	1.360 0.507
2	0.363 0.153	0.035 0.008	0.015 0.009	0.978	0.700	299 0.000	2.026 0.256	0.674 0.412	1.531 0.234	1.349 0.510
3	0.206 0.167	0.040 0.008	0.026 0.009	0.976	0.823	299 0.000	3.900 0.105	1.679 0.195	0.090 0.768	2.247 0.325

Source: Authors' calculation.

Table 5

Estimates of Parameters and Testing Results of the Russia-India Trade Turnover Model

N	*const*	Y_{2t-1}	D_t	P_t	R^2_{adj}	*sigma*	*F*	*GQ*	*LM*	*RESET*	*JB*
1	2.319 0.623	0.005 0.001	−0.091 0.008	−1.000 1.054	0.915	0.922	65.590 0.000	2.949 0.263	0.011 0.917	0.001 0.999	0.721 0.698
2	2.409 0.659	0.006 0.001	−0.094 0.009	−1.042 0.903	0.918	0.974	60.090 0.000	5.582 0.726	0.123 0.412	1.531 0.234	1.248 0.536
3	1.912 0.635	0.005 0.001	−0.077 0.025	−1.600 0.904	0.898	1.038	59.970 0.000	5.317 0.101	0.642 0.423	0.016 0.902	1.067 0.587

Source: Authors' calculation.

Table 6

Estimates of Parameters and Testing Results of the Trade Turnover Model between Russia and China

N	FT_{3t-1}	Y_{t-1}	Y_{3t-1}	D_t	Oil_t	P_t	*s*	*F*	*GQ*	*LM*	*RESET*	*JB*
1	0.305 0.162	−0.028 0.001	0.009 0.001	−0.421 0.074	0.589 0.038	−8.0 4.15	3.1	1464 0.00	2.67 0.22	2.53 0.11	0.26 0.62	0.13 0.94
2	0.321 0.164	−0.027 0.005	0.009 0.001	−0.406 0.074	0.579 0.038	−5.5 3.64	3.2	1599 0.00	1.36 0.39	2.40 0.12	1.17 0.29	0.51 0.77
3	0.49 0.21	−0.036 0.005	0.008 0.001	−0.387 0.099	0.615 0.049	−12 4.29	4.3	1076 0.00	3.32 0.13	5.10 0.02	2.47 0.14	0.29 0.86

Source: Authors' calculation.

of the Gauss-Markov theorem are met. The parameter estimates obtained from the sample data of the first and second cumulative intervals are almost identical and differ only slightly from the estimates for the third interval, indicating the stability of the model. The coefficient of determination indicates the high quality of the model, and the F-test indicates its statistical significance.

Table 7

Estimates of Parameters and Testing Results of the Trade Turnover Model between Russia and South Africa

Const	Y_{4t-1}	Y_{t-1}	D_t	Oil_t	P_t	s	F	GQ	LM	RESET	JB
−0.486 0.115	−0.002 0.001	0.001 0.000	0.017 0.004	0.009 0.003	0.17 0.11	0.1	55 0.00	6.91 0.07	0.98 0.32	2.03 0.18	0.99 0.61
−0.496 0.113	−0.002 0.001	0.001 0.000	0.017 0.004	0.009 0.003	0.12 0.10	0.1	55 0.00	4.57 0.09	0.88 0.35	2.54 0.14	0.99 0.61
−0.442 0.106	−0.001 0.001	0.001 0.000	0.013 0.002	0.006 0.002	0.13 0.09	0.1	62 0.00	4.71 0.08	3.15 0.08	1.08 0.32	1.68 0.43

Source: Authors' calculation.

The analysis of the trade model between India and Russia is presented in *Table 5*.

The current volume of India's trade with Russia is significantly influenced by the lagged value of India's GDP and the current exchange rate of the dollar. The results of the evaluation over all three intervals showed the significance of parameter estimates, the significance of the regression as a whole, high model quality, and the fulfilment of all its assumptions. In the third observation period, covering the years from 2000 to 2021, the constant reflecting the influence of factors related to macroeconomic instability changed significantly.

The results of the statistical study on the trade volume between China and Russia are summarized in *Table 6*.

The volume of trade between China and Russia was significantly influenced by the lagged values of the GDPs of China and Russia, the current value of the dollar exchange rate, the current value of oil prices, and the pandemic index. It is worth noting that the model is statistically significant and of high quality across all research intervals:

$$R^2_{adj1} = 0{,}998\,,\ R^2_{adj2} = 0{,}998\,,\ R^2_{adj3} = 0{,}997\,.$$

However, in the third interval, there is a change in parameter estimates and the emergence of autocorrelation in the random disturbance.

The assessment of the South African foreign trade model, based on the export of minerals and the import of equipment and mineral fuels, is presented in *Table 7*.

From the *p-value* values provided under the estimates of the diagnostic test statistics, it is evident that all the assumptions of the Gauss-Markov theorem for the behavioral equation are met. The model is statistically significant and of high quality across all research intervals ($R^2_{adj1} = 0{,}938$, $R^2_{adj2} = 0{,}940$, $R^2_{adj3} = 0{,}938$). As with the trade models of Russia with Brazil, India, and China, the estimates of the parameters of the trade model between Russia and South Africa in the third interval have changed.

Based on the estimated behavioral equations, forecasts of trade volumes for each BRICS country with Russia were constructed for each study period. *Table 8* presents the results of point and interval forecasts,[2] used to verify the adequacy of the models. To construct forecasts of Russia's trade turnover, the identity of the system of simultaneous equations was used (1).

[2] lwr — lower bound of the confidence interval, upr — upper bound of the confidence interval.

Table 8

Point and Interval Forecasts of Trade Turnover between Russia and the BRICS Countries

Training sample: 2000–2019 Control sample 2020					
Country	**Forecasts**	**True values**	**Forecast errors**	**lwr**	**upr**
Brazil	4.260	4.01	−0.25	2.525	5.999
India	11.480	9.26	−2.22	8.688	14.272
China	98.502	103.97	5.47	88.687	108.318
South Africa	1.089	0.98	−0.11	0.793	1.384
Russia	115.33	118.22	2.89		
Training sample: 2000–2020 Control sample 2021					
Country	**Forecasts**	**True values**	**Forecast errors**	**lwr**	**upr**
Brazil	5.014	7.3	2.286	3.300	6.729
India	9.876	12.00	2.124	7.390	12.362
China	129.778	145.70	15.922	120.369	139.188
South Africa	1.244	1.04	−0.204	0.903	1.585
Russia	145.91	166.036	20.45		
Training sample: 2000–2021 Control sample 2022					
Country	**Forecasts**	**True values**	**Forecast errors**	**lwr**	**upr**
Brazil	7.21	9.9	2.69	5.496	8.638
India	11.44	43.50	32.06	9.003	13.880
China	173.35	190.20	16.85	155.521	191.182
South Africa	1.252	0.84	−0.412	0.986	1.524
Russia	193.25	244.436	51.186		

Source: Authors' calculation.

From the analysis of the results presented in *Table 8*, it follows that the values of the trade turnover of BRICS countries in 2020 are covered by confidence intervals estimated based on sample data from 2000–2019. The estimated models correspond to the sample data at a significance level of 5%. Interval estimates constructed based on turnover forecasts for 2021 using the training sample from the second interval (data from 2000–2020) for Brazil and China do not include the true values. Of the models evaluated based on the training sample of the third interval (data from 2000–2021), only the model of trade turnover between China and Russia was adequate, but as the diagnostic data (*Table 6*) showed, it exhibits autocorrelation. The maximum forecast error for the volume

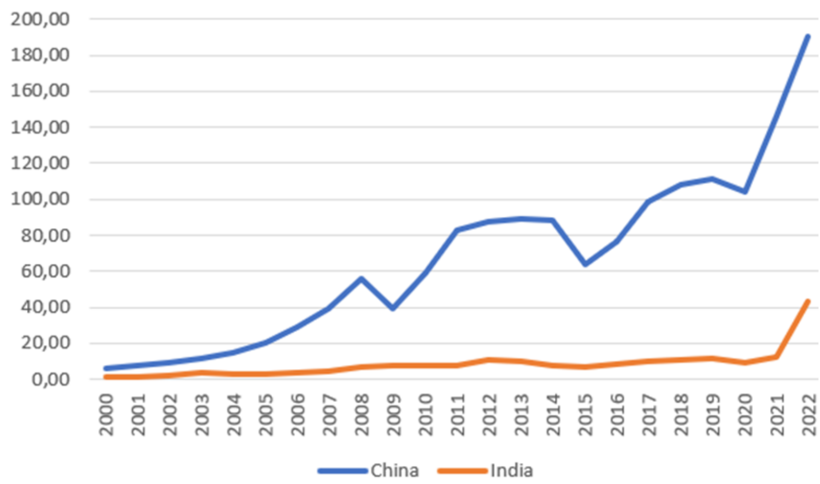

Fig. **Dynamics of Trade Turnover Volumes between Russia and India and China in 2000–2022, Billion US Dollars**

Source: Compiled by the authors according to *Table 2*.

of trade between Russia and India is 32.06 billion dollars. Therefore, a log-linear model was selected and estimated based on data from 2000 to 2021:

$$\underset{(t)}{\ln FT_{2t}} = \underset{(7{,}418)}{0{,}859} + \underset{(10{,}340)}{0{,}001} \cdot Y_{2t} - \underset{(-3{,}998)}{0{,}09} \cdot D_t + \underset{(0{,}209)}{e_t}, \quad R^2_{adj3} = 0{,}892, \quad F = 87{,}952. \qquad (5)$$

The justification for the choice between linear and log-linear models was carried out using the Zaremba method [22]. The specification change allowed for a reduction in the forecast error (20.58 billion dollars), however, the interval estimate of the endogenous variable of model (5) at the 5% significance level (*fit* = 22.92, *lwr* = 13.943, *upr* = 37.688), also does not cover the value of the turnover in 2022 (43.5 billion rubles). Apparently, the reason for the inadequacy of the log-linear model lies in the structural changes in the data related to the influence of the geopolitical situation.

TESTING OF STRUCTURAL SHIFT

The volumes of mutual trade between Russia and China and India sharply increased in the post-pandemic period (see *Fig.*).

China's trade turnover with Russia in 2022 compared to 2021 increased by 1.3 times. As of the end of 2023, it amounted to 240.11 billion dollars, which was a record result. Compared to 2022, the trade volume between the countries increased by 26.3%. This is evidenced by the data released in January 2024 by the General Administration of Customs of the People's Republic of China. Russia and India increased their trade turnover in 2023 to 65 billion dollars. This is 1.5 times more than the trade volume in 2022 and 5.4 times more than the volume in 2021. The record growth in trade between Russia and India during the period 2022–2023 is explained by the increase in energy supplies from Russia against the backdrop of reduced purchases by EU countries, caused by unprecedented sanctions from Western countries [23, 24]. Linear models used for the periods 2000–2019 and 2000–2020 (*Tables 5, 6*) are unable to "capture" the emerging nonlinear patterns. One of the approaches developed for such cases is regime-switching models — time series models characterized by different behavior over different time intervals (Markov-switching models and structural break models) [25, 26]. Markov switching models are used to describe frequent shifts that occur at random points in time. In structural break models, the shifts are rare and fully exogenous, which better corresponds to the behavior of the studied data in the post-pandemic period.

For testing the statistical significance of structural changes, the Chow breakpoint test is used. The test statistic follows a Fisher distribution and is calculated using the formula [27]:

$$F_{Чоу_b} = \frac{\left[ESS_0 - \left(ESS_1 + ESS_2\right)\right]/k}{\left(ESS_1 + ESS_2\right)/\left(n-2k\right)} \sim F(k, n-2k), \tag{6}$$

where k —number of model parameters, n — sample size, ESS_0 — sum of squared residuals of the model estimated on the sample of size n (all sample data), ESS_1 and ESS_2 — sum of squared residuals of the model estimated on subsamples formed with consideration of assumptions about structural changes. The sizes of the subsamples are — n_1 и n_2, where $n = n_1 + n_2$. To formulate the null and alternative hypotheses of the test, we will write the specification of the multiple regression model (4) for the first and second subsamples [28]:

$$Y_t = \beta_1' X_{1t} + \ldots + \beta_i' X_{it} + \ldots + \beta_k' X_{kt} + \varepsilon_t', \tag{7}$$

$$Y_t = (\beta_1' + \gamma_1) X_{1t} + \ldots + (\beta_i' + \gamma_i) X_{it} + \ldots + (\beta_k' + \gamma_k) X_{kt} + \varepsilon_t''. \tag{8}$$

Thus, taking into account (7) and (8), the null and alternative hypotheses of the Chow test take the form:

$$H_0 : \gamma_1 = \gamma_2 = \ldots = \gamma_k = 0, \; H_1 : \gamma_1^2 + \gamma_2^2 + \ldots + \gamma_k^2 > 0. \tag{9}$$

In the case where there are insufficient observations to compute statistic (6) after structural changes, the work [29] shows that the sum of squared residuals of the model estimated on the second subsample can be neglected, and thus statistic (6) of the Chow test is transformed into the statistic of the predictive Chow test

$$F_{Чоу_f} = \frac{\left(ESS_0 - ESS_1\right)/\left(n - n_1\right)}{ESS_1/\left(n_1 - k\right)} \sim F(n - n_1, n_1 - k). \tag{10}$$

The Chow breakpoint test (10) for the model assessing the volumes of mutual trade between Russia and China over the period from 2000 to 2022 takes values

$$F_{Чоу_f} = 7{,}520 > F_\alpha = 3{,}806,$$

and for trade with India

$$F_{Чоу_f} = 5{,}910 > F_\alpha = 3{,}190.$$

Thus, at the significance level $\alpha = 0{,}05$ structural changes in the economies of trading countries significantly affected the stability of the estimates of the parameters of their behavioral equations.

TWO-PHASE LINEAR MODEL

The statistical significance of structural changes means that the null hypothesis (9) is not accepted, and when transitioning from the first part of the observation interval (before

Table 9

Residuals of the Two-Phase Model

t	1	2	3	4	5	6	7
e_t	– 0.278	2.125	1.179	–1.798	–5.040	–3.351	2.182
t	8	9	10	11	12	13	14
e_t	1.918	4.718	2.075	1.134	0.057	–1.992	–1.143
t	15	16	17	18	19	20	21
e_t	–3.731	–0.252	4.762	–1.692	–2.569	2.569	**0.000**

Source: Authors' calculation.

structural changes) to the second (after structural changes), at least one parameter, or some part of them, changes. If we denote the first n_1 observations from the available data Y, X as Y_1, X_1, and the remaining ones as — Y_2, X_2, then the system of equations for the observations of the regression model (4) in matrix form, taking into account structural changes, takes the form[3]

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}. \quad (11)$$

The specification of type (11) is called a *two-phase linear regression model* or a *switching regression model* [25]. If the constant and one or more slope coefficients change, but some parameters remain unchanged, then the regressor matrix in the two-phase linear model may include the following variables:

$$X = \begin{bmatrix} i_{pre} & 0 & Z_{pre} & 0 & W_{pre} \\ 0 & i_{post} & 0 & Z_{post} & W_{post} \end{bmatrix},$$

where the first two columns of the regressor matrix X — dummy variables indicating the observation periods before and after structural changes; Z — values of the regressors whose coefficients change; W — values of the regressors whose coefficients do not change. The index *pre* denotes the values of the variables before the structural changes, *post* — after.

When assessing the volume of mutual trade between Russia and China based on data from 2000–2021, taking into account the insignificance of the constant term in the behavioral equation, a regressor matrix of the form was used:

$$X = \begin{bmatrix} Z_{pre} & 0 & W_{pre} \\ 0 & Z_{post} & W_{post} \end{bmatrix},$$

where

$$Z_{pre} = FT_{3t-1\,pre},\ Z_{post} = FT_{3t-1\,post},\ W_{pre} = (Y_{t-1}, Y_{3t-1}, D_t, Oil_t, P_t)_{t=pre},$$

$$W_{post} = (Y_{t-1}, Y_{3t-1}, D_t, Oil_t, P_t)_{t=post}.$$

[3] Green, W.G. Econometric Analysis. Textbook for university students. Vol. 1; trans. from English; under the scientific editorship of S.S. Sinelnikov and M. Yu. Turuntseva. Moscow: Publishing House "Delo" of RANEPA; 2016. 760 p.

Table 10

Forecast of Russia's Trade Turnover with BRICS Partners for 2022 Taking Into Account the Structural Shift

Training sample: 2000–2021 Control sample 2022					
Country	**Forecasts**	**True values**	**Forecast errors**	***lwr***	***upr***
Бразилия	8.102	9.9	1.798	5.830	10.373
Индия	31.193	43.50	12.307	14.882	65.387
Китай	183.230	190.20	6.970	168.677	197.784
ЮАР	1.150	0.84	–0.310	0.813	1.486
Россия	223.675	244.436	20.765		

Source: Authors' calculation.

Below are the results of estimating a two-phase linear regression model of mutual trade between Russia and China based on data from 2000 to 2021

$$FT_{3t} = \underset{(0,164)}{0,321} \cdot FT_{3t-1} \cdot d_{1t} + \underset{(0,158)}{0,474} \cdot FT_{3t-1} \cdot d_{2t} - \underset{(0,005)}{0,027} \cdot Y_{t-1} + \underset{(0,001)}{0,009} \cdot Y_{3t-1} -$$

$$- \underset{(0,074)}{0,406} \cdot D_t + \underset{(0,038)}{0,579} \cdot Oil_t - \underset{(3,641)}{5,480} \cdot P_t + \underset{(3,164)}{e_t}, \quad R^2_{adj3} = 0,998, \quad F = 1673,17, \tag{12}$$

where

$$d_{1t} = \begin{cases} 1 \; on\,the\,interval\,[2000-2020] \\ 0 \qquad in\;2021 \end{cases}, \quad d_{2t} = \begin{cases} 0 \; on\,the\,interval\,[2000-2020] \\ 0 \qquad in\;2021 \end{cases}$$

— dummy variables (switch). For the two-phase model (12), all assumptions are satisfied: *GQ* = 1.023, *p-value* = 0.513; *LM test* = 2.525, *p-value* = 0.112; *RESET* = 1.172, *p-value* = 0.299; *X-squared*: 0.395, *p-value*= 0.821.

Table 9 presents the residuals of model (12), confirming the equality of the residual to zero at the switching moment, valid for two-phase models.[4]

Similar studies have been conducted in the construction of a two-phase model of mutual trade between Russia and India:

$$\underset{(t)}{\ln(FT_{2t})} = \underset{(7,482)}{0,903} + \underset{(10,510)}{0,001} \cdot Y_{2t} \cdot d_{1t} + \underset{(8,301)}{0,0011} \cdot Y_{2t} \cdot d_{2t} - \underset{(-4,201)}{0,021} \cdot D_t + \underset{(0,207)}{e_t},$$

[4] Green W.G. Econometric Analysis. Textbook for university students. Vol. 1, trans. from English; under the scientific editorship of S.S. Sinelnikov and M. Yu. Turuntseva. Moscow: Publishing House "Delo" of RANEPA; 2016. 760 p.

$$R^2_{adj3} = 0{,}894\,,\ F = 60{,}266\,,$$

where

$$d_{1t} = \begin{cases} 1 & on\,the\,interval\,[2000-2020] \\ 0 & in\,2021 \end{cases},$$

$$d_{2t} = \begin{cases} 0 & on\,the\,interval\,[2000-2020] \\ 1 & in\,2021 \end{cases}.$$

The prerequisites of the model have been met: *GQ* = 0.331, *p-value* = 0.916; *LM test* = 0.359, *p-value* = 0.549; *RESET* = 7.367, *p-value* = 0.02; *X-squared*: 0.711, *p-value*= 0.701.

Forecasts of Russia's foreign trade turnover depending on the foreign trade turnover of BRICS countries, taking into account the impact of structural changes over the study period, are presented in *Table 10*.

As empirical studies have shown, using the example of modelling Russia's mutual trade with BRICS members (*Tables 8, 10*), one of the advantages of two-phase models, in the presence of structural changes during the study period, is a higher degree of correspondence with sample data. The forecasting error for the volumes of mutual trade between Russia and BRICS partners for 2022 decreased by 2.5 times.

CONCLUSION

As the basic model for forecasting the level of Russia's foreign trade turnover with BRICS countries, an econometric model of a system of simultaneous equations (SSE) was chosen for the study. The behavioral equations of this model describe the current levels of foreign trade turnover of each country (other than Russia) depending on the current or lagged values of a number of macroeconomic factors, such as GDP, the price of Brent crude oil, the exchange rate of the US dollar, and the indicator of the presence of a pandemic in the current period. The independence of the system of behavioral equations allowed them to be evaluated in isolation for each country. To select the form of specification for the regression equations, standard *t*-tests and *F*-tests for nested models were used, as well as Ramsey and Zaremba tests for non-nested models. The verification of model assumptions was carried out using tests implemented in the *R* programming environment.

To analyse the stability of the forecasting model for the volume of mutual trade between Russia and its BRICS partners, the sample data for the study period were divided into three cumulative intervals. For each of the intervals, behavioral equations were constructed for all BRICS countries (except Russia) based on the training sample, and their adequacy was tested using the control sample data. It should be noted that the models are stable over the periods 2000–2019 and 2000–2020. Parameter estimates changed slightly. The true values of trade volumes were covered by confidence intervals. The periods 2000–2021 and 2000–2022 are characterized by significant structural changes associated with the intensification of sanctions pressure from Western countries. To test the statistical significance of the impact of these changes on the trade volumes of BRICS countries with Russia, the Chow test was used, which is applied in cases of insufficient observations in the period after structural changes (one or two observations).

The Chow breakpoint test for the model assessing the volumes of mutual trade between Russia and China, and Russia and India, over the period from 2000 to 2022, showed statistical significance of structural changes. Russia and India increased their trade turnover in 2022 to 43.5 billion dollars, while Russia and China reached 190.2 billion dollars. For modelling structural changes, two-phase models were used, applied to individual behavioral equations of the SSE. This approach allowed for a 2.5-fold increase in the accuracy of the forecast for the volume of trade turnover in the Russian Federation.

As follows from the results of the conducted research, economic instability can have a significant impact on the robustness of an econometric model (and consequently on its predictive capabilities). Therefore, under current conditions (pandemic, sanctions pressure), the diagnostic stage should include checking the significance of structural changes, and if they are significant, making adjustments.

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Conflicts of Interest Statement: The authors have no conflicts of interest to declare.

The article was submitted on 07.11.2024; revised on 18.11.2024 and accepted for publication on 26.11.2024. The authors read and approved the final version of the manuscript.

Translated by V.I. Timonina