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Investment Portfolio Optimization on Russian Stock Market in Context of Behavioral Theory

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ABSTRACT

The paper investigates possible investment portfolio optimization considering behavioral errors. The research rationale is due to the adaption of the investment recommendations for unqualified investors on the Russian stock market. In economic literature, the consequences of behavioral effects are not detailed enough when making a portfolio of Russian securities. The aim of the article is to make the most optimal portfolio based on the risk/reward ratio. The author made a hypothesis on applying various periods of profitability analysis to improve profitability indicators and increase the subjective probability of its achievement. To build a portfolio model, the behavioral portfolio theory and its optimization through linear programming were used. The study was based on modeling the investment portfolio of the most liquid stocks on the Russian stock market. Modified elements of the cumulative prospect theory with behavioral coefficients were used as indicators of profitability and probability. Based on the analysis results, the model of semi-annual portfolio analysis was proposed as a tool for portfolio optimization. The investor review of the portfolio semi-annual rate of profitability led to its best final index of effectiveness. In the medium-term assessment of portfolio profitability, the influence of behavioral factors decreases while maximizing returns with medium high risk. The research result is consistent with the basics of behavioral economics as the prospect theory regarding risk and loss aversion. Moreover, the factor of frequency of access to information and the degree of naive portfolio diversification with high profitability are promising areas for the development of research in behavioral finance. However, determining by the investor the objective probability to achieve the expected return level by using specific benchmarks is controversial.

Keywords: behavioral finance; behavioral portfolio theory; portfolio optimization; cumulative prospect theory; stocks portfolio; Russian stock market

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INTRODUCTION

The modern economic theory provides two points of view on the premise of human rationality in economic models. According to the expected utility theory, on which the modern portfolio theory [1] is based, the investor is inclined to independently calculate all the risks. The behavioral theory followers in economics believe that an investor tends to make mistakes in evaluating information, probabilities, and estimating losses and gains. The behavioral theory, in particular, was considered in the context of portfolio optimization in finance. On this basis, behavioral models were presented to compile an investment portfolio [2–4].

The current investment portfolio models use an approach where it is divided into "rational" and "irrational" parts, depending on the type of the investor or his investment goal. However, the questions remain:

- To what extent is the use of the standard modern portfolio theory optimal, if behavioral errors of a person's perception are also reflected while drawing up the "rational" part of the portfolio?
- Since these errors are highly probable to come up even in rational portfolio based on optimization of the risk / return ratio, what methods can improve the results of the portfolio considering behavioral errors when overestimating or underestimating probabilities and risks?

Behavioral models are an interpretation of the modern portfolio theory. However, the meaning of risk assessment, profitability and utility is different. In behavioral models, objective parameters are replaced by subjective ones. As a result, the obtained utility is not the product of the risk and return on the asset, but the perception of profitability and the probability of its receipt. At the same time, the numerical value of the profitability of the irrational investor differs from the subjective understanding of profitability, since the latter includes a distorted understanding of the "gains" and "losses" ("profits" and "losses"). Collectively, behavioral distortion affects the fact that the modern private investor tends to ignore rational recommendations to compile a portfolio. It is necessary to reformulate the basis of the rational portfolio theory in adopting the model by a non-rational private investor subject to behavioral errors.

Due to the crises of 1998, 2008 and 2014 in Russia, the population faced the phenomena of devaluation in the financial market, currency depreciation and revocation of bank licenses. Together, these phenomena led to the desire of private investors to minimize the risks of investing savings. The stock market with a high level of risk of financial investments and no guarantees of investment insurance seemed to be an unreliable direction of investment for this category of the population. Therefore, private investors did not consider investing in the stock market as a way to increase passive income. As a result, investments in the stock market often began to be considered solely in terms of the high risk and speculative nature of trading. The lack of experience of operations in the stock market and the developing ideas about the risk nature of stock investments led to a low level of financial literacy of private investors. To overcome the behavioral effect of avoiding losses through a complete rejection of investment is a pressing issue to improve financial literacy. For this, it is necessary to change the perception of risk level in the stock market. The development of behavioral portfolio theory can help develop a mechanism for selecting such assets that together will narrow the boundaries of the accepted risk. Considering behavioral factors will allow to satisfy investors' requests for risk and at the same time will enable them to gradually explore the mechanisms of the stock market.

The above problems of differences in the understanding of behavioral utility and the presence of pronounced behavioral effects of attitude to risk (in the context of financial crises) determine the importance of studying options to optimize and improve the investment portfolio efficiency at a low level of financial literacy of private investors.

LITERATURE REVIEW

The seminal paper of the irrational nature of human behavior in the economy was the article by D. Kahneman and A. Tversky [5], dedicated to the presentation of the "prospect theory". The paper proved the relationship between the behavior in case of risk and uncertainty of prospects (probability). So, people tend to underestimate the situation with an uncertain probability and overestimate the situation with an exact probability. This leads to the fact that there is a "certainty effect" which means rejecting risk in situations with guaranteed income and seeking for risk in situations with guaranteed losses. Moreover, there is an "isolation effect" which ignores all possible prospects and leads to the choice dependence on the question. Later, this effect was developed into the explanation of the "framing effect" [6, 7], an assessment of preferences depending on the wording of the offer of a product or service.

The probabilities in the utility theory by D. Kahneman and A. Tversky are replaced by estimated "decision weights", and the utility function is replaced by losses and gains, in contrast to the expected utility function. The decision weights are usually lower than the real probability, except for cases with a low probability. It was the reevaluation of low probability compared with high probability where D. Kahneman and A. Tversky saw the appeal of insurance and gambling in general for consumers. The utility function by D. Kahneman and A. Tversky generally resembled the approach by H. Markovitz in the welfare function.

Thus, D. Kahneman and A. Tversky suggested that a distorted perception of information (ignoring some alternatives) and overestimation of one's own confidence leads to underestimating real probability and violating the axioms about a rational choice between alternatives. Accordingly, two key points of the utility theory in formulating the prospect theory are the errors of logic in simplifying information and overestimating the known information (and, accordingly, underestimating the unknown information). That is, besides the change in the sense of utility, the estimates of gains or losses, the theory made it dependent on the assessment of probability, not the risk level.

Later, the prospect theory by D. Kahneman and A. Tversky was supplemented by "diminishing sensitivity" and "loss aversion" effects. The utility function by A. Tversky and D. Kahneman [8] in the "cumulative prospect theory" included three main features:

1. The function is constructed relative to the gains and losses and a certain "reference point" that refracts it.

2. The function has the property of "diminishing sensitivity" which indicates the dependence — the bigger the sums of money are, the smaller the psychological difference between equal intervals of the sums of money.

3. The tendency to avoid losses (losses are perceived to be more substantial than gains).

The ratio of risk and probability in the extended theory included two phenomena:

1. Seeking for risk in case of losses and avoidance of risk in case of gains at high probability of loss or gain.

2. Seeking for risk in case of gains and avoidance of risk in case of losses at low probability of loss or gain.

This was due to the fact that people tend to overestimate low probabilities and underestimate moderate and high probabilities of events (losses or gains). The functions of the decision weights are located side by side, but the function of estimating the probability of gains is a little more curved than the probability function of losses. Therefore, avoiding of risk for gains is more specified than seeking for risk for losses in case of moderate and high probabilities of these events (gain or loss).

The development of the portfolio theory is associated with the work by J. Williams [9] "The Theory of Investment Value". It introduced the concept of discounted future growth, which was a discounted cash flow of future dividends. That is, the company's stock prices were determined by future dividends. J. Williams was the first who tried to use mathematical tools to calculate future value.

Based on his work, H. Markowitz [1] presented the development of the theory for assessing the profitability of an investment portfolio, which was later called the "Modern Portfolio Theory".

His work was devoted to the analysis of the most optimal investment portfolio in terms of increasing profitability and reducing risk. He refuted the portfolio theory only from the point of view of maximizing the discounted expected income and added dependence on income dispersion to the model, as the investor seeks to reduce the risk of loss of income at the end of investments. According to this theory, a diversified portfolio is anyway more preferable for the investor than a portfolio without diversification. Compiling a portfolio itself involved two steps: a retrospective assessment of the returns on securities and then an assessment of their potential future returns.

For a long time, this interpretation of the asset utility valuation theory in terms of risk / return (CAPM model) was the main practice in compiling investment portfolios.

The theory of risk / return assessment through the utility theory was developed in the work by H. Shefrin and M. Statman [3], who considered the problem of compiling a portfolio from a behavioral point of view. In the developed behavioral portfolio theory, utility theories were analyzed through profitability and risk considering the prospect theory and "mental accounting" proposed by R. Thaler [10]. Mental accounting meant the concept of dividing the numerical expression of utility in the consumer's mind into separate "accounts", i.e. independent target sections in the consumer budget. A specific feature of the "accounts" was their independent assessment in terms of profitability or loss-making relative to the past periods. Based on the theory by psychologist Lopes [11] in the SP / A formula (investors evaluate the categories of "security" and "potential", limited by the level of the investor's desire), the authors propose two portfolio options: with one and with two mental accounts. The portfolio is determined based on the ratio of probability (risk and level of expectation by Lopes) and expected welfare (according to H. Markowitz). In case of a portfolio with two mental accounts, it is proposed to add a pyramidal structure (one account is to accumulate savings for large acquisitions, the other is to save for a rainy day, with no specific aim). The effective boundary line of H. Shefrin and M. Statman portfolios does not coincide with the effective average dispersion boundary according to the theory by H. Markowitz. Subsequently, H. Shefrin and M. Statman [12] applied the criterion proposed earlier [13] when studying the impact of psychological factors on the design and marketing of structured financial products.

Psychological foundations in the portfolio theory are also reflected in works by other scientists. Thus, J. Lakonishok [14] documented the frequency of using various types of option strategies; A. Poteshman and V. Serbin [15] revealed the irrationality of options strategies; and J. McConnell and E. Schwartz [16] presented how individual investors use rates from bank accounts to fund options. S. Das and M. Statman [17] applied the theory to options and structured products.

Later, a generalization of the Markowitz theory with the mental accounting theory was proposed by S. Das [18]. It was suggested to formulate a portfolio as a whole according to the Markowitz mean-variance portfolio theory, but to separate the portfolios according to their intended purpose (according to the mechanism of mental accounting) and keep track of each portfolio according to the theory by H. Shefrin and M. Statman.

The subsequent development of the theory is associated with the adjusting indicators of the behavioral model. In the work by S. Das [19], the concept of risk in the modern portfolio theory instead of the standard deviation was replaced by achieving the goal by investors. Researcher E. De Giorgi [20] showed the importance of the prospect theory instead of the mean-variance analysis in the portfolio optimization considering the principles of integrated private capital management (including real and financial assets). Later, E. De Giorgi [21] proposed breaking

down the investment process into two stages: setting goals and investing for each goal according to a specific strategy by separating the goals into short-term and long-term ones. Moreover, E. De Giorgi and S. Legg [2] applied the model by N. Barberis and M. Huang [22] using "narrow framing" and "loss avoidance" to construct a mathematical portfolio model. E. De Giorgi [23] also gave a mathematical formulation of the "naive diversification" model (that is, the phenomenon of preference for uniform diversification between all assets or preferences of a certain type of known assets).

Changes in the parameters of the behavioral portfolio theory are presented in *table 1*.

As it is seen from *table 1*, the introduction of behavioral factors in the portfolio theory took place in stages. Certain aspects of the behavioral theory were used to justify such effects as loss aversion, framing (dependence of the perception of information on its presentation form), mental accounting, and behavioral finance phenomena such as a section of behavioral economics that studies behavioral effects in the stock market (for example, " naive diversification"). In this case, the first stage of implementation was characterized by a key replacement of indicators with behavioral, estimated values, and subsequently the behavioral theory was considered as an integral part of the general portfolio optimization theory.

The study of the behavioral effects in the Russian market was based on the identification of the phenomena already seen in the US stock market. Therefore, the behavioral finance was developed to a greater extent. Thus, V.R. Evstigneev considered the decision-making mechanism in the foreign exchange market based on the expectations of other participants [24]. Decision models in the foreign exchange market based on the Bayesian procedure were also proposed by Yu.V. Yeltsov [25]. In the securities market V.R. Evstigneev proposed to formalize the cognitive dissonance effect through the matrix operator of the observed securities yield vector [26]. Also V.R. Evstigneev proposed a prediction model by an investor based on a predictable random process leading to the rejection of maximizing utility in favor of attempts to hit the jackpot in each individual case [27]. In her work, V.A. Goretskaya noted the importance of applying the prospect theory as the basis of the behavioral finance for decision-making in the stock market [28]. The issues of information asymmetry in the financial market of Russia were also considered by V.P. Ivanitsky and V.A. Tatyannikov [29].

Thus, to analyze the situation on the Russian market, the assessment of the behavioral model is of particular interest. This is explained by the need to assess the impact of behavioral errors of the private investor who tends to overestimate the probabilities of losses and gains on compiling an investment portfolio. The main aim of the research was to study the parameters of portfolios compiled considering behavioral errors. The objectives of the study were to find the most optimal variant of the behavioral portfolio and to identify the main parameters of the best portfolios in terms of efficiency and risk / return ratio. The paper simulates possible options for compiling portfolios based on the distorted perception of recommendations according to the modern portfolio theory. After the impact of the behavioral effects on the final result was identified, the options were calculated with an assessment of portfolio returns for long periods of time: quarter, half a year, a year.

AXIOMATICS

Before we proceed to the model presentation, it is necessary to determine the basic axiomatic provisions on the utility theory and the type of investor for whom the portfolio is compiled. It should be noted that the behavioral portfolio models differ from the standard economic theory in terms of the utility. In case of the average variance model, the final value is the utility of a rational investor, i.e. expected utility. In case of behavioral models, another utility value is considered, namely: the irrational investor utility, i.e. distorted rational utility according to the perception of objective utility (profitability) relative to the reference point (reference point) and

Table 1

Development of behavioral aspects in portfolio theory

Behavioral portfolio theories	Criteria			
	Portfolio optimization factors	**Portfolio diversification**	**Optimal portfolio**	**Basic utility theories**
J. Williams (1938)	Discounted future dividend flow	Missing	Dividend yield maximization	Utility maximization
H. Markowitz (1952)	Risk and return	Based on the correlation between asset returns	Minimization of the average variance and maximization of profitability	Maximization of utility while minimization of risk
H. Shefrin and M. Statman (2000)	Probability (risk and aspiration level) and expected wealth	Pyramid structure of 2 mental accounts	Curve of coincidence of the level of expected wealth and the level of risk (probability), with different levels of aspiration	Prospect theory
S. Das, H. Markowitz, J. Sheid and M. Statman (2010)	Probability and return	Markowitz theory for the total portfolio, Shefrin and Statman theory for mental accounts	A number of optimums of behavioral portfolio theory on a common risk-return curve	Prospect theory and utility maximization while minimization of risk
S. Das, D. Ostrov, A. Radhakrishnan and D. Srivastav (2018)	Probability and goals' achievements	Achievements of goals by time	Minimization of the average variance and intersection with the wealth function	Maximization of wealth while minimization of risk
E. De Giorgi (2011)	Use of framing, loss aversion and naive diversification	Achievements of goals by different time periods with different strategies	Minimization the Kahneman and Tversky utility curve and maximization of return	Prospect theory and utility maximization using subjective probability

Source: compiled by the author based on the analysis of references [1, 3, 9, 18, 19, 21].

probability distribution. Thus, comparing the objective values of the average variance theory with the indicators of the subjective utility of the behavioral models is not correct due to the different nature of the studied utilities.

Moreover, the model specification in question does not include the option of short sales (thus, only positive shares of assets in the portfolio are considered); there are no restrictions on the maximum number of assets in the portfolio, and an uneven probability distribution of future returns is used.

The need to apply the mentioned restrictions is due to the axiomatics of the investor's behavioral type who has subjective preferences and errors in perceiving information ("inexperienced" investor). The investor has the following characteristics:

- lack of qualified investor status (according to the Russian standards);
- lack of access to professional programs of asset managers (Bloomberg, Reuters terminals);
- low level of financial literacy;
- little or no trading experience.

The stated characteristics of the investor type are necessary to analyse a clean, distorted utility curve inherent to an irrational investor. Changing these parameters to greater investor awareness will distort the initial position of the utility curve and the psychological perception of gain and loss. This is a special case and is not a part of the behavioral theory considering the lack of information about the market as a fundamental point in decision making.

One should also distinguish between the concepts of "objective", "expected" profitability and risk (standard deviation) and the concepts of "subjective", "distorted", "behavioral" profitability and "subjective", "behavioral" probability. These terms are used in the work to distinguish between the calculation of the expected return and the magnitude of the risk and their alternatives perceived as "profitability" and "probability" of its achievement reduced or increased by the value of the behavioral coefficients. One should also distinguish between "objective" probability, which is the size of the probability distribution between past maximum and minimum values, and "subjective" probability, i.e. the value of the "objective" probability, which is perceived considering the behavioral coefficients. The terminological distinction between these concepts is essential to understand the logic of action of the behavioral model and its essential difference from similar terms in the expected utility theory.

SURVEY SAMPLE

The fixed parameters of models from the original prospect theory and the cumulative prospect theory were used in the work. The values of the coefficients are shown in *table 2*.

The difference between the behavioral models is the use of factors γ, δ as parameters of the probability estimate distortion (re-evaluation of low probabilities and underestimation of high probabilities, respectively), as well as α, β as risk aversion and search factors relative to gain (loss) and λ as a loss aversion factor that affect perception of the objective profitability, as well as the use of subjective probability as a risk factor, rather than variance. The cumulative prospect theory was used as the underlying behavioral model, which includes all the above-mentioned behavioral parameters (both probability estimates and risk estimates).

The behavioral parameters reflect the bend of the yield perception curve and probability estimates. Their values are constant and determined according to the empirical data in the prospect theory proof. The values are widely used in the behavioral analysis and are axiomatic in the behavioral theory.

The models were based on the data on bank interest rates for individuals of the Bank of Russia, on the volume of circulation of securities on the Moscow Exchange and on FINAM quotes for each month in 2011–2018. The risk-free rate was calculated based on the retrospective data on the average interest rate on deposits of less than a year, except demand deposits. The same rate was taken as a reference point (reference) in calculating profitability according to the prospect theory. Quotations of 48 most liquid shares

Table 2

Fixed parameters of behavioral models

Parameter	Value
λ, loss aversion factor	2.25
α, β, risk aversion and search factors	0.88
δ, small probability factor	0.61
γ, high probability factor	0.69
k, level of separation of probabilities into "large" (above level k) and "small" (below level k)	0.33

Source: compiled by the author based on data [5, 8].

(the first quotation list) at the end of the month for the same period were taken as assets. These included both ordinary shares and preferred shares. Other categories of assets were not considered in this study due to the shorter market circulation time. For comparison purposes, only shares of Russian companies were selected over the long period.

The research methodology was based on the interpretation of the modern portfolio theory in a behavioral form. It should be noted that such an element of the Markowitz theory, as diversification by reducing the correlation between assets, was not used in the behavioral model. This is a drawback and an important feature of the behavioral model, since in this case the object of the analysis is not the criterion for reducing portfolio risk through a decrease in the dependence of assets, but the perceived portfolio return relative to a certain benchmark (also in dynamics) and assessing the probability of achieving such a return in the future. Moreover, the analysis of information is limited to the data that are strictly accessible to the irrational investor: the average bank interest rates whose changes are felt by private investors (preferring deposits), the profitability of an asset relative to its price in the same period a year ago (caused by the mental peculiarity to calculate profitability for a round number — a year, ignoring seasonal factors at the beginning and the end of the year), and the probability "to break the price level" in the past. In the study, the price level in the past has the same strong influence as the price level of the benchmark.

Thus, the study is based on the existing parameters of behavioral distortions of the return and risk curves, as well as on the empirical data of the parameters of the average rate on deposits for the year and asset prices for 96 periods.

MODEL

The main aim of this work was to consider the possible consequences of the behavioral errors in assessing profitability in portfolio optimization on the example of the Russian stock market. The cumulative prospect theory (CPT) was taken as the basis of the behavioral model [8, 30]. This theory takes into account such behavioral factors as loss avoidance, weighting prospects, risk aversion and search, and use of a reference point.

As a portfolio returns assessment method, a behavioral portfolio model was applied considering the behavioral theory parameters for each period. Each model consisted of a sample of indicators of annual profitability (a month of the current year to the same month of the previous one) with a frequency of analysis once a month, once every 3 months, once every six months and once a year. This use of samples was explained by the need to analyze the dependence of the behavioral model effectiveness on the number of times the investor visits the portfolio return statistics.

According to the cumulative prospect theory, the optimization model is the following:

$$CPT(x)=\sum_{s=1}^{S}\pi(p_s)v\left(\sum_{i=1}^{N}r_{si}\omega_i\right)\to max, \qquad (1)$$

where

$$\overline{r}(x)=\sum_{i=1}^{N}\overline{r}_i\omega_i\geq d, \qquad (2)$$

$$\sum_{i=1}^{N}\omega_i=1\ ,\text{at}\ \omega_i\geq 0, \qquad (3)$$

where $\pi(p_s)$ is a function of the subjective probability dependence (a subjective assessment of the probability to reach the asset price considering behavioral parameters γ, δ of the objective probability distortion) on the objective probability (probability to reach the asset price based on its maximum and minimum prices in the period); r_{si} is the average return on the asset; ω_i is its shares in the portfolio; s is a perception indicator of the current price gain relative to the previous value of the price so that (r_0 is a reference point indicator):

- if $s=0$, i.e. $r_s=r_0$, then the investor's gain is 0;
- if $s>0$, i.e. $r_s>r_0$, then the investor considers the outcome as a gain from this investment;
- if $s<0$, i.e. $r_s<r_0$, then the investor is aware of the loss.

In case of the following function of value from the return on an asset:

$$v(r(x))=\begin{cases}(r_i-r_0)^{\alpha}, \text{if } r(x)\geq r_0,\\ -\lambda(r_0-r_i)^{\beta}, \text{if } r(x)<r_{0,}\end{cases} \qquad (4)$$

where r_0 is the value of the reference point in a given period; r_i is an asset return; α, β are the risk aversion and search factors relative to the gain (loss); λ is the loss aversion factor.

In this case, the subjective probability $\pi(p_s)$ is a function of the objective probability (p_s) by formula π

$$\pi(p_s)=\begin{cases}\pi^{-}\left(\sum_{j=1}^{s}p_j\right)-\pi^{-}\left(\sum_{j=1}^{s-1}p_j\right), & s=1,\ldots,k,\\ \pi^{+}\left(\sum_{j=s}^{S}p_j\right)-\pi^{+}\left(\sum_{j=s+1}^{S}p_j\right), & s=k+1,\ldots,S,\end{cases} \qquad (5)$$

where the bends of the dependence function of the subjective probability are determined through the relations

$$\pi^{-}(p_j)=\frac{p_j^{\delta}}{\left(p_j^{\delta}+(1-p_j)^{\delta}\right)^{1/\delta}},\ \text{where } j=1,\ldots,k, \qquad (6)$$

$$\pi^{+}(p_j)=\frac{p_j^{\gamma}}{\left(p_j^{\gamma}+(1-p_j)^{\gamma}\right)^{1/\gamma}},\ \text{where } j=k+1,\ldots,S, \qquad (7)$$

where γ, δ are the distortion parameters of the objective probability estimate.

Profitability was determined by the formula of logarithmic profitability:

$$r_i = \ln\left(\frac{P_i}{P_0}\right)\times 100, \quad (8)$$

where r_i is the annual return for the i-th period;

P_1 — stock price at the end of the i-month;

P_0 — stock price at the end of the i-period in the previous year.

Dispersion (standard deviation) and the Sharpe ratio were calculated by the generally accepted formulas [31].

Therefore, in the behavioral models, the risk assessment was calculated based on the probability weights (with the parameters of relation to probability), and the profitability was calculated considering the relation to the reference point (deposit rate) and the loss aversion and risk search (aversion) parameters.

The essence of comparing models is to distort the perception of profitability and risk when applying the parameters from the cumulative prospect theory. In case of parameters α, β, λ in the prospect theory, it was enough to replace profitability by a function of subjective estimation of profitability for each period (revaluation at the end of the month, every three months, every six months, the year). However, the integration into the cumulative prospect theory model with parameters γ, δ seemed to be a more difficult task.

Previously, there were used the models considering the algorithms for selecting values by the Monte Carlo method. A similar approach was previously applied taking into account the heuristic method of calculation [32]. However, this approach is a difficult option from the point of view of the algorithm for calculating probability indicators. In the behavioral model, it is possible to apply an approach where the calculation will be based on benchmarks known to the irrational investor; for example, the maximum and minimum return on an asset in a given time period.

The work suggests the following formula to estimate objective probability based on the current value of an asset at the end of the month. This formula is an interpretation of the objective probability estimation in the expected utility theory by M. Hayes [33]. The difference is that the maximum value in the past is taken as the limit value, and the average value of return for the entire period is taken as a guideline.

$$p_j = 1 - \left|\frac{2\left(r(x) - \overline{r}(x)\right)}{r_m - r_n}\right|, \quad (9)$$

where the conditions are met:

$$r_m = \begin{cases} r_{max}, & \text{if } r_{max} - \overline{r}(x) > \overline{r}(x) - r_{min} \\ \overline{r}(x) + \left(\overline{r}(x) - r_{min}\right), & \text{if } r_{max} - \overline{r}(x) \le \overline{r}(x) - r_{min} \end{cases} \quad (10)$$

$$r_n = \begin{cases} r_{min}, & \text{if } r_{max} - \overline{r}(x) > \overline{r}(x) - r_{min} \\ \overline{r}(x) - \left(r_{max} - \overline{r}(x)\right), & \text{if } r_{max} - \overline{r}(x) \le \overline{r}(x) - r_{min}, \end{cases} \quad (11)$$

where r_m and r_n — maximum and minimum asset return;

$r(x)$ — current asset return;

$\overline{r}(x)$ — asset return average value for the entire period;

r_{max} and r_{min} — maximum and minimum return for the entire period.

The calculation by this formula is due to the need to impose a probability distribution on the possible spread of the asset value in a given period. The largest return corresponds to the equality of the asset return average value for the period, the smallest — to the value of the maximum difference between the price of the asset and its average value (relative to the maximum or minimum profitability of the asset for the entire period). To simplify the calculations, the maximum return was taken as 100%, the minimum (maximum or minimum value) was taken as 0%. In fact, the probability parameter will only be as close as possible to this value, but not equal to it.

Further, subjective probability is calculated by the formulas given above. Maximizing or minimizing the overall utility and variance is the optimization task (when compiling many effective portfolios). In this study, we used the search for solutions to nonlinear problems by the generalized reduced gradient OLS method in the Excel application.

Таблица 3 / Table 3

Comparison of average risk, return and portfolio quality in behavioral models

Model	σ, risk, % per annum	*r*, return, % per annum	Sharpe ratio	Return on bank deposit, % per annum	Excess return, % per annum
CPT	18.09	20.89	0.79	6.59	14.3
CPT-3	16.95	19.22	0.74	6.58	12.64
CPT-6	17.56	22.05	0.88	6.79	15.26
CPT-12	17.13	18.12	0.67	7.28	10.84
Avg. value	17.21	20.07	0.77	6.81	13.26

Source: compiled by the author.

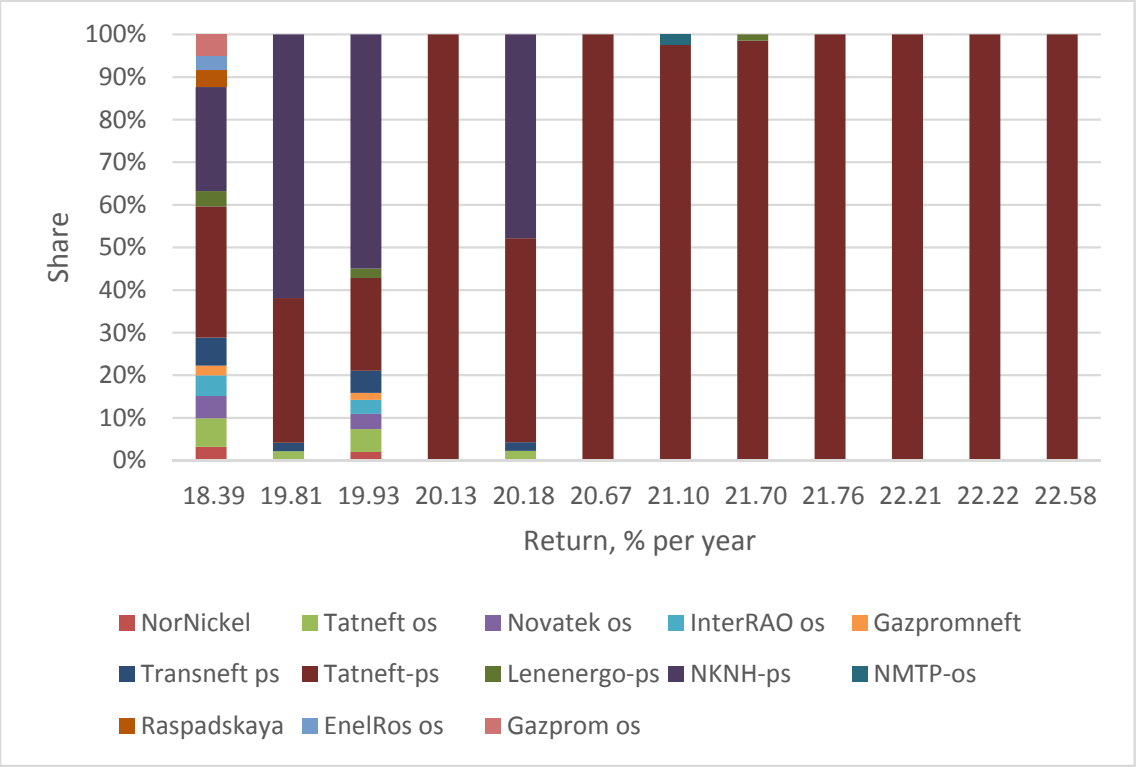

Fig. 1. **Optimized stock portfolios structure of the CPT model**

Source: compiled by the author.

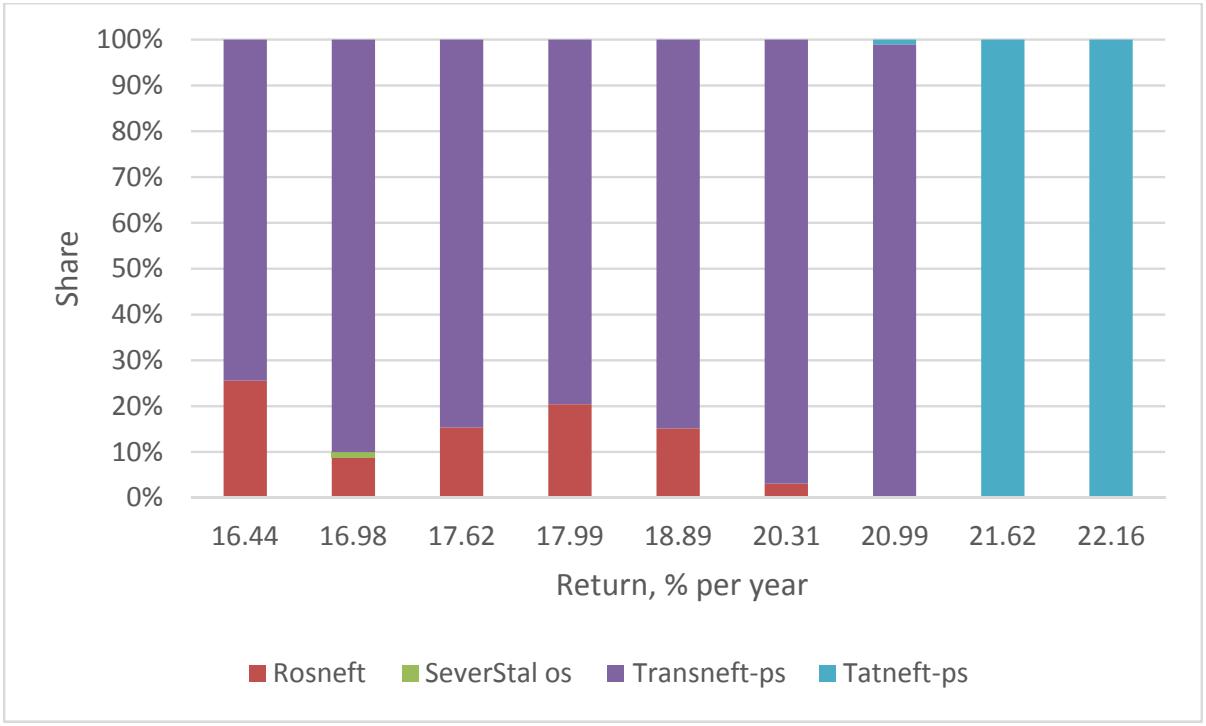

Fig. 2. **Optimized stock portfolios structure of CPT-3 model**

Source: ccompiled by the author.

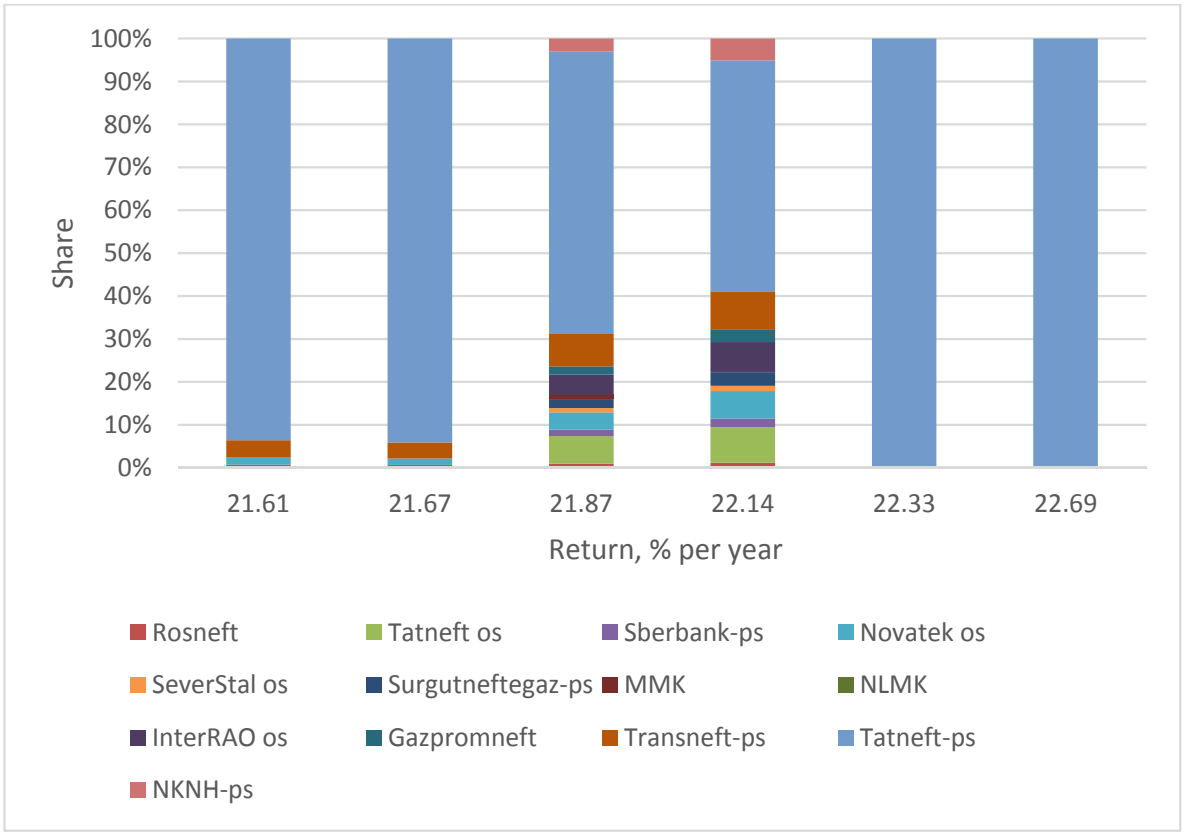

Fig. 3. **Optimized stock portfolios structure of CPT-6 model**

Source: compiled by the author.

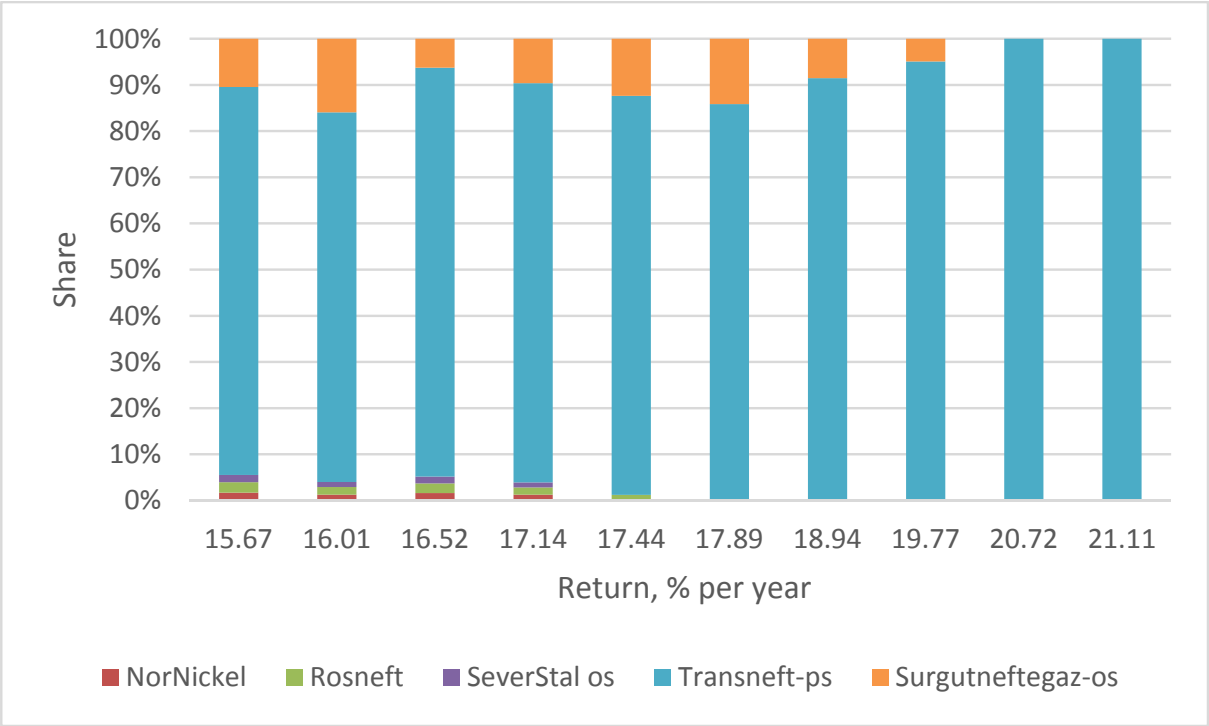

Fig. 4. **Optimized stock portfolios structure of CPT-12 model**

Source: compiled by the author.

RESEARCH RESULTS

Modeling resulted in an effective set of portfolios in each model: the behavioral portfolio theory when calculating the return in each month (CPT), and also every 3, 6, 12 months (CPT-3, CPT-6, CPT-12).

Table 3 shows the main indicators of return, risk and portfolio management efficiency (Sharpe ratio). It should be noted that these indicators are recalculated considering undistorted risk indicators (standard deviation of profitability) and profitability according to the H. Markowitz model, since in this case objective values are compared. The average values of indicators among all effective portfolios of each group are indicated.

As can be seen from the data above, the average return in the behavioral model portfolios varies with time. At the same time, the magnitude of the accepted risk first decreases, and then it remains at an average level. The highest risk values remain with a monthly portfolio review. In this case, the semi-annual revision model has the highest Sharpe ratio. The same model has the highest average return. This indicates a preference for higher returns and higher risk on average when choosing by a behavioral portfolio model.

Experimental stock portfolios were also modeled based on the data above. The results are presented (*Fig. 1–4*) by the ratio of the shares of assets in the portfolio according to different levels of return of the border of the effective portfolio set. An effective portfolio set in the behavioral theory is a set of portfolios with minimal risk at each level of return.

According to the distribution of shares in the portfolios, it can be noted that behavioral portfolios tend to concentrate on a particular stock in the portfolio, which increases with the investor's desire to acquire greater returns (*Fig. 2, 4*).

Within the CPT model, portfolio diversification is observed at lower returns. However, with an increase in the desired return, the same orientation occurs towards investment in the same share. CPT and CPT-6 models show a higher level of return with more uneven diversification (*Fig. 1, 3*).

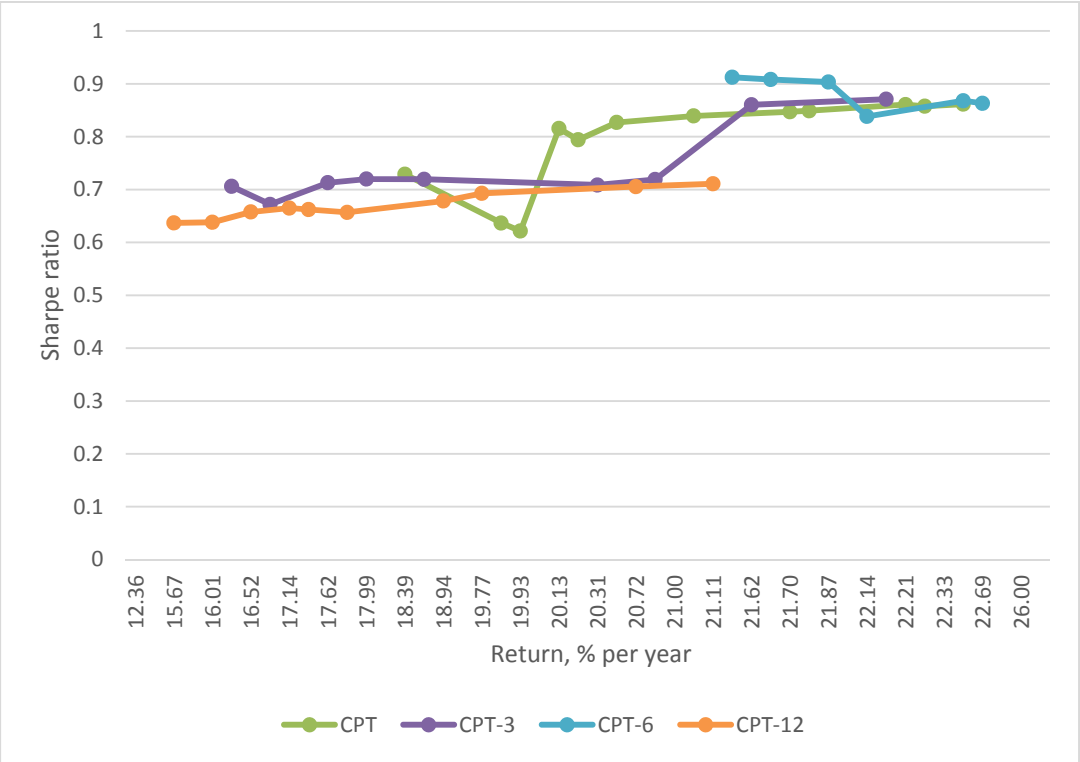

Fig. 5. **Sharpe ratio indicators depending on portfolio returns**

Source: compiled by the author.

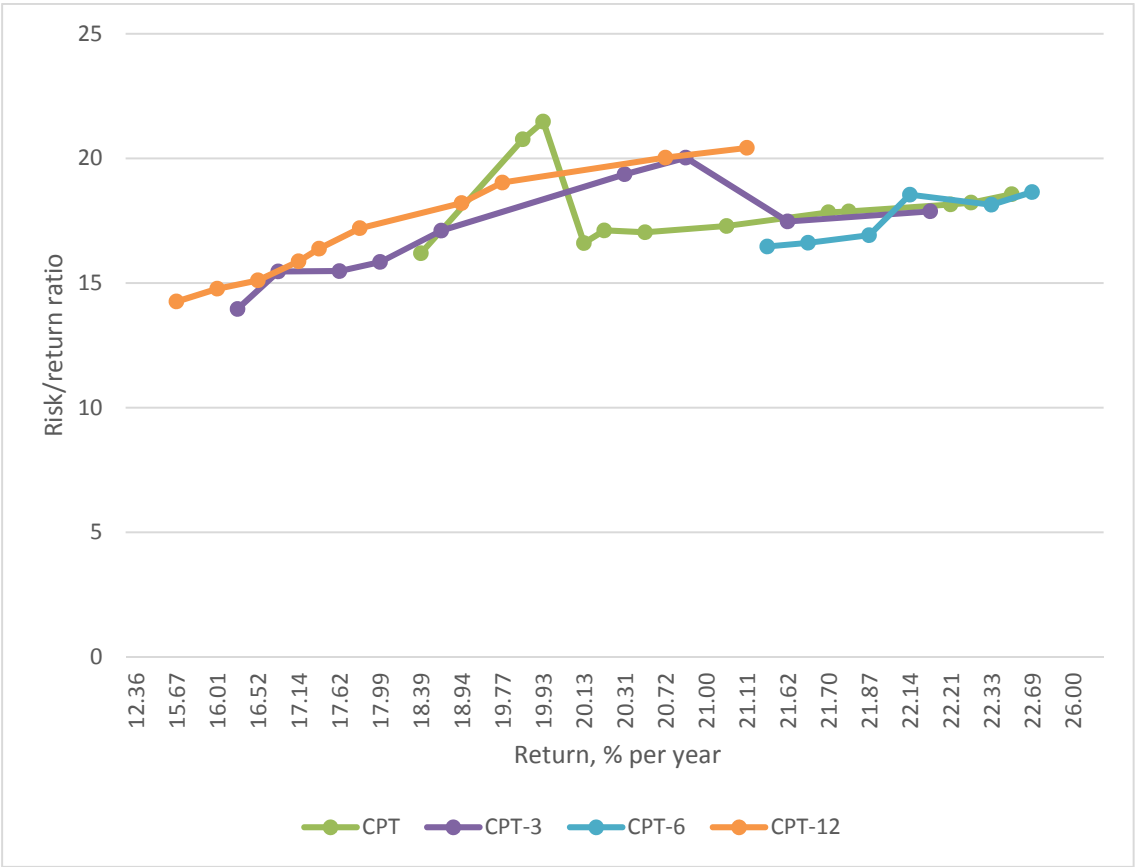

Fig. 6. **Portfolio Risk / Reward Ratios**

Source: compiled by the author.

In CPT model (*Fig. 1*) with a monthly portfolio assessment, diversification is achieved at return levels at 18–19% returns per year (starting from 13 securities to 4 securities at 20% return). As the return in the model grows, the number of securities included in the portfolio decreases. Already with a 20% return, the portfolio model focuses on investing in only one security (Tatneft-p).

In CPT-3 model (*Fig. 2*), portfolio diversification is lower than in CPT model. The number of securities in the portfolio is 2–3 types of shares. At a return rate of above 21% per year, diversification also disappears from the model. The main investment in this model is Transneft-p.

The diversification of securities analysis in the portfolio with the portfolio assessment every six months (*Fig. 3*) allows to conclude that it mostly diversifies at the level of 21–22% per year (13 securities). At return levels below and above this interval, there is no diversification. In this case, the largest share, in any case, also belongs to the investments in Tatneft-p.

In the model with a portfolio review once a year (*Fig. 4*), the largest share of investments also belongs to the Transneft-p shares, as well as in CPT-3 model. In general, the diversification model resembles CPT-3 model. However, 15–17% return per year, a larger number of assets (up to 5 shares) are included in the portfolio.

The common feature for all models is the tendency to choose one dominant security for investment with high return (over 20%) and a greater level of diversification at lower levels of return. However, CPT-6 model, the leader in terms of returns and the Sharpe ratio, is characterized by a high level of diversification with a high return of 21–22%. For most investment, Transneft-p or Tatneft-p shares were preferred in the models.

Considering the portfolio efficiency level, it can be established that among the behavioral ones, semi-annual CPT-6 model has the highest indicator of the Sharpe ratio (*Fig. 5*).

Looking at the ratio of risk and return, a lower level of risk in the optimal model can be noted in general. It increases significantly with higher return, while behavioral models CPT and CPT-6 are more advantageous at return over 22%, i.e. assessment models for a month and six months (*Fig. 6*).

Thus, the behavioral models have apparent defects in diversification balance. Unlike the average variance model, they do not include an indicator responsible for the level of covariance between asset returns. Nevertheless, the behavioral models show the general tendency of the irrational investor to choose a higher portfolio return in the future with a higher level of risk. Thus, the Sharpe ratio (model efficiency) in CPT-6 model is characterized by the highest value (*Fig. 6*) with the highest risk / return ratio (*Fig. 5*). It should also be noted that the assets selected as the sole stock in the portfolio have a tendency to grow steadily for several years, which indicates the maximum desire of the investor to avoid losses at volatility and to choose an obvious trend. As a rule, such stocks have a growth chart with no visible drawdowns. This attracts inexperienced investors who are oriented towards stable growth and "guaranteed income". There is definitely no diversification of assets in this case. To minimize objective risk, the expected return for inexperienced investors should be limited.

The next direction to study behavioral effects may be changing the objective probability indicator by correlating the prices of maximum and minimum for different periods (in this case, the entire time series is taken), including additional behavioral financial effects (naive diversification identified in the portfolio analysis results), as well as including in a bond portfolio. Besides, determining the reference point remains debatable. In this work, the average rate on deposits with banks was used as the main benchmark for the private investor. However, it is possible to further consider a comprehensive benchmark indicator, including the key rate, refinancing rate, and other monetary policy instruments of the Central Bank.

CONCLUSIONS

The analysis of the behavioral investment portfolio models results from the shares of Russian companies on the Moscow Exchange allowed to formulate the most optimal model of the irrational private investor's preferences in the Russian market. The model of reviewing and adjusting the portfolio every six months leads to decreasing the influence of behavioral effects. As a result, it brings large profits at the highest portfolio efficiency. This model is also characterized by a sufficient level of diversification (13 types of securities).

Using these models from the point of view of risk reduction is controversial, since they do not consider the correlation index of assets among themselves. However, these models are intended to reflect the consumer's desires and increase the consumer's utility in terms of their ideas about risk and loss aversion. In terms of the model validity, its irrational nature should be considered, which at the same time is more preferable for this type of the investor and will subsequently lead to more strict adherence to the investment plan.

According to the simulation results, the following points can be noted:

- When calculating the behavioral model for a six-month period, the indicators came as close to optimal as possible.
- The behavioral models are less diversified with high returns, but give greater returns with the same level of diversification (in models CPT, CPT-6).
- The behavioral models are more effective at high returns and risk.

The authors suggested a hypothesis on the need to change the term for evaluating the return on assets in the portfolio for behavioral effects on the medium-term, since the effect of behavioral errors in the cumulative prospect theory is ultimately leveled off due to the lack of attention to frequent price fluctuations leading to an increased distortion in the perception of objective statistical indicators. Risk assessment, therefore, is not distorted at each iteration of the profitability assessment of each period and the standard deviation as a risk measure. It is important to note the inability of the quarterly and annual models to approach the indicators of the optimal model.

It should be considered that errors in diversification and perceptions of subjective probability and estimated profitability can show the preferences of investors with behavioral deviations towards the ratio of greater risk and greater profitability. On average, the risk in behavioral models varies less than in case of the optimal model, which also indicates the importance of the risk aversion tool. It is possible that a focus on naive diversification and the predominance of a particular asset in the portfolio during semi-annual or monthly monitoring may make the portfolio more profitable at moderate to high risk.

The study results can be used in studying the modern behavioral economic theory to compare differences in assessing the results of the expected utility theory and subjective behavioral utility. Moreover, the features of compiling a portfolio under the conditions of behavioral errors in the perception of portfolio profitability can be used to compile investment portfolios for unqualified investors by investment advisers in the brokerage business.

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