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New Models for Analyzing Changes in Company Value Based on Stochastic Discount Rates

P.E. Zhukov

Financial University, Moscow, Russia
<http://orcid.org/0000-0003-0951-7164>
 Scopus ID: 57200148155

ABSTRACT

We propose new models for analyzing changes in the value of the company using stochastic discount rates. It is shown that for the majority of the companies under study, local changes in the rate of the company value growth (percentage changes to the previous level) are not explained by the corresponding changes neither in the weighted average cost of capital (WACC), nor in the cash flows. This fact, as well as the research results by J. Cochrane, who proved that discount rates volatility is the main contributor to price volatility, became initial prerequisites for building models based on stochastic discount rates. The work presents three models built on stochastic discount rates, where cash flows are assumed to be growing with a certain trend, and the factors affecting the price of the company are described by stochastic discount factors. These models are alternative in relation to the commonly used traditional cash flow discounting (DCF) models where the free cash flow is discounted through the WACC, or the free flow to capital at the opportunity cost of equity. The first model is used to analyze the dependence of the company value on investments. It uses free cash flow subject to zero growth. The second model uses net cash flow from operating activities plus interest, minus the minimum investment subject to zero growth. The third model uses net cash flow from operating activities plus interest adjusted to taxes. This model requires to estimate the rates of the company downsizing subject to zero investment. The third model is applicable for companies with volatile investments, where it is difficult to reliably estimate free cash flow in case of zero growth. The models are designed for analysis of the factors influencing the value of the company for value-based management. Another application of the models is the evaluation of investment value of the company and the answer to the question of its possible overestimated or underestimated value. The third way to apply this model is the empirical evaluation of the weighted average cost of capital applicable to the company's investment projects, alternative to WACC, assessed by standard methods.

Keywords: enterprise value; financial risks; free cash flow to the firm; weighted average cost of capital; stochastic discount rates; generalized method of moments

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INTRODUCTION

The relevance of the company value analysis is explained by the fact that it is necessary at least to solve two interrelated tasks: the value-based management and the future investment valuation of the company by an investor¹. These tasks are usually set when choosing a company's capital structure, issuing stocks and bonds, investments, mergers and acquisitions, financial policies, considering investment projects, etc. One of the key issues is also the capital valuation (price, opportunity costs) and its impact on the company value.

From the Modigliani-Miller theory (MM) [1–3], it follows that the weighted average cost of capital is the discount rate applicable to the free cash flow to the firm [4]:

$$EV(0) = \frac{\sum_{t=1}^{\infty} FCF(t, 0)}{(1+r(0))^t}. \quad (1)$$

At the same time, $FCF(t, 0)$ and $r(0)$ in (1) denote free cash flow to the firm and the weighted average cost of capital estimated at time $t = 0$. It should be noted that the MM theory does not at all imply that the expected amount of cash flows and the discount rate are constant, that is, they do not depend on the zero moment when the investor makes decisions about investments. However, as a rule, it is implied (by default) that the free cash flow to the firm is a parametric set of random variables depending on t (as a parameter) with an expected value that is also dependent on t , but not dependent on the evaluation time. Moreover, the stiffness of the structure and cost of capital are regarded as an obvious assumption, and instead of (1) a simplified model is usually considered [5]:

$$EV(0) = \frac{\sum_{t=1}^{\infty} FCF(t)}{(1+r)^t}. \quad (2)$$

Note that the generally accepted model (2) differs from model (1) in that the expected cash flows and discount rate do not depend on the moment of investment valuation [6], that is, the moment when the investor makes investment decisions.

Here r is the weighted average cost of capital (WACC), which is calculated as the average weighted by the share in the total value of the firm, the value of the average required return: equity, preferred shares and interest debt [6]:

$$WACC = \left[\frac{MV(S)Re + MV(ND)Rd(1-T) + MV(PS)Rps}{EV} \right]. \quad (3)$$

Here $MV(S)$ is the capitalization (market value of ordinary shares), $MV(PS)$ is the market value of preferred shares, $MV(ND)$ is the cost of net debt, Re , Rd and Rps are the required return of the indicated components of the company's capital, and T is the effective corporate income tax rate. EV means the total value of a company — the sum of all its capital components [6]:

$$EV = MV(S) + MV(ND) + MV(PS).$$

At the same time, net debt usually means only long-term interest-bearing debt with the deduction of cash and short-term investments, although it is sometimes stipulated [5] that short-term debt, a permanent part of a company's capital (that is, not dependent on seasonal variations), can be adopted for calculation (3). The basis for such a long-term interpretation of the WACC in (2) and (3) is that this discount rate should be applicable to free cash flows to the firm, valued for a very long time. At the same time, the possibility to change the capital structure of a firm is usually not considered. Representation (1) — (3) is not suitable for companies whose cash and securities exceed the debt (for

¹ It is necessary to distinguish a balance sheet valuation and an investment valuation. Balance sheet valuation determines the current market value of assets and is carried out in accordance with International Financial Reporting Standard (IFRS 13). Investment valuation refers to the future (estimated) value of the company and is made by investors in accordance with their internal regulations and methods.

example, Google or Surgutneftegaz). In these cases, it can be assumed that the company's activities, in addition to the main business, also include "financial business", and formulas (1) – (3) are modified in accordance with this assumption, but this goes beyond the framework of the classical MM theory.

One of the key shortcomings of the MM theory, which was immediately noticed by its critics (in particular, Stiglitz [7]), is ignoring the costs of a possible default and financial instability of a company.

In accordance with the traditional theory of capital structure, these costs are the main factor preventing the increase in financial leverage (the level of a company debt). In any case, the level of debt and the risk of default are always considered by the company managers and banks that provide loans.

Despite this, most theorists (including Modigliani, Miller, Myers and Merton) insist that these costs are insignificant, since for most large companies the risk of default is usually insignificant. However, if such companies start increasing their debt, literally following the conclusions of the MM theory (the company's value grows with debt growth), then their risk of bankruptcy becomes significant. This was illustrated by the 2008 crisis when a number of major companies were on the verge of bankruptcy. The main reason for reluctance to recognize the significance of the default risk for choosing a capital structure is that theorists usually come from rather artificial assumptions about an ideal market where bankruptcy really does not change the value of a company's assets. A striking example can be found in the classic work by R. Merton [8], where a stochastic model for the company's value is given and, based on the Black-Scholes model, a clearly contradictory practice and common sense concludes that the MM theory retains its significance even in potential event of default².

² This article, unfortunately, contains a number of serious errors that have not been critically evaluated and analyzed in the literature, except [18]. In particular, the use of the Black-Scholes model is possible only for stationary random processes.

Later, the traditional approach of the MM theory was questioned, and there arose the problem of choosing the optimal capital structure, resulting in later theories — trade-off and pecking order [9, 10]. The latter theory proposed by Myers is based on the fact that the transaction costs arising from the placement of new issues of debt or stocks can play a key role in choosing the capital structure. As an empirical base, this work used the conclusions by Donaldson [11].

In works by many authors, in particular, Strebulaev and others [12–14], stochastic modelling of changes in company value was carried out to find the optimal capital structure and default risk. The stochastic Merton model was used (but not theoretical conclusions) with heteroscedasticity and GARCH models.

An empirical test of the trade-off and pecking order theories applicability was made by Eu. Fama and K. French [15]. The result was that both of them have certain confirmations. Although this result is hardly satisfactory, since the two theories mentioned above contradict each other to a significant extent.

The works of the scholars belonging to V. Brusov's scientific school [16] consider a different approach to the development of the optimal capital structure of the company, taking into account the finite life of the company and suggests an alternative mechanism to develop the optimal capital structure of the company, different from the trade-off and pecking order theories.

R. Merton's theory critical analysis [8] was carried out in work [17] based on the empirical data. A theory similar to the MM one is being developed in work [18], however, it considers the corrections for default risk and transaction costs. The MM theory is shown to be correct when debt increases to a certain limit where there is no significant increase in financial risks for investors, including the risk of default. When debt increases this limit, the costs of default risk prevail over the benefits of tax shields. However, financial leverage is not the only factor affecting default risk, and therefore the capital structure in fact depends largely on such factors as the specifics

of the company, the success of its business, as well as macroeconomic, country and other risks.

Analyzing the factors that the company value depends on, according to model (1) or (2), two main groups can be distinguished: the factors affecting expected cash flows and the factors affecting the assessment of discount rates. In particular, although the MM theory refers to the overall impact of financial policy on the company value, the main group of considered factors is financial risks reflected in the WACC discount rate (3).

Basically, any risk factors can be reflected in calculating the expected cash flows in (1) or (2), or discount rates in (3). Both principles are theoretically equivalent, but in practice they usually give various results due to the variety in calculation methods.

For example, the WACC is usually calculated by means of model (3), where CAPM model is used to estimate the required return on equity that evaluates it solely from systematic risks [5, 6]. At the same time, methods for calculating beta and premiums for market risk can vary greatly among different researchers, and especially among different investment companies that use different empirical “corrections” to beta and market risk premiums — premiums for low liquidity, CDS, etc. The assessment of the required return on equity often includes adjustments for the company’s individual risks, or country risks. Strictly speaking, the introduction of such amendments contradicts the theory of CAPM, since in the fully diversified portfolio of securities the effect of individual and country risks is zero in the limit. However, it corresponds better to the risk assessment by investors [19]. Numerous studies [20–23] have shown that it is the reassessment of risks that gives rise to fluctuations in the company value. This is consistent with the principles of risk assessment analysis in the Basel-3 standard for banks [24].

The study is based on the WACC calculated in the Bloomberg system, which usually experiences significant fluctuations associated with the systematic risks assessment, the risk-free rate and the premium for a systematic risk. How-

ever, changes in the WACC do not fully reflect the risks considered by investors in the discount rate. For example, it was shown in [25] that the WACC fluctuations calculated in the Bloomberg system often do not relate to the company value change, or this relationship exists, but it describes only about 10–20% of the variance.

Apparently, the main reason is that the WACC actually does not always and in full reflect the investors’ assessment of the real risks of the company. In particular, investor risk assessment is subjective, unlike the WACC, which calculation is usually standard and objective.

From the rational investor expectations hypothesis it follows that, at least, professional investors use an estimate of the benefits derived from owning an asset during an investment horizon (for example, 1 year) and from its actual or potential sale after this period. This leads to a discounted cash flow (DCF) model for estimating the future value of assets, reflected by equality (1).

The relevance of the rational investor expectations hypothesis is challenged by the school of behavioral economics, including R. Thaler [26]. However, this mainly concerns the behavior of households and small investors. Indeed, it is difficult to assume that a housewife applies complex procedures to estimate future utility of consumption and chooses between current and future consumption by maximizing the Hamiltonian expressing the overall utility of consumption. However, there are qualified investors in the financial markets who usually use models like (1) to evaluate the future benefits of investment.

Theoretically, the influence of risks in model (1) can be considered either through cash flows or through discount rates and these approaches are equivalent and interchangeable. However, in real life it is not so: the expected cash flows are usually estimated based on the past experience, and the discount rates reflect future risks assessment by the investor.

An important conclusion was made in J. Cochrane’s work [27], where the long-term correlation between the volatility of wide

indexes and their profitability was studied. This paper shows that the main role in the volatility of indexes reflecting the asset pricing (of companies) is precisely the volatility of the discount rate, and the volatility of cash flows contribution is close to zero. This conclusion by Cochrane was the starting point of this study.

Since the discount rates turn out to be responsible for the volatility of market prices, it should be assumed that they are stochastic in nature. Numerous works by various foreign authors [28–33] have considered models that use stochastic discount rates to analyze price changes in stock markets and product markets.

Accordingly, in the work the following research tasks were set:

- to determine the degree of dependency of stock prices of companies on the volatility of their cash flows and discount rates;
- to build cash flows and stochastic discount rates applicable for analyzing company prices,
- to develop a method for analyzing the company value based on appropriate stochastic discount rates.

MODELS AND METHODS OF RESEARCH

Going back to the MM model (2) and (3), we note that here the WACC discount rate depends mainly on the capital structure, interest rates, as well as on the systematic risks reflected in the beta coefficient and market risk premium.

At the same time, in (2) it is assumed that the discount rate and cash flows are determined at the time of investment valuation and for all the years ahead (to the investment horizon and even further). The discount rate is determined in accordance with the current WACC (3), and the expected cash flows are also estimated at the present time, and this estimate is maintained in the future (that is, it does not depend on the estimation point).

If we assume that this estimate of the expectation and discount rates in (2) may change over time, then we turn to model (1), and also, possibly, to non-stationary cash flows and future discount rates outside the MM and CAPM theories applied at the time of investment valuation.

For example, in conditions of nonstationarity, the basic principle on which the MM theory is based is the impossibility of arbitration. If the expected cash flows of two companies today are equal, but they are not stationary, then the next day (month, year, etc.), they can already be different and then the companies can no longer be considered equivalent.

On the contrary, in general model (1) the discount rate and cash flows can depend on both time and the moment of their assessment by the investor. It is assumed that the assessment of discount rates is changeable and substantially depends on the models used by the investor.

In work [25], low sensitivity of investors to random fluctuations of the company cash flows at intervals from one quarter to three years was established. At the same time, there is no doubt that for fast-growing companies, a significant change in the trend of the expected future cash flow (or, equivalently, the expected growth rate of the cash flow) influences the estimation of the company value by investors. However, this can be attributed to the expected rate of growth of cash flow in the future, and random fluctuations in cash flow themselves do not seem to have a noticeable effect on price, unless investors perceive them as a change in trend.

Based on the empirical data and results by J. Cochrane [27], it can be assumed that investors are guided by a certain average level of cash flow, considering the expected average growth rate. At the same time, the average growth rate can also experience stochastic fluctuations affecting the price of the company. Further it will be shown that these fluctuations are indistinguishable from fluctuations of the discount rate, and therefore, they can be considered as one parameter — the stochastic discount rate of cash flows given at some expected level with a constant average growth rate.

For the beginning, a specific model of generalized moments was chosen, in the variant proposed by Cochrane [35] as the most general model for analyzing the company value. It is based on the general concepts of the utility function and the useful return on an asset:

$$p = M \times CF. \quad (4)$$

Here, p — is the expected total company value (or its capitalization), M — is the vector of infinite dimension of stochastic discount rates (points) in the future moments of time, and CF — is the vector of expected cash flows from the asset (investor's gain on ownership of the asset). Model (4) is similar to model (2), but at the same time, discount rates may depend on time (in the MM theory they are constant), and free cash flows to a company or capital do not necessarily act as cash flows from an asset, as is commonly believed (this issue is discussed below).

The method of “generalized moments” in model (4) does not necessarily set the task of statistical estimation of the best parameters of econometric models classically (see L. Hansen and T. Sargent [36]). Similar to classical method [36], stochastic discount rates are found by minimax methods, as parameters of the C-CARM economic model, and describe intertemporal investor preferences associated with changes in the relative value of consumption and savings. To justify the stochastic discount factors, J. Cochrane also uses the Arrow-Debreu macroeconomic theory of consumption, based on the choice between future and present consumption. This theory is fundamental in modern economic theory, and Jean Tirole [37], in particular, notes that the MM and C-CARM theories can be obtained from the Arrow-Debreux theory of macroeconomic equilibrium. However, Cochrane [35] also notes that model (4) is of more general nature and the stochastic discount rates in (4) are not necessarily related to the C-CARM.

The model proposed in this paper considers stochastic discount rates as a reflection of the assessment of macroeconomic, systematic and systemic risks common to the entire industry where the company belongs. As a result, general model (4) takes the following form:

$$EV(0) = \frac{\sum_{t=1}^{\infty} CF(t,0)}{(1+r(t,0))^t}. \quad (5)$$

Here $CF(t, 0)$ — is the investor's expected future cash flow that develops the company value (in model (1) and in the MM theory, there is only free cash flow), and $r(t, 0)$ — is the predicted discount rate. Note that in model (5), in contrast to (2), expected future cash flow and discount rates depend on future periods and on a zero point in time — this is the moment when the investor analyzes investments and makes decisions about investment and financial policies.

As already noted, R. Thaler criticizes the hypothesis of rational expectations of investors and proves that the concept of intertemporal preferences of investors, based on optimizing their expression (4) with discount factors that do not depend on the reference point, does not correspond to the real behavior of investors. He especially emphasizes that the discount rates may change (shift over time) with a change in the point of reference (the moment of investment valuation). For example, in the case of hyperbolic discounting, an investor always uses a higher rate for more distant points in time.

Model (5) considers the possibility of hyperbolic discounting. Moreover, with a choice of expected cash flows $CF(t, 0)$ and discount rates $r(t, 0)$, model (5) will exactly match any individual investor model. The reason for this universality is that $CF(t, 0)$ in model (5) can correspond to any factors reflecting the benefits of the investor or related to the company value. Also, any method of assessing risks not included in the estimation of the expectation of cost or benefit factors can be considered in the discount rates $r(t, 0)$.

Research results [18, 25] showed that short-term fluctuations in the value of the companies reviewed, measured as a percentage change, are usually not associated with the same fluctuations in free cash flow, net cash flow, or the WACC. It also shows for fast-growing companies, for example, such as PAO Novatek, the trend of growth in cash flows entails a similar trend for the company value. At the same time, random deviations of cash flows and company value from this trend are in no way connected. Also, for companies with a stable ratio of free cash

flow to net cash flow, model (1) can be used to discount the free cash flow.

As a result, we can define three postulates underlying the model:

1. In his assessment the investor uses the expected cash flow, which changes with a stable trend. At the same time, the trend assessment may also change, that is, the trend may depend on the moment of evaluation.

2. The investor considers the constantly changing assessment of risks in discount rates, which as a result change in an unpredictable way, that is, they are essentially stochastic factors.

3. At the same time, changing risks can be reflected either in a change in the cost of capital, or in a change in the trend of growth of cash flows. Both factors are stochastic in nature and independent by default.

What follows is the key to the proposed method, the division of cash flows into minimum cash flows and growth flows. Namely, expression (5) for the company value can be written as the total of two components, where the first represents the company value with minimal investment, ensuring that the current business is maintained at a constant level, but does not ensure the growth of the company, and the second is development investment:

$$EV(0) = \frac{\sum_{t=1}^{\infty} CF0(t,0)}{(1+r(t,0))^t} + \frac{\sum_{t=1}^{\infty} CF1(t,0)}{(1+r(t,0))^t}. \quad (6)$$

Here, minimum cash flow $CF0(t, 0)$ includes the minimum investment required to maintain the company cash flow at a constant level (the growth rate is zero). The second part of $CF1(t, 0)$ includes additional investments and cash flows expected from investments.

Further, for simplicity (but without loss of generality), general model (6) will be considered for the particular case of the standard method of calculating the total company value for free cash flows to a company. In this case, the first part is an investment project to support the current business with a zero growth rate, and the second — is an investment project to accelerate the

growth of the company, leading to an increase in growth rate, or to its decline to negative values. The first part is the company value subject to the minimum investment, and the second — is the net present value of the additional investment (positive or negative).

Both projects are optional — the owners and managers of the company may reject the second project, or even both projects. For example, a project with an investment below the minimum level may be accepted, resulting in a negative growth rate. Moreover, a project of curtailing activities by withdrawing funds from the company can be adopted. In particular, it can be realized by repurchasing shares, or paying dividends through loans. In this case, the cash flow of the second project $FCF1(t, 0)$ will be negative, and the project will contribute negative value added. Further, for simplicity, we will assume that the second project always has a positive value added (that is, the company has a positive average growth rate).

We will write down (6) as:

$$V(0) = EVmin(0) + PV(0),$$

$$EVmin(0) = \frac{\sum_{t=1}^{\infty} FCF0(t,0)}{(1+r(t,0))^t},$$

$$PV(0) = \frac{\sum_{t=1}^{\infty} FCF1(t,0)}{(1+r(t,0))^t}. \quad (7)$$

For the first part, the expectation of cash flows is constant. We will base on the standard method derived from the MM theory — the calculation of the total company value for free cash flow and denote the cash flow corresponding to the zero growth rate, by $FCF(0)$:

$$EVmin(0) = \frac{\sum_{t=1}^{\infty} FCF0(0)}{(1+R(0))^t}. \quad (8)$$

Here $R(0)$ is the average discount rate for a cash flow of a firm with zero growth rate.

For the second part, cash flows become positive after the investment period. Instead of the time-dependent discount rate $r(t, 0)$, we can go to the constant rate:

$$PV(0) = \frac{\sum_{t=1}^{\infty} FCF_1(t, 0)}{(1+r(0))^t}. \quad (9)$$

Expression (9) determines the present value of an investment growth project with a positive growth rate. At the same time, it follows from (7) and (9) that the cash flows $FCF(t, 0)$ in model (6) grow with a certain average growth rate $g(0)$ relative to cash flow $FCF_0(0)$ with a zero growth rate, which means we can write down the expression (6) as:

$$EV(0) = \frac{FCF_0(0)}{R(0) - g(0)}.$$

As a result, for the moment of analysis τ , we obtain the final model to analyze the company value $EV(\tau)$ by use of the stochastic discount rate $r(\tau)$, satisfying postulates 1–3 (see above):

$$EV(\tau) = \frac{FCFe(\tau+1)}{r(\tau)}, \quad (10)$$

$$r(\tau) = \frac{FCFe(\tau+1)}{EV(\tau)} = R(\tau) - g(\tau).$$

Here τ — is the moment when the investor analyzes the investment and makes decisions about investment and financial policy, $FCFe(\tau)$ — is the expected free cash flow to the company with zero growth, $R(\tau)$ — is the calculated discount rate of the company cash flow at zero growth rate, and $g(\tau)$ — is the average growth rate of the company, estimated at time τ .

For free cash flow at zero growth rate in (10), its expression can be used through the expected operating cash flow CFO_{exp} (this does not consider profit from financial and investment activities, which is assumed to be insignificant):

$$FCFe(\tau+1) = CFO_e(\tau+1) + Int(1-T) - Inv_0, \quad (11)$$

$$r(\tau) = \frac{FCFe(\tau+1)}{EV(\tau)} = R(\tau) - g(\tau).$$

Here, free cash flow to a firm is expressed in terms of the average expected operating cash flow to a CFO_{exp} firm ($\tau+1$), deducting the minimum investment required for zero growth of investment Inv_0 plus interest after taxes $Int(1-T)$. At the same time, free cash flows in models (10) and (11) depend on the moment of analysis τ , since expressions (8) and (9) are defined for zero moment of investment analysis. To estimate the minimum investment required for zero growth, we can take depreciation adjusted for the replacement value of assets.

All parameters of models (10), (11) are independent stochastic values. However, for the analysis of the company value, only the difference between the cost of capital with zero growth $R(\tau)$ and the cash flow growth rate $g(\tau)$ is important, not each value separately.

If the expected cash flows from operating activities, the minimum investment, the average growth rate and the cost of capital can be separately estimated empirically, then models (10), (11) can be used for factor analysis of changes in the company value compared to its expected value. It is assumed that even a zero growth rate requires some minimal investments that can be assessed.

The problem with free cash flow to a firm is that its relationship with the company value is usually not obvious. For companies with a high growth rate, this flow can be negative for a long time, or its average value can hardly change at a very high growth rate of the company. Therefore, for companies with unstable investments, free cash flow to a zero-trend firm may be difficult to determine analytically.

In these cases, it is preferable to use the net cash flow from operating activities plus interest (that is, the condition of zero investment). This may correspond to a free cash flow, but it is possible that it is not with zero, as assumed in model (8), but with a negative growth trend. For this case, model (10) is as:

$$R(\tau) = \frac{CFO(\tau+1) + Int(1-T)}{EV(\tau)} = R(\tau) + e(\tau) - g(\tau). \quad (12)$$

The designations in (12) are similar to (11). Here $g(t)$ — is the growth rate of the company-value, but an additional variable $e(t)$ is also added, describing the decline in the company value under the condition of zero investment.

Models (10), (11) and (12) differ from model (4) by J. Cochrane [35], as well as from the method of generalized moments by L. Hansen and T. Sargent [36] in the fact that the stochastic discount rates are not associated with intertemporal investor preferences. Instead, they describe the stochastic discount rate, which depends on:

- first, the cost of capital depending on macroeconomic and financial risks,
- second, the changes in the expected future rate of growth of cash flows.

At the same time, both parameters — the cost of capital and the rate of growth — can be considered as stochastic variables associated with both systematic and individual risks of a company, country, or industry. Here, at least two approaches to models (10) or (11) (but not (12)) are possible:

1. If it can be assumed that the estimated cost of capital based on its weighted average value WACC calculated by standard methods (3)³ corresponds to the return required by the investor, then model (10) or (11) can be used to estimate the company stochastic average growth rate reflecting volatility of the company value.

2. On the contrary, it may be assumed that the investor can reliably estimate the average rate of a company future growth, calculated, for example, from long-term macroeconomic forecasts. At the same time, it is obvious that the estimate of the past growth rate according to statistical data can be interpolated into the future only if there is a reason to assume that the trend will continue (as, for example, with oil companies). Then, from model (10) or (11), an empirical esti-

mate of the stochastic cost of capital is obtained, considering the risks that were not taken into account when analyzing the expected cash flows and their growth rate.

Note that in the second case, a reasonable empirical assessment of the discount rate is obtained, applicable to the company's investment projects, which gives an alternative WACC rating, preferable to the common methods based on the CAPM and MM.

Model (12) can be used with the same two objectives, provided that it is possible to reliably estimate the rate of “closing” of the company's activity at zero investment. This model is designed for companies whose free cash flow is insignificant (or even negative) due to high investment in development — for example, companies in a stage of rapid growth or who make significant investments to modernize their business. For such companies, the effects of investment can occur only in the distant future, but the cash flow from operating activities allows to reliably estimate the value of the business.

RESULTS OF EMPIRICAL ANALYSIS OF STOCHASTIC DISCOUNTING BETS AT INDIVIDUAL COMPANIES

Empirical research was conducted on the example of several companies in the oil and gas sector (including BP, Shell, Novatec, Lukoil, Gazprom), since in this sector companies usually have moderate growth rates (close to zero) and the company value changes dependence on the trend does not change the results significantly. Later, several non-primary sector companies with opposite properties regarding systematic risks were added to them (including Coca-Cola, Sony, Apple, Nike). *Table 1* shows selected results of a study on the independence of relative changes in the company value in the oil and gas sector from changes in cash flows and the WACC. The analysis of changes in the relative growth of these indicators to the previous value was used, the essence of which can be illustrated by the expression⁴:

³ At the same time, investment companies apply various amendments to the classical theories of CAPM and MM — see [4–6, 19].

⁴ Panel analysis of changes in absolute values often leads to false regressions due to their nonstationarity. However, the relative change in percent growth, as a rule, is of TS nature.

Table 1

Independent changes in the value of oil companies and Coca-Cola from cash flow and WACC

| Company | FCF (p-val.) | CFO (p-val.) | WACC (p-val.) | R ² | F-stat (p-val.) | MCAP (p-val.) | R ² for the Mcap |
|-----------|-----------------|-----------------|------------------|----------------|--------------------|------------------|--------------------------------|
| BP | 0.64 | 0.6 | 0.78 | 0.01 | 0.89 | 10E-57 | 0.97 |
| Shell | 0.35 | 0.38 | 0.5 | 0.13 | 0.07 | -331.57 | 0.94 |
| Coca-cola | 0.61 | 0.65 | 0.95 | 0.01 | 0.97 | 0.98 | 1.6E-05 |
| Rosneft | 0.63 | 0.32 | 0.14 | 0.07 | 0.4 | 2.6E-27 | 0.94 |
| Lukoil | 0.31 | 0.71 | 0.4 | 0.02 | 0.68 | 5.1E-45 | 0.96 |
| Gazprom | 0.85 | 0.24 | 0.38 | 0.07 | 0.23 | 2.62E-27 | 0.94 |

Source: compiled by the author.

Table 2

Comparison of WACC values and stochastic discount rates by models (11) and (12) for BP company

| | WACC | CFO mln \$ | FCF mln \$ | R _{cfo} | R _{fcf} | EV mln \$ | Mcap mln \$ |
|---------|-------|---------------|---------------|------------------|------------------|-----------|-------------|
| Median | 0.088 | 539 | 135 | 0.012 | 0.003 | 144,000 | 111,000 |
| St.Var. | 0.19 | 1.84 | 0.46 | 0.21 | 0.21 | 0.27 | 0.38 |

Source: compiled by the author.

$$d \ln(x) = \frac{dx}{x} = \frac{\Delta x}{x}.$$

Table 1 shows that local changes in cash flows and the WACC do not affect the changes in the value of the oil companies and the Coca-Cola Company. Thus, two conclusions can be made:

1. For many companies, relative WACC changes are not a reliable measure of risk change. This statement is true, at least for the given sample, but presumably for a significant number of companies.

It is for this reason that the method of stochastic discount rates has an undoubted advantage over the traditional methods of estimating the WACC. Moreover, in real life, these methods almost always include subjective additives on country risk, liquidity risk and other individual risks (which do not correspond to the classical CARM theory).

It should be noted that for some of the studied companies not included in the table 1 (for example, Apple), the significance of dependence (p-val.) is within acceptable values (less than 0.05). However, the coefficient R², characterizing

the fraction of the variance explained, is in all cases too low to speak of a significant dependence (its value, as a rule, does not exceed 0.15).

2. Local changes in the rate of growth of cash flows also do not affect the changes in the rate of growth of the company value.

This does not apply to a change in the trend of growth of cash flows, reflected in stochastic discount rates in models (10), (11) and (12). In this case, models (10), (11), (12) are not based on actual current values of cash flows, but on their expected values obtained from historical data, after smoothing the effects of fluctuations.

Table 2 shows the results of the WACC study for the BP company.

Table 2 shows that for the BP company the real discount rates are significantly lower than the WACC. For example, if calculated according to model (12) on a net cash flow plus interest (without investment), the WACC value is 7.6% higher than the estimated stochastic discount rate.

This difference cannot be explained by the expected growth rate of the company, which

Table 3

Mathematical expectation and variance of WACC and stochastic discount rates by models (11) and (12) for Sony and Shell companies in 2000–2016

| Company | | WACC | CFO mln \$ | FCF mln \$ | Rcfo | Rfcf | EV mln \$ |
|---------|----------|-------|------------|------------|-------|---------|-----------|
| Sony | Median | 0,084 | 1252 | 503 | 0,05 | 0,02 | 30901 |
| Sony | St. Var. | 0,026 | 1,1 | 2,8 | 1,2 | 3 | 0,59 |
| Shell | Median | 0,085 | 8000 | 2880 | 0,055 | 0,00002 | 1200 |
| Shell | St. Var. | 0,14 | 0,4 | 1,2 | 0,36 | 1,4 | 2,6 |

Source: compiled by the author based on quarterly data from Bloomberg.

for BP (as well as for most other major oil companies) is close to zero. It seems that using the WACC as an approximation to the cost of capital for such companies as BP is unreasonable. It can be concluded that for BP (as for other studied oil companies) the WACC does not reflect the cost of capital and one should use the stochastic cost of capital derived from (11) for a given average rate of growth of cash flows. With regard to other sectors of the economy (e.g., IT), such a conclusion cannot be made unambiguously.

It may be concluded is that for oil companies the cost of capital is overestimated due to the overestimated values of the equity cost according to the CARM model. The main problem, apparently, is that the historical assessment of beta does not provide an adequate approximation for macroeconomic risks.

A similar assumption can be made for companies in non-resource sectors — the CARM model gives an overestimated equity cost, but the reasons for this effect are a separate issue that requires analysis, which are not the objectives of this study.

Moreover, for these companies the use of free cash flow gives distorted results due to the fact that it includes investments, highly volatile and discretionary by nature. Thus, for BP (as for most other reviewed oil companies, except Novatek), it is preferable to consider not the free cash flow

to the company, but the net cash flow from operating activities plus interest, as well as to use model (12).

This choice is even more preferable for such fast-growing companies as, for example, Apple and Facebook. For fast-growing companies, free cash flow to the firm is usually not suitable as a factor for analyzing the value because it is usually very low due to high investments and almost never reflects the growth rate of the company. Moreover, in these cases quite often negative free cash flow to the company is observed.

There are also exceptions to this rule — companies such as Novatek and Coca-Cola have stable investments in relation to cash flow, for them free cash flow and net cash flow from operating activities are closely related and any of models (10), (11) or (12) can be considered. However, even in these cases, the cash flow of operating activities plus interest and the model (12) is more appropriate as a base cash flow.

For other reviewed oil and gas companies, using the WACC calculated in accordance with the MM and CAPM theories (or C-CAPM) as a discount rate for cash flow also usually gives an estimated growth rate that is too high compared with empirical data. At the same time, this effect of overestimation is different for different companies. For example, for Sony and Shell companies, this effect is no longer so obvious (Table 3),

if we take the discount rate for the net cash flow from operating activities.

The growth rate for these companies is about 3%, which is slightly higher than the estimated average rate of growth based on actual data. Another approach can also be used — to calculate the expected growth rate using empirical data, or even assume it is constant. In this case, the cost of capital can be only considered as a stochastic variable.

In fact, in accordance with (2), investments have an impact on the growth rate of cash flows, and therefore in model (6) they are automatically considered in the stochastic discount rate.

The result is the following method for analyzing the company value by discounted cash flow.

1. One of the models is selected as the basis:

- free cash flow to the company, subject to minimal investment, ensuring zero growth — model (10);
- net cash flow from operating activities plus interest, minus the minimum investment — model (11);
- net cash flow plus interest — model (12).

Cash flows are assumed to be stochastic, with a fixed, expected value at the current moment and a stochastic growth rate in the future (reflected in the discount rate). The average current cash flow growth rate is determined by empirical data that can be adjusted according to the scenario analysis of future opportunities.

2. The statistical characteristics of stochastic discount rates are determined based on models (10), (11) or (12). These parameters can be adjusted by sampling, corresponding to the expected stage of the business cycle, or by another method (for example, in accordance with the VaR estimation methods proposed in Basel-3). Further, the discounted rates are applied to analyze the company value, its growth rate, or the weighted average cost of capital.

SUMMARY

The paper proposed three models (10), (11) and (12) for empirical calculation of stochastic dis-

count rates applicable to analyze the factors that influence changes in the company value.

Model (10) is applicable for analyzing the dependence of the company value on investments and it applies free cash flow subject to zero growth.

Model (11) is applicable for the empirical calculation of the weighted average cost of capital at a known average future growth rate of the company. It uses net cash flow from operating activities plus interest less the minimum investment subject to zero growth.

Model (12) is applicable for analyzing the company value by net cash flow from operating activities plus interest. This model can be used by companies with unstable investments, for whom it is difficult to reliably estimate free cash flow under conditions of zero growth. In this model, an assessment of the negative growth rate of the company (collapse of its activities) is required, subject to zero investment.

All three models are intended for empirical evaluation of factors affecting the company value, in order to manage it. These models are alternative to traditional DCF models, where free cash flow is discounted through the WACC, or free flow to capital at the opportunity cost of equity.

It is shown that for most of the studied companies, local changes in the rate of growth of company value (percentage changes to the previous level) are not explained by the corresponding changes either in the WACC or in the cash flows. This fact, as well as the results by J. Cochrane [27], became the presuppositions for the construction of models based on stochastic discount rates (10), (11) and (12).

In models (10), (11), (12), cash flows are assumed to grow with a certain trend, and stochastic factors affecting the company value are described by stochastic discount factors.

Other possible applications of models (10), (11) and (12) are the analysis of the company investment value, the answer to the question whether the company is overvalued (or undervalued) by the market and the estimated (empirical) weighted average cost of capital applicable for investment projects.

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ABOUT THE AUTHOR



Pavel E. Zhukov — Cand. Sci. (Econ.), Associate Professor, Department of Corporate Finance and Corporate Management, Financial University, Moscow, Russia
paul-joukov@yandex.ru

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